

Unit 7 Practice Solutions: The Right Triangle

Chapter 9 THE PYTHAGOREAN THEOREM

Pages 368–369 (Section 9.1)

1 a 2 b $3\sqrt{3}$ c $6\sqrt{2}$ d $4\sqrt{2}$ e $7\sqrt{2}$ f $10\sqrt{2}$ g $2\sqrt{5}$ h $2\sqrt{6}$

2 a $5\sqrt{9}\sqrt{2} = 15\sqrt{2}$ b $\sqrt{13}$ c $\sqrt{9+16} = \sqrt{25} = 5$

d $\sqrt{25+144} = \sqrt{169} = 13$ e $\frac{1}{6} \cdot 4\sqrt{3} = \frac{2\sqrt{3}}{3}$ f $7\sqrt{3}$

3 a $\frac{\sqrt{2}}{2}$ b $\frac{\sqrt{5}}{5}$ c $2\sqrt{2}$ d $2\sqrt{3}$ 4 a $11\sqrt{3}$ b $8\sqrt{2} + 7\sqrt{3}$

c $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$ d $6\sqrt{2} + 5\sqrt{3} - 4\sqrt{3} = 6\sqrt{2} + \sqrt{3}$ 5 a ± 3

b ± 12 c ± 13 d $\pm \frac{1}{2}$ e $\pm 2\sqrt{3}$ f $\pm 3\sqrt{2}$ 6 a $x^2 = 9$, $x = \pm 3$

b $x^2 = 64$, $x = \pm 8$ c $x^2 = 25$, $x = \pm 5$ d $x^2 + 27 = 36$, $x^2 = 9$, $x = \pm 3$ e $5 + 11 = x^2$, $16 = x^2$, $x = \pm 4$ f $x^2 = 75 + 5$, $x^2 = 80$, $x = \pm\sqrt{16+5} = \pm 4\sqrt{5}$

7 a $(x-6)(x+1) = 0$ b $(x+6)(x-2) = 0$

x = 6, -1

x = -6, 2

c $(x-5)(x-3) = 0$

d $(x-6)(x+3) = 0$

x = 3, 5

x = 6, -3

e $x^2 - 9x - 36 = 0$

f $(-x+9)(x+4) = 0$

(x-12)(x+3)

x = 9, -4

x = 12, -3

8 a $x(x-4) = 0$ b $x^2 - 10x = 0$

x = 0, 4

x(x-10) = 0

x = 0, 10

c $x^2 - 13x = 0$

d $0 = x^2 - 8x$

x(x-13) = 0

0 = x(x-8)

x = 0, 13

x = 0, 8

9 a $x^2 + (21)^2 = (29)^2$ b $(2)^2 + y^2 = (4)^2$

$x^2 = 841 - 441 = 400$

$y^2 = 16 - 4$

x = 20

$y^2 = 12 = 4 \cdot 3$

$y = 2\sqrt{3}$

c $(4.1)^2 + (7.1)^2 = r^2$ 10 a $2x^2 - 3x - 35 = 0$

$16.81 + 50.41 = r^2$

$(2x+7)(x-5) = 0$

$67.22 = r^2$

$x = 5, -\frac{7}{2}$

$8.2 = r$

b $x^2 - 15 = -11x$

c $x^2 - 7x + 9 = 2x^2 + 6x + 7$

$x^2 + 11x - 15 = 0$

$x^2 + 13x - 2 = 0$

$x = \frac{-11 \pm \sqrt{121 - 4(1)(-15)}}{2(1)}$

$x = \frac{-13 \pm \sqrt{169 - 4(1)(-2)}}{2}$

$x = \frac{-11 \pm \sqrt{181}}{2}$

$x = \frac{-13 \pm \sqrt{177}}{2}$

11 $7(3x-3) = (x+1)(2x+4)$

12 OB = 10

$21x - 21 = 2x^2 + 6x + 4$

OA = 7

$0 = 2x^2 - 15x + 25$

$(OA)^2 + (OB)^2 = (AB)^2$

$0 = (2x-5)(x-5)$

$100 + 49 = (AB)^2$

$x = \frac{5}{2}, 5$

$12.2 \approx AB$

13 a $|h| = -h$ since $h < 0$ b $|x-3| = 3-x$ since $x < 3$

c $|p||q| = (-p)(-q) = pq$ since $p < 0$ and $q < 0$

d $|x||y|\sqrt{x} = -xy\sqrt{x}$ since $x > 0$ and $y < 0$

Pages 374–376 (Section 9.2)

1 a $C = 2\pi r = 2\pi(9.8) = 19.6\pi \approx 61.58$

A $= \pi r^2 = \pi(9.8)^2 = 96.04\pi \approx 301.72$

2 a $m\widehat{CD} = \frac{1}{4}(360) = 90$

b $\widehat{CD} = \frac{1}{4}(2\pi(5)) = 2.5\pi \approx 7.85$

3 a $m\widehat{EF} = 180$ b $m\widehat{EF} = \frac{1}{2}C = \frac{1}{2}(2\pi(4)) = 4\pi \approx 12.6$

4 a $(0, 4)$

b Area of sector DOG $= \frac{1}{4}$ Area $\odot O$
 $= \frac{1}{4}(\pi(4)^2)$
 $= 4\pi \approx 12.57$

5 Area of sector AOB $= \frac{1}{2}$ Area $\odot O$

$= \frac{1}{2}(\pi(5)^2)$
 $= 12.5\pi \approx 39.27$

6 $m\angle F = \frac{1}{2}(m\widehat{DE})$

$= \frac{1}{2}(90) = 45$

7 a $m\angle A = \frac{1}{2}(m\widehat{BC}) = \frac{1}{2}(80) = 40$

b $m\angle D = \frac{1}{2}(m\widehat{BC}) = \frac{1}{2}(80) = 40$

8 $m\widehat{BC} = 180 - m\widehat{AB} = 180 - 50 = 130$

$m\angle BCA = \frac{1}{2}(m\widehat{AB}) = \frac{1}{2}(50) = 25$

9 O = mdpt \overline{AB}

O = $(\frac{1+5}{2}, \frac{9-3}{2}) = (3, 3)$

10 Q = mdpt \overline{DF}

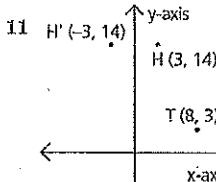
$= (\frac{1+7}{2}, \frac{3-5}{2}) = (4, -1)$

Slope $\overline{DE} = \frac{3-2}{1-8} = -\frac{1}{7}$

Slope $\overline{EF} = \frac{2-(-5)}{8-7} = 7$

So $\overline{DE} \perp \overline{EF}$, $\angle DEF$ is a rt \angle .

$\therefore \triangle DEF$ is a rt \triangle .



a H' is at $(-3, 14)$ b Slope $\overline{TH'} = \frac{3-14}{8+3} = -\frac{11}{11} = -1$

12 a $m\widehat{JK} = 180 - m\widehat{HJ} - m\widehat{MK}$

$= 180 - 20 - 40 = 120$

b $\widehat{JK} = \frac{120}{360}(27\pi) = 9\pi$

c HM = $\frac{C}{\pi} = \frac{27\pi}{\pi} = 27$

13 O is midpt of \overline{GH} , so

$$\frac{3+x}{2} = 5 \text{ and } \frac{7+y}{2} = 4$$

$$3+x = 10 \quad 7+y = 8$$

$$x = 7 \quad y = 1$$

O at (7, 1)

G' at (-3, 7)

14 $\angle AEB \cong \angle DEC$, because they are vertical \angle s. Since $\angle B$ and $\angle C$ both intersect \overline{AD} , they are \cong and \sim .

$\therefore \triangle ABE \sim \triangle DCE$ by AA~.

15 $\triangle ABE \sim \triangle DCE$, so

$$\frac{AB}{CD} = \frac{BE}{CE} \frac{4}{\frac{5}{3}} = \frac{5}{3}, CD = \frac{12}{5} = 2\frac{2}{5}$$

$$16 \text{ a } m\widehat{JK} = 2m\angle JHK = 2(45) = 90$$

$$\text{b } r = 3; \widehat{JK} = \frac{90}{360}(2\pi(6)) = \frac{1}{4}(12\pi) = 3\pi \approx 9.42$$

17 Replace x with 5 and y with 8 in $(x-2)^2 + (y-4)^2 = 25$.

$$(5-2)^2 + (8-4)^2 = 25$$

$$3^2 + 4^2 = 25$$

$$9 + 16 = 25$$

So (5, 8) is on $\odot O$.

Pages 379–383 (Section 9.3)

1 a $\frac{7}{HF} = \frac{HF}{3}$ b $\frac{7}{EF} = \frac{EF}{11}$ c $\frac{HG}{6} = \frac{6}{9}$

$$HF^2 = 21 \quad EF^2 = 77 \quad 9 \cdot HG = 36$$

$$HF = \sqrt{21} \quad EF = \sqrt{77} \quad HG = 4$$

2 a $\frac{4}{x} = \frac{x}{5}$ b $\frac{4}{y} = \frac{y}{9}$ c $\frac{5}{z} = \frac{z}{9}$

$$x^2 = 20 \quad y^2 = 36 \quad z^2 = 45$$

$$x = \sqrt{20} \quad y = 6 \quad z = 3\sqrt{5}$$

$$x = 2\sqrt{5} \quad \frac{1}{2}y = 3 \quad z + 8 = 3\sqrt{5} + 8$$

$$2x = 4\sqrt{5}$$

3 a $\frac{AD}{CD} = \frac{CD}{AB}$ Alt to hyp to rt Δ is the mean

$$\frac{4}{CD} = \frac{CD}{9} \text{ prop to the seg of hyp.}$$

$$CD^2 = 36$$

$$CD = \sqrt{36}$$

$$CD = 6$$

b $\frac{AD}{CA} = \frac{CA}{AB}$ Each leg of a rt Δ is the mean

$$\frac{4}{CA} = \frac{CA}{16} \text{ prop btwn the adj seg of the}$$

$$CA^2 = 64 \text{ hyp and the total hyp.}$$

$$CA = \sqrt{64}$$

$$CA = 8$$

c $\frac{BD}{CB} = \frac{CB}{BA}$ Each leg of a rt Δ is the mean

$$\frac{6}{CB} = \frac{CB}{8} \text{ prop btwn the adj seg of the}$$

$$CB^2 = 48 \text{ hyp and the total hyp.}$$

$$CB = \sqrt{48}$$

$$CB = \sqrt{16} \cdot \sqrt{3}$$

$$CB = 4\sqrt{3}$$

d $\frac{BD}{CD} = \frac{CD}{AD}$ Alt to the hyp of rt Δ is the mean prop to the seg of the

$$\frac{16}{8} = \frac{8}{AD}$$

$$16AD = 64 \text{ hyp.}$$

$$AD = 4$$

e $\frac{AD}{AC} = \frac{AC}{AB}$ Each leg of a rt Δ is the mean prop bwtn the

$$\frac{3}{AC} = \frac{AC}{27} (24+3)$$

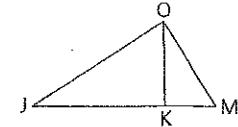
$$AC^2 = 81 \text{ adj seg of the hyp and}$$

$$AC = \sqrt{81} \text{ the total hyp.}$$

$$AC = 9$$

f Impossible

4 Given: $\angle JOM = 90^\circ$



\overline{OK} is an alt.

a $\frac{JK}{OK} = \frac{OK}{KM}$

$$\frac{12}{OK} = \frac{OK}{5}$$

$$60 = OK^2$$

$$OK = \sqrt{60}$$

$$OK = 2\sqrt{15}$$

b $\frac{JK}{OK} = \frac{OK}{KM}$

$$\frac{9}{3\sqrt{5}} = \frac{3\sqrt{5}}{KM}$$

$$9(KM) = 9(5)$$

$$9KM = 45$$

$$KM = 5$$

c $\frac{JK}{JO} = \frac{JO}{JM}$

$$\frac{3}{3\sqrt{2}} = \frac{3\sqrt{2}}{JM}$$

$$9(JM) = 9(2)$$

$$3JM = 18$$

$$JM = 6$$

d $KM + JK = JM$

$$\frac{KM}{OM} = \frac{OM}{JM}$$

$$5 + 6 = JM$$

$$\frac{5}{OM} = \frac{OM}{11}$$

$$11 = JM$$

$$55 = OM^2$$

$$\sqrt{55} = OM$$

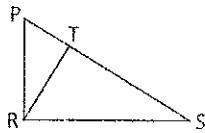
5 a. $\frac{3}{3\sqrt{3}} = \frac{3\sqrt{3}}{a}$
 $3a = (3\sqrt{3})(3\sqrt{3})$
 $3a = 9\sqrt{9}$
 $3a = 9 \cdot 3$
 $a = 9$

b. $\frac{3}{b} = \frac{b}{3+a}$
 $a = 9$
 $\frac{3}{b} = \frac{b}{12}$
 $b^2 = 36$
 $b = \sqrt{36}$
 $b = 6$
 $ab = 9 \cdot 6$
 $ab = 54$

6 Given: \overline{RT} is an alt.
 $\angle PRS$ is a rt \angle .
Concl: $\frac{PR}{RS} = \frac{RT}{ST}$

- 1 \overline{RT} is an alt.
2 $\angle PRS$ is a rt \angle .
3 $\triangle PRS$ is rt \triangle .
4 $\triangle PRT \sim \triangle RST$

5 $\frac{PR}{RS} = \frac{RT}{ST}$



- 1 Given
2 Given
3 A \triangle with a rt \angle is a rt \triangle .
4 Alt to hyp of rt \triangle —2 \triangle s formed are \sim to given rt \triangle and each other.
5 Corr sides of \sim \triangle s are proportional.

7 Given: \overline{SY} is an alt.
 $\angle VSX$ is a rt \angle .

Prove: $XY \cdot SV = XS \cdot YS$

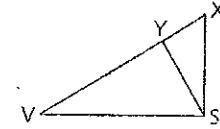
- 1 \overline{SY} is an alt.
2 $\angle VSX$ is a rt \angle .
3 $\triangle VSX$ is a rt \triangle .

4 $\triangle XSV \sim \triangle XYS$

5 $\frac{XS}{XY} = \frac{SV}{YS}$

6 $XY \cdot SV = XS \cdot YS$

8 P is mdpt of \overline{CD} .
 $P = (\frac{10-4}{2}, \frac{0+2}{2}) = (3, 1)$



- 1 Given
2 Given
3 A \triangle that contains a rt \angle is a rt \triangle .
4 Alt to hyp of rt \triangle —2 \triangle s formed are \sim to given rt \triangle and each other.
5 Corr sides of \sim \triangle s are proportional.
6 Means-extremes products theorem

9 $m\angle HJP = \frac{1}{2}m\widehat{HP} = 25$
 $m\angle HKP = \frac{1}{2}m\widehat{HP} = 25$
 $m\angle HMP = \frac{1}{2}m\widehat{HP} = 25$

10 $m\widehat{RH} = 2(m\angle RGH) = 2(90) = 180$

11 Area of sector DOG = $\frac{90}{360}(\pi(10)^2) = 25\pi \approx 78.54$

12 a $(0, -12)$

b $m\widehat{ABC} = 360 - m\widehat{AC}$
 $m\widehat{ABC} = 360 - \frac{90}{360}(360)$

$m\widehat{ABC} = 270$

c $\widehat{ABC} = \frac{270}{360}(2\pi(12))$

$\widehat{ABC} = 18\pi \approx 56.55$

13 $m\widehat{EF} = 180 - m\widehat{FG} - m\widehat{DE}$

$m\widehat{EF} = 180 - 80 - 40$

$m\widehat{EF} = 60$

$m\angle EDF = \frac{1}{2}(m\widehat{EF}) = \frac{1}{2}(60) = 30$

14 $\frac{CB}{DB} = \frac{DB}{BA}$
 $\frac{CB}{6} = \frac{6}{3}$

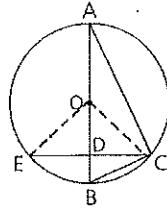
$3 \cdot CB = 36$

$CB = 12$

12 meters from B to C

15 Given: $\odot O, \overline{CD} \perp \overline{AB}$

$\angle ACB$ is a rt \angle .
Concl: $\frac{AD}{CD} = \frac{CD}{BD}$
 $\frac{AD}{ED} = \frac{ED}{BD}$



- 1 $\odot O$
2 $\angle ACB$ is a rt \angle .
3 $\triangle ACB$ is a rt \triangle .
4 $\triangle ADC \sim \triangle CDB$
- 5 $\frac{AD}{CD} = \frac{CD}{BD}$
- 6 Draw \overline{OE} and \overline{OC}
7 $\overline{OE} \cong \overline{OC}$
8 $\overline{CD} \perp \overline{AB}$
9 $\angle ADE$ is a rt \angle .
10 $\angle ADC$ is a rt \angle .
- 1 Given
2 Given
3 A \triangle that contains a rt \angle is a rt \triangle .
4 Alt to hyp of rt \triangle —2 \triangle s formed are \sim to given rt \triangle and each other.
5 Corr sides of \sim \triangle s are proportional.
6 Two pts determine a line.
7 All radii of \odot are \cong .
8 Given
9 \perp lines intersect to form rt \angle s.
10 Same as 9

11 $\overline{OD} \cong \overline{OD}$

12 $\triangle ODE \cong \triangle ODC$

13 $\overline{ED} \cong \overline{CD}$

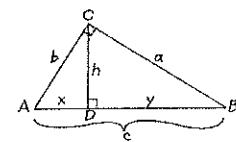
14 $\frac{AD}{ED} = \frac{ED}{BD}$

11 Reflexive prop

12 HL

13 CPCTC

14 Substitution prop

20 Given: Rt $\triangle ABC$ withlegs a and b and
hypotenuse c ;
alt h .Prove: $ab = ch$

1 $a^2 = cy$

1 Each leg of a rt Δ is the
mean prop betwn the adj
seg of the hyp and the
total hyp.

2 Same as 1

3 Multiplication prop

4 Alt to hyp of rt Δ is mean
proportional betwn segs
of hyp.

5 Substitution

6 Square root of both sides

16 a $HG = 4$

$EF = 3\sqrt{5}$

$EG = x + 4$

$$\frac{EH}{EF} = \frac{EF}{EG}$$

$$\frac{x}{3\sqrt{5}} = \frac{3\sqrt{5}}{x+4}$$

$x(x+4) = 9(5)$

$x^2 + 4x = 45$

$x^2 + 4x - 45 = 0$

$(x-5)(x+9) = 0$

$x = 5$

$EH = 5$

b $GF = 6$ $EH = 9$

$GH = x$ so $EG = 9+x$

$$\frac{GH}{GF} = \frac{GF}{EG}$$

$$\frac{x}{6} = \frac{6}{9+x}$$

$x(9+x) = 36$

$9x + x^2 - 36 = 0$

$x^2 + 9x - 36 = 0$

$(x-3)(x+12) = 0$

$x = 3,$

$EG = 9+x$

$EG = 9+3$

$EG = 12$

17 a $AD = 7, AB = 11$

$AB - AD = DB$

$DB = 4$

$$\frac{4}{CD} = \frac{CD}{7}$$

$$\frac{8}{CD} = \frac{BC}{12}$$

$CD^2 = 28$

$CD = \sqrt{28}$

$CD = 2\sqrt{7}$

c $AB = 12, AD = 4$

$BD = AB - AD$

$$\frac{BD}{BC} = \frac{BC}{AB}$$

$$\frac{8}{BC} = \frac{BC}{12}$$

$BC^2 = 96$

$BC = \sqrt{96}$

$BC = 4\sqrt{6}$

d $AC = 7, AB = 12$

$AD = AB - BD$

$$\frac{AD}{CD} = \frac{CD}{DB}$$

$$\frac{6}{8} = \frac{8}{AB-6}$$

$\frac{12-BD}{7} = \frac{7}{12}$

$144 - 12BD = 49$

$6AB - 36 = 64$

$6AB = 100$

$AB = 16\frac{2}{3}$

18 $\frac{AD}{CD} \neq \frac{BD}{CD}$

$\frac{AD}{CD} = \frac{CD}{BD}$

$\frac{AD}{BD} \neq \frac{CD}{CD}$

Probability = $\frac{\text{equal}}{\text{possible}} = \frac{1}{3}$

19 To find AC

$$\frac{AD}{AC} = \frac{AC}{AB}$$

$$\frac{4}{AC} = \frac{AC}{9}$$

$AC^2 = 36$

$AC = 6$

To find DC

$$\frac{AD}{DC} = \frac{DC}{BD}$$

$$\frac{4}{DC} = \frac{DC}{5}$$

$DC^2 = 20$

$DC = \sqrt{20}$

To find DE

$$\frac{AD}{AC} = \frac{DE}{EC}$$

$$\frac{4}{6} = \frac{x}{2\sqrt{5}-x}$$

$6x = 8\sqrt{5} - 4x$

$10x = 8\sqrt{5}$

$x = \frac{4}{5}\sqrt{5} \approx 1.8$

20 Given: Rt $\triangle ABC$ withlegs a and b and
hypotenuse c ;alt h .Prove: $ab = ch$

1 $a^2 = cy$

2 $b^2 = cx$

3 $a^2b^2 = c^2xy$

4 $h^2 = xy$

5 $a^2b^2 = c^2h^2$

6 $ab = ch$

21 $AB = x, \frac{AB}{AD} = \frac{AD}{AC}$

$$\frac{x}{6} = \frac{6}{x+5}$$

$$x(x+5) = 36$$

$x^2 + 5x = 36$

$(x-4)(x+9) = 0$

$x = 4$

$AB = 4$

$$\frac{AB}{DB} = \frac{DB}{BC}$$

$$\frac{4}{DB} = \frac{DB}{5}$$

$DB^2 = 20$

$DB = \sqrt{20}$

$DB = 2\sqrt{5}$

22 Given: $\overline{FG} \perp \overline{GH}$

Prove: $\angle 1 \text{ comp } \angle 3$

$$\frac{JH}{GH} = \frac{GH}{HF}$$

1 $\overline{FG} \perp \overline{GH}$

2 $\angle 1 \text{ comp } \angle 3$

3 $\angle 1 \cong \angle 2$

4 $\angle 2 \text{ comp } \angle 3$

5 $m\angle 2 + m\angle 3 = 90$

6 $m\angle 2 + m\angle 3 +$

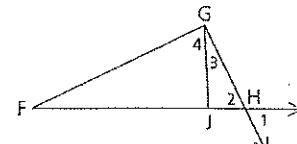
$m\angle GJH = 180$

$7 m\angle GJH = 90$

8 $\overline{GJ} \perp \overline{JH}$

9 $\angle FGH$ is a rt \angle .

10 $\triangle FGH$ is a rt Δ .



1 Given

2 Given

3 Vert \angle s are \cong .

4 Substitution

5 If two \angle s are comp, then
the sum of their measures
is 90.6 Sum of \angle s of Δ is 180.

7 Subtraction prop

8 If two lines intersect to
form rt \angle s, then they are \perp .9 \perp lines intersect to form
rt \angle s.10 If a Δ contains a rt \angle , then
it is a rt Δ .

11 $\overline{GJ} \perp \overline{FH}$

11 If a ray is drawn from a vertex, \perp to a side, then it is an alt.

12 $\frac{JH}{GH} = \frac{GH}{HF}$

12 Each leg of a rt \triangle is the mean propor btwn the adj seg of the hyp and the total hyp.

23 Given: HKMO is a rect.
 $\overline{PK} \perp \overline{HM}$
 $\overline{PJ} \perp \overline{HK}$

Prove: $ab = fg$

1 HKMO rect

2 $\overline{PK} \perp \overline{HM}, \overline{PJ} \perp \overline{HK}$

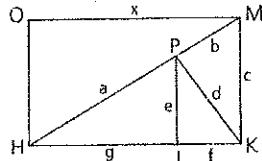
3 $d^2 = f(f+g)$

4 $f+g = x$

5 $d^2 = fx$

6 $d^2 = ab$

7 $ab = fx$



1 Given

2 Given

3 Each leg of a rt \triangle is the mean propor btwn the adj seg of the hyp and the total hyp.

4 Opp sides of a rect are \cong .

5 Substitution

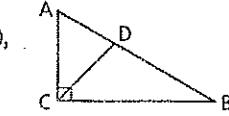
6 Alt to hyp of rt \triangle is mean propor btwn segs of hyp.

7 Transitive prop

24 Arithmetic Mean = $\frac{1}{2}(a+b)$,

Geometric Mean: $\frac{AD}{CD} = \frac{CD}{DB}$

a 1 AM = $\frac{1}{2}(2+8)$



GM: $\frac{2}{x} = \frac{x}{8}, x^2 = 16$

AM = 5 GM is 4.

2 AM = $\frac{1}{2}(3+12)$ GM: $\frac{3}{x} = \frac{x}{12}, x^2 = 36$

AM = $7\frac{1}{2}$ GM is 6.

3 AM = $\frac{1}{2}(4+25)$ GM: $\frac{4}{x} = \frac{x}{25}, x^2 = 100$

AM = $14\frac{1}{2}$ GM is 10.

1 HM: $\frac{2}{\frac{1}{2} + \frac{1}{8}} = \frac{2}{\frac{5}{8}} = \frac{16}{5} = 3.2$

2 HM: $\frac{2}{\frac{1}{3} + \frac{1}{12}} = \frac{2}{\frac{5}{12}} = \frac{24}{5} = 4.8$

3 HM: $\frac{2}{\frac{1}{4} + \frac{1}{25}} = \frac{2}{\frac{29}{100}} = \frac{200}{29} = 6\frac{26}{29}$

b Two positive numbers are unequal in the same order as their squares.

$$\begin{aligned} \left(\frac{a+b}{2}\right)^2 &\stackrel{?}{=} (\sqrt{ab})^2 \\ \frac{a^2 + 2ab + b^2}{4} &\stackrel{?}{=} ab \end{aligned}$$

$$a^2 + 2ab + b^2 \stackrel{?}{=} 4ab$$

$$a^2 - 2ab + b^2 \stackrel{?}{=} 0$$

$$(a-b)^2 \stackrel{?}{=} 0$$

$(a-b)^2 \geq 0$ for real numbers a, b,

$$\text{so } \left(\frac{a+b}{2}\right)^2 \geq (\sqrt{ab})^2$$

Pages 387-391 (Section 9.4)

1 a $x^2 + y^2 = r^2$	b $x^2 + y^2 = r^2$
$4^2 + 5^2 = r^2$	$15^2 + y^2 = 17^2$
$16 + 25 = r^2$	$225 + y^2 = 289$
$41 = r^2$	$y^2 = 64$
$r = \sqrt{41}$	$y = 8$
c $x^2 + y^2 = r^2$	d $x^2 + y^2 = r^2$
$x^2 + 9^2 = 15^2$	$12^2 + y^2 = 13^2$
$x^2 + 81 = 225$	$144 + y^2 = 169$
$x^2 = 144$	$y^2 = 144$
$x = 12$	$y = 5$
e $x^2 + y^2 = r^2$	f $x^2 + y^2 = r^2$
$5^2 + (6\sqrt{3})^2 = r^2$	$5^2 + y^2 = (\sqrt{29})^2$
$25 + 25\sqrt{9} = r^2$	$25 + y^2 = 29$
$25 + 75 = r^2$	$y^2 = 4$
$100 = r^2$	$y = 2$
	$r = 10$
g $x^2 + y^2 = r^2$	
$(2\sqrt{5})^2 + y^2 = (\sqrt{38})^2$	
$20 + y^2 = 38$	
$y^2 = 18$	
$y = \sqrt{18} = 3\sqrt{2}$	

2 All sides of a square are \cong , so each side = $\frac{12}{4} = 3$

All \angle s of a square are rt \angle s

If a and b are sides of the square and c is the diagonal,

$$c^2 = a^2 + b^2$$

$$c^2 = 3^2 + 3^2$$

$$c^2 = 9 + 9$$

$$c^2 = 18$$

$$c = \sqrt{18} = 3\sqrt{2}$$

- 3 In a rhombus, diagonals are \perp bis of each other.

$$BE = ED = 8, AE = EC = 6$$

$$\overline{AC} \perp \overline{BD}$$

$$AB^2 = 6^2 + 8^2$$

$$AB^2 = 36 + 64$$

$$AB^2 = 100$$

$$AB = \sqrt{100} = 10$$

In a rhombus all sides are \equiv , so
perimeter $= 4(10) = 40$.

4 $15^2 + b^2 = 17^2$

$$225 + b^2 = 289$$

$$b^2 = 64$$

$$b = 8$$

$$\text{Perimeter} = 2(15) + 2(8)$$

$$P = 30 + 16 = 46$$

- 5 $\triangle JGF \cong \triangle JGH$ (AAS), therefore $FG = 12$.

Since $\angle JGF$ is a rt \angle (Pythagorean Theorem)

$$FG^2 + JG^2 = JF^2$$

$$12^2 + JG^2 = 15^2$$

$$144 + JG^2 = 225$$

$$JG^2 = 81$$

$$JG = \sqrt{81} = 9$$

- 6 In Section 3.7, problem 7 proved that the altitude to the base of an equilateral \triangle is the median to the base. An altitude forms rt \angle s, so the equilateral \triangle is divided into 2 rt \triangle s and the sides are 2 and x . The hypotenuse is 4.

$$2^2 + x^2 = 4^2 \quad (\text{Pythagorean Theorem})$$

$$4 + x^2 = 16$$

$$x^2 = 12$$

$$x = \sqrt{12} = 2\sqrt{3}$$

- 7 Draw the dotted segment.

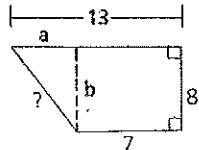
$$a = 13 - 7 = 6, b = 8$$

$$x^2 = a^2 + b^2$$

$$x^2 = 6^2 + 8^2$$

$$x^2 = 36 + 64$$

$$x = \sqrt{100} = 10$$



- 8 Since the wall is perpendicular to the ground, the \triangle formed is a rt \triangle . Use the Pythagorean Theorem.

$$48^2 + x^2 = 50^2$$

$$2304 + x^2 = 2500$$

$$x^2 = 196$$

$$x = \sqrt{196} = 14$$

The foot of the ladder is 14 dm from the wall.

9 a $C(2, 3)$

b $AC = 11 - 3 = 8$

$$CB = 8 - 2 = 6$$

c $(AB)^2 = 6^2 + 8^2$

$$(AB)^2 = 36 + 64$$

$$(AB)^2 = 100$$

$$AB = 10$$

d Yes

10 $PQ = \sqrt{(13 - 1)^2 + (3 - 8)^2}$

$$PQ = \sqrt{(12)^2 + (-5)^2}$$

$$PQ = \sqrt{144 + 25}$$

$$PQ = \sqrt{169} = 13$$

11 a $AC = x, BC = y$

$$AC^2 + BC^2 = AB^2$$

$$x^2 + y^2 = AB^2$$

$$AB = \sqrt{x^2 + y^2}$$

c $AC = 3a, BC = 4a$

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = (3a)^2 + (4a)^2$$

$$AB^2 = 9a^2 + 16a^2$$

$$AB^2 = 25a^2$$

$$AB = 5a$$

b $AC = 2, BC = x$

$$AB^2 = AC^2 + BC^2$$

$$AB^2 = 2^2 + x^2$$

$$AB^2 = 4 + x^2$$

$$AB = \sqrt{4 + x^2}$$

d $AB = 13c, AC = 5c$

$$AB^2 = AC^2 + BC^2$$

$$(13c)^2 = (5c)^2 + BC^2$$

$$169c^2 = 25c^2 + BC^2$$

$$BC^2 = 144c^2$$

$$BC = 12c$$

12 a $\frac{AD}{CD} = \frac{CD}{DB}$

$$(CD)^2 = (7)(4)$$

$$CD^2 = 28$$

$$CD = \sqrt{28} = 2\sqrt{7}$$

c $\frac{DB}{CB} = \frac{CB}{AB}$

$$\frac{2}{8} = \frac{8}{AB}$$

$$2AB = 64$$

$$AB = 32$$

b $(CD)^2 + (DB)^2 = (CB)^2$

$$64 + 36 = (CB)^2$$

$$100 = (CB)^2$$

$$CB = 10$$

d $(AC)^2 + (CB)^2 = (AB)^2$

$$21^2 + (CB)^2 = 29^2$$

$$441 + (CB)^2 = 841$$

$$(CB)^2 = 400$$

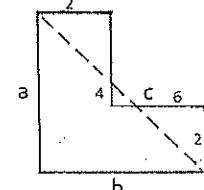
$$CB = 20$$

13 a $a = 6, b = 8$

$$c^2 = 6^2 + 8^2$$

$$c^2 = 36 + 64$$

$$c = \sqrt{100} = 10 \text{ km}$$

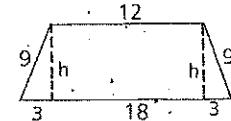


- 14 Draw in altitudes, making a rectangle and two rt \triangle s, 18 - 12 = 6. The \triangle s are \cong , so the shorter legs are 3.

$$h^2 + 3^2 = 9^2$$

$$h^2 + 9 = 81$$

$$h = \sqrt{72} = 6\sqrt{2}$$



16 Perimeter of $\triangle ACE = ?$

Using information given and the Pythagorean Theorem,

$$FD = AB = 24, BC = 7$$

$$\text{Then } AC^2 = 24^2 + 7^2$$

$$AC = \sqrt{576 + 49} = 25$$

A mdpt divides a seg into 2 \cong segs.

$$\text{Then } FE = ED = 12 \text{ and } CD = 16 - 7 = 9$$

$$CE^2 = 12^2 + 9^2$$

$$CE = \sqrt{144 + 81} = 15$$

Opp sides of a rectangle are \cong , so

$$AF = BD = 16 \text{ and}$$

$$AE^2 = 16^2 + 12^2$$

$$AE = \sqrt{256 + 144} = 20$$

$$\text{Perimeter of } \triangle ACE = 25 + 15 + 20 = 60$$

16 Since $\triangle ACB$ is a rt \triangle , $AB = 10$, $DB = y$, so

$$6^2 + 8^2 = AB^2 \quad AD = 10 - y, \frac{AD}{AC} = \frac{AC}{AB}$$

$$36 + 64 = AB^2 \quad \frac{10-y}{6} = \frac{6}{10}$$

$$AB = \sqrt{100} = 10$$

$$10(10-y) = 36$$

$$100 - 10y = 36$$

$$10y = 64, y = 6.4$$

$$DB = 6.4, AD = 3.6$$

$$\frac{AD}{CD} = \frac{CD}{DB}$$

$$\frac{3.6}{CD} = \frac{CD}{6.4}$$

$$CD^2 = (3.6)(6.4)$$

$$CD = \sqrt{23.04}$$

$$CD = 4.8$$

17 Starting with the bottom \triangle ,

$$z^2 = 1^2 + 1^2$$

$$z = \sqrt{2}$$

In the second \triangle ,

$$y^2 = z^2 + 1^2$$

$$y^2 = (\sqrt{2})^2 + 1 = 3$$

$$y = \sqrt{3}$$

In the third \triangle ,

$$w^2 = 1^2 + y^2$$

$$w^2 = 1^2 + (\sqrt{3})^2$$

$$w^2 = 1 + 3 = 4$$

$$w = 2$$

In the last \triangle ,

$$x^2 = w^2 + 1^2$$

$$x^2 = 2^2 + 1^2$$

$$x^2 = 5$$

$$x = \sqrt{5}$$

18 $P = 8\sqrt{5}$

$$\text{each side} = \frac{1}{4}(8\sqrt{5}) = 2\sqrt{5}$$

$$\text{one diagonal} = 4\sqrt{2}$$

$$\frac{1}{2} \text{diagonal} = 2\sqrt{2}$$

$$(2\sqrt{2})^2 + b^2 = (2\sqrt{5})^2$$

$$8 + b^2 = 20$$

$$b^2 = 12$$

$$b = \sqrt{12} = 2\sqrt{3}$$

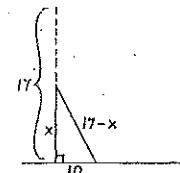
$$\text{Other diagonal} = 4\sqrt{3}$$

$$19 x^2 + 10^2 = (17 - x)^2$$

$$x^2 + 100 = 289 - 34x + x^2$$

$$34x = 189$$

$$x = \frac{189}{34} = 5\frac{19}{34} \text{ m} = 5.56 \text{ m}$$



20 Let $x = \text{leg of } \triangle$,

$$x^2 + x^2 = 6^2$$

$$2x^2 = 6^2$$

$$x^2 = 18$$

$$x = \sqrt{18} = 3\sqrt{2}$$

21 $P = 20$, each side = 5

The diagonals bisect each other, so

the \triangle has legs $x, \frac{x}{2}$ and hypotenuse is 5.

$$x^2 + \left(\frac{x}{2}\right)^2 = 5^2$$

$$x^2 + \frac{x^2}{4} = 25$$

$$4x^2 + x^2 = 100$$

$$5x^2 = 100$$

$$x^2 = 20$$

$$x = \sqrt{20} = 2\sqrt{5}, 2x = 4\sqrt{5}, \text{ sum} = 6\sqrt{5}$$

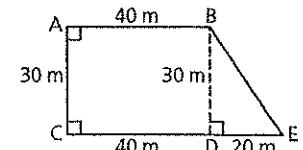
22 Obtuse; Impossible ($3 + 5 > 10$)

23 a Draw altitude BD. Then

$$A = A_{ABCD} + A_{BDE}$$

$$A = (30 \cdot 40) + \frac{1}{2}(30 \cdot 20)$$

$$A \approx 1,200 + 300 = 1,500$$



$$b (BE)^2 = (30)^2 + (20)^2 = 900 + 400$$

$$BE \approx 36$$

24 First find CB.

$$CD^2 + CA^2 = DA^2$$

$$12^2 + CA^2 = 20^2$$

$$144 + CA^2 = 400$$

$$CA = \sqrt{256} = 16$$

$$CA - BA = CB$$

$$CB = 6$$

$$P = 12 + 6\sqrt{5} + 6 = 18 + 6\sqrt{5}$$

Now find DB.

$$CD^2 + CB^2 = DB^2$$

$$12^2 + 6^2 = DB^2$$

$$144 + 36 = DB^2$$

$$\sqrt{180} = DB$$

$$6\sqrt{5} = DB$$

25 a Let $GF = x$, $HF = y$

$$FE = 21 - x$$

$\triangle GHF$ is a rt \triangle , so $\triangle HFE$ is a rt \triangle , so

$$10^2 = x^2 + y^2 \quad 17^2 = (21 - x)^2 + y^2$$

$$100 - x^2 = y^2 \quad 289 - (21 - x)^2 = y^2$$

Then:

$$100 - x^2 = 289 - (21 - x)^2$$

$$100 - x^2 = 289 - 441 + 42x - x^2$$

$$100 = -152 + 42x$$

$$252 = 42x$$

$$6 = x$$

$$\therefore GF = 6$$

$$\text{In } \triangle GHF, GF^2 + HF^2 = 10^2$$

$$36 + HF^2 = 100$$

$$HF = \sqrt{64} = 8$$

b If $\triangle EHF \sim \triangle HGF$, their corr sides

must be in proportion

$$\frac{EH}{HF} ? \frac{HG}{GF}$$

$$\frac{17}{8} ? \frac{10}{6}$$

$$102 \neq 80$$

Their corr sides are not in proportion,
so the \triangle s are not \sim .

26 $\overline{AB} = \overline{AC}$ (An isos \triangle has 2 legs \cong .)

$BC = 32 - 2x \quad \triangle ABD \cong \triangle ACD$ (ASA)

$$BD = DC = 16 - x \quad AD^2 + BD^2 = AB^2$$

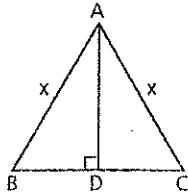
$$8^2 + (16 - x)^2 = x^2$$

$$64 + 256 - 32x + x^2 = x^2$$

$$320 - 32x + x^2 = x^2$$

$$320 = 32x$$

$$10 = x$$



27 In $\triangle JOM$, $JM^2 = JO^2 - MO^2$ $PO = x$, $JK = 2x$

$$JM^2 = 25^2 - 20^2 \quad KM = 15 - 2x$$

$$JM^2 = 225 \quad PM = 20 + x$$

$$JM = 15$$

$$\text{In } \triangle PMK, 25^2 = (20 + x)^2 + (15 - 2x)^2$$

$$625 = 400 + 40x + x^2 + 225 - 60x + 4x^2$$

$$0 = -20x + 5x^2$$

$$0 = 5x(x - 4)$$

$$4 = x$$

$$KM = 15 - 2x, KM = 15 - 2(4) = 7$$

28 In $\triangle ACD$, $(2a)^2 + (b)^2 = (2\sqrt{13})^2$

$$4a^2 + b^2 = 52$$

In $\triangle BCE$, $(a)^2 + (2b)^2 = (\sqrt{73})^2$

$$a^2 + 4b^2 = 73$$

Solving both equations,

$$(4a^2 + b^2 = 52) \text{ Mult } -4 \quad -16a^2 - 4b^2 = -208$$

$$a^2 + 4b^2 = 73$$

$$\underline{-15a^2 = -135}$$

$$a^2 = 9$$

$$a = 3$$

$$(2a)^2 + (b)^2 = (2\sqrt{13})^2 \quad AB^2 = (2a)^2 + (2b)^2$$

$$36 + b^2 = 52$$

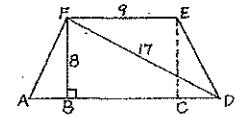
$$AB^2 = 36 + 64$$

$$b^2 = 16$$

$$AB^2 = 36 + 64$$

$$b = 4$$

$$AB = \sqrt{100} = 10$$



29 In $\triangle FBD$, $8^2 + (BD)^2 = 17^2$,

$$BD^2 = 289 - 64$$

$$BD = 15.$$

Since $BC = FE = 9$,

$$CD = AB = 6$$

$$\text{In } \triangle ECD, ED^2 = 8^2 + 6^2$$

$$ED^2 = 64 + 36$$

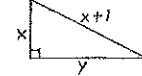
$$ED = \sqrt{100} = 10 \text{ and } \overline{ED} \cong \overline{FA}, \text{ so}$$

$$P = 9 + 2(10) + (15 + 6) = 50$$

30 a one leg = x

hypotenuse = $x + 1$

second leg = y



$$\text{Then } (x + 1)^2 = x^2 + y^2$$

$$x^2 + 2x + 1 = x^2 + y^2 \text{ and } y^2 = 2x + 1$$

b One counterexample

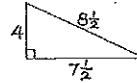
would be a rt \triangle with

$$\text{legs } 4, 7\frac{1}{2}$$

$$\text{hypotenuse } 8\frac{1}{2}$$

($7\frac{1}{2}, 8\frac{1}{2}$ are not integers.)

$$4^2 = 7\frac{1}{2} + 8\frac{1}{2}$$



31 a slope $\overline{QU} = \frac{15}{8}$

slope $\overline{DA} = \frac{15}{8}$

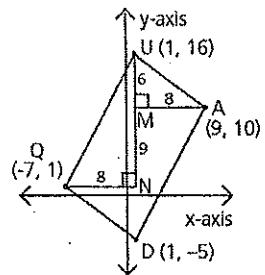
$\therefore \overline{QU} \parallel \overline{DA}$

Slope $\overline{QD} = \frac{-3}{4}$

Slope $\overline{AU} = \frac{-3}{4}$

$\overline{QD} \parallel \overline{AU}$

$\therefore \text{QUAD is a } \square$



b $(UA)^2 = 6^2 + 8^2$

$UA = 10$

$(QU)^2 = 8^2 + 15^2$

$QU = 17$

Perimeter QUAD = $17 + 17 + 10 + 10 = 54$

32 By theorem, AC is the mean prop between

AB and AD. $AB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$AC^2 = (AB)(AD)$

$3^2 = 5(AD)$

$\frac{9}{5} = AD$ and

$DE = 5 - 2 - \frac{9}{5} = \frac{6}{5}$

In $\triangle ACD$, $AC^2 = CD^2 + AD^2$

$3^2 = CD^2 + (\frac{9}{5})^2$

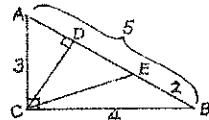
$CD = \sqrt{9 - \frac{81}{25}} = \sqrt{\frac{225}{25} - \frac{81}{25}}$

$CD = \sqrt{\frac{144}{25}} = \frac{12}{5}$

In $\triangle CDE$, $CE^2 = CD^2 + DE^2$

$CE^2 = (\frac{12}{5})^2 + (\frac{6}{5})^2 = \frac{144}{25} + \frac{36}{25}$

$CE = \frac{6\sqrt{5}}{5}$



33 Draw \overline{RP} and \overline{VX}

$\overline{SP} \cong \overline{XT}$, so $SP \cong \frac{1}{2}(ST - RV)$.

$SP = 3$. Draw $\overline{TQ} \perp \overline{SW}$

$\angle Q \cong \angle P$ (rt \angle s are \cong .)

$\angle S \cong \angle S$ (Reflexive prop)

$\triangle SQT \sim \triangle SPR$ (AA~)

$$\frac{SR}{ST} = \frac{SP}{SQ}$$

$$\frac{9}{18} = \frac{3}{SQ}$$

$ST^2 = SQ^2 + QT^2$

$18^2 = 6^2 + QT^2$

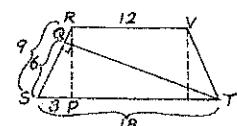
$9(SQ) = 54$

$324 = 36 + QT^2$

$SQ = 6$

$\sqrt{288} = QT$

$12\sqrt{2} = QT$



34 In rt $\triangle PRT$, $15^2 + (RT)^2 = 25^2$

$RT = \sqrt{625 - 225}$

$RT = \sqrt{400} = 20$

If $ST = x$, $SR = 20 - x$. In rt $\triangle PRS$,

$15^2 + (20 - x)^2 = (x + 12)^2$

$225 + 400 - 40x + x^2 = x^2 + 24x + 144$

$-64x = -481$

$x = \frac{481}{64}$

$SR = 20 - x$, $SR = 20 - \frac{481}{64}$

$SR = \frac{799}{64}$

35 Let x = shorter side

y = longer side

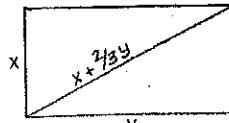
$x^2 + y^2 = (x + \frac{2}{3}y)^2$

$x^2 + y^2 = x^2 + \frac{4}{9}xy + \frac{4}{9}y^2$

$\frac{5}{9}y^2 = \frac{4}{3}xy$

$\frac{5}{9}y = \frac{4}{3}x$

$$\frac{x}{y} = \frac{\frac{5}{9}}{\frac{4}{3}} = \frac{5}{12}$$



36 a Show $(BP)^2 + (PD)^2 = (AP)^2 + (CP)^2$

$(BP)^2 = y^2 + (TB)^2$

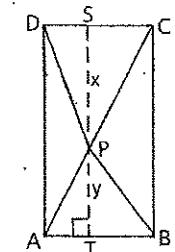
$(PD)^2 = x^2 + (DS)^2$

$(BP)^2 + (PD)^2 = x^2 + y^2 + (TB)^2 + (DS)^2$

$(AP)^2 = y^2 + (AT)^2$

$(CP)^2 = x^2 + (SC)^2$

$(AP)^2 + (CP)^2 = x^2 + y^2 + (AT)^2 + (SC)^2$



If \overline{ST} is drawn through P, parallel to CB, STBC is a rectangle and $\overline{TB} \cong \overline{SC}$ and $\overline{DS} \cong \overline{AT}$.

b Yes, the same procedure should be followed.

Pages 394-400 (Section 9.5)

1 a $\sqrt{(6-4)^2 + (0-0)^2} = 2$

b $\sqrt{(2-2)^2 + (3-(-1))^2} = 4$

c $\sqrt{(7-4)^2 + (5-1)^2} = \sqrt{9+16} = 5$

d $\sqrt{(4-(-4))^2 + (-8-(-2))^2} = \sqrt{8^2 + 6^2} = 10$

e $\sqrt{(2-0)^2 + (5-0)^2} = \sqrt{4+25} = \sqrt{29}$

f $\sqrt{(6-2)^2 + (3-1)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

2 $AB = \sqrt{(5-2)^2 + (10-6)^2} = \sqrt{9+16} = 5$

$BC = \sqrt{(5-0)^2 + (10-13)^2} = \sqrt{25+9} = \sqrt{34}$

$CA = \sqrt{(2-0)^2 + (6-13)^2} = \sqrt{4+49} = \sqrt{53}$

$P = AB + BC + CA = 5 + \sqrt{34} + \sqrt{53} \approx 18.1$

3 a Let A = (8, 4); B = (3, 5); C = (4, 10)

$AB = \sqrt{(8-3)^2 + (4-5)^2} = \sqrt{25+1} = \sqrt{26}$

$BC = \sqrt{(4-3)^2 + (10-5)^2} = \sqrt{1+25} = \sqrt{26}$

$AC = \sqrt{(8-4)^2 + (4-10)^2} = \sqrt{16+36} = \sqrt{52}$

$AB^2 + BC^2 = AC^2$

$(\sqrt{26})^2 + (\sqrt{26})^2 = AC^2$

$26 + 26 = AC^2$

$\sqrt{52} = AC$

b $m(\overline{AB}) = \frac{5-4}{3-8} = -\frac{1}{5}$

$m(\overline{BC}) = \frac{10-5}{4-3} = \frac{5}{1} = 5$

The slopes are negative inverses, so the sides are \perp .

4 DO = $\sqrt{(6-0)^2 + (0-0)^2} = 6$

$OC = \sqrt{(3-0)^2 + (3\sqrt{3}-0)^2} = \sqrt{9+27} = 6$

$DG = \sqrt{(3-6)^2 + (3\sqrt{3}-0)^2} = \sqrt{9+27} = 6$

$\therefore \triangle DOG$ is isosceles since all 3 sides are \cong .

5 $r = \sqrt{(9 - (-3))^2 + (-4 - 5)^2} = \sqrt{144 + 81} = \sqrt{225} = 15$
 $A = \pi r^2 = \pi(15)^2 = 225\pi$

6 a Midpt $\overline{RV} = (7, -1)$
 distance from T to midpt $\overline{RV} =$
 $\sqrt{(9 - 7)^2 + (8 - (-1))^2} = \sqrt{85}$

b The length of the segment joining the midpoints of two sides of a \triangle is $\frac{1}{2}$ of the length of the third side.

$$RV = \sqrt{(13 - 1)^2 + (2 - (-4))^2} = \sqrt{144 + 36} = \sqrt{180} = 6\sqrt{5}$$

length of the segment = $\frac{RV}{2} = 3\sqrt{5}$

7 $(BD)^2 + (DC)^2 = (BC)^2$ Let $AD = x$
 $6^2 + 8^2 = (BC)^2$ So $\frac{8}{10} = \frac{10}{8+x}$
 $36 + 64 = (BC)^2$ $8(8+x) = 100$
 $100 = (BC)^2$ $64 + 8x = 100$
 $10 = BC$ $8x = 36$
 $AD = x = 4.5$

8 a A(0, 2b) B(2a, 2b) C(2a, 0) O(0, 0)
 b $M = \left(\frac{0+0}{2}, \frac{2b+0}{2}\right) = (0, b)$ $P = \left(\frac{2a+2a}{2}, \frac{2b+0}{2}\right) = (2a, b)$
 $N = \left(\frac{2a+0}{2}, \frac{2b+2b}{2}\right) = (a, 2b)$ $Q = \left(\frac{2a+0}{2}, \frac{0+0}{2}\right) = (a, 0)$
 c slope $\overline{MN} = \frac{2b-b}{a-0} = \frac{b}{a}$ slope $\overline{MQ} = \frac{0-b}{a-0} = -\frac{b}{a}$
 slope $\overline{QP} = \frac{b-0}{2a-a} = \frac{b}{a}$ slope $\overline{NP} = \frac{2b-b}{a-2a} = -\frac{b}{a}$

Since $\overline{MN} \parallel \overline{QP}$ and $\overline{MQ} \parallel \overline{NP}$, MNPQ is a \square .

d $MN = \sqrt{(a-0)^2 + (2b-b)^2} = \sqrt{a^2 + b^2}$
 $QP = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$
 $MQ = \sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$
 $NP = \sqrt{(2a-a)^2 + (b-2b)^2} = \sqrt{a^2 + b^2}$

$MN = QP = MQ = NP$, \therefore MNPQ is a rhombus.

9 a $PQ = \sqrt{(-a+c)^2 + (0-b)^2} = \sqrt{c^2 - 2ac + a^2 + b^2}$
 $SR = \sqrt{(a-c)^2 + (0-b)^2} = \sqrt{a^2 - 2ac + c^2 + b^2}$
 $PQ = SR$, \therefore PQRS is an isos trap.
 b $PR = \sqrt{(c+a)^2 + (b-0)^2} = \sqrt{c^2 + 2ac + a^2 + b^2}$
 $QS = \sqrt{(a+c)^2 + (0-b)^2} = \sqrt{c^2 + 2ac + a^2 + b^2}$
 $PR = QS$, $\therefore \overline{PR} \approx \overline{QS}$.

10 Yes, since $m\angle E = 90$ (by def of \square), $m\widehat{RTC} = 180$; $\therefore \overline{RC}$ is a diameter.

11 a $\triangle REC$ is a 5-12-13 \triangle , so $\overline{RC} = d = 13$.

C = $\pi d = 13\pi$

b $A = \pi r^2 = \pi(\frac{13}{2})^2 \approx 132.7$

12 $BD = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$
 $AC = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$
 slope $\overline{BD} = \frac{a}{-a} = -1$
 slope $\overline{AC} = \frac{a}{a} = 1$
 $\therefore \overline{AD} \perp \overline{BC}$

13 a $m\widehat{TV} = 180 - m\widehat{VW}$
 $m\widehat{TV} = 180 - 120 = 60$
 Area of sector TRV = $\frac{60}{360} (\text{Area of } \odot R)$
 $= \frac{1}{6} (\pi(9)^2) = 13.5\pi$

≈ 42.4
 b $\overline{TW} = 18$
 $\widehat{VW} = \frac{120}{360} (2\pi(9)) \approx 18.8$
 $\widehat{VW} - \overline{TW} \approx 18.8 - 18 \approx 0.8$

14 The distance from the pts to the center would all be equal.
 $\sqrt{(7-2)^2 + (11-(-1))^2} = \sqrt{25 + 144} = \sqrt{169} = 13$
 $\sqrt{(7-2)^2 + (-13-(-1))^2} = \sqrt{25 + 144} = \sqrt{169} = 13$
 $\sqrt{(14-2)^2 + (4-(-1))^2} = \sqrt{144 + 25} = \sqrt{169} = 13$
 15 $AB = \sqrt{(7-2)^2 + (3-1)^2} = \sqrt{25 + 4} = \sqrt{29}$
 $BC = \sqrt{(12-7)^2 + (1-3)^2} = \sqrt{25 + 4} = \sqrt{29}$
 $CD = \sqrt{(12-7)^2 + (1-(-4))^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$
 $DA = \sqrt{(7-2)^2 + (-4-1)^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$
 $P = 2\sqrt{29} + 10\sqrt{2} \approx 24.9$; Kite (pairs of adj sides =)

16 d(-1, -3) to (3, -2) = $\sqrt{(3-(-1))^2 + (-2-(-3))^2} = \sqrt{17}$
 d(2, 1) to (3, -2) = $\sqrt{(3-2)^2 + (-2-1)^2} = \sqrt{10}$
 Consecutive sides are not congruent; therefore the parallelogram is not a rhombus.

17 d from (-2, 1) to (5, 5) = $\sqrt{(5-(-2))^2 + (5-1)^2} = \sqrt{65}$
 d from (-2, 1) to (-1, -7) = $\sqrt{(-1-(-2))^2 + (-7-1)^2} = \sqrt{65}$

18 One diag has length = $\sqrt{(6-0)^2 + (8-0)^2} = 10$
 Since in a \square both diagonals are equal, the sum of the two is 20.

19 Let A = (1, 2); B = (4, 6); C = (10, 14)

a $AB = \sqrt{(4-1)^2 + (6-2)^2} = 5$

$BC = \sqrt{(10-4)^2 + (14-6)^2} = 10$

$AC = \sqrt{(10-1)^2 + (14-2)^2} = 15$

$AB + BC = AC$ They are collinear.

b $m(\overline{AB}) = \frac{6-2}{4-1} = \frac{4}{3}$

$m(\overline{BC}) = \frac{14-6}{10-4} = \frac{8}{6} = \frac{4}{3}$

They are collinear.

$$\begin{aligned}
 20 \quad & \sqrt{(5-1)^2 + (y-4)^2} = \sqrt{(10-5)^2 + (-3-y)^2} \\
 & \sqrt{16 + (4-y)^2} = \sqrt{25 + (y+3)^2} \\
 & 16 + (4-y)^2 = 25 + (y+3)^2 \\
 & 16 + 16 - 8y + y^2 = 25 + y^2 + 6y + 9 \\
 & -2 = 14y \\
 & -\frac{1}{7} = y
 \end{aligned}$$

$$\begin{aligned}
 21 \quad & h^2 + 9^2 = 41^2 \\
 & h^2 + 81 = 1681 \\
 & h = \sqrt{1600} = 40
 \end{aligned}$$

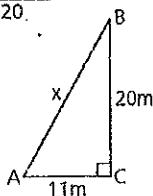


$$22 \quad x^2 = 20^2 + 11^2$$

$$x^2 = 521$$

$$x \approx 23$$

$$AB + BC \approx 20 + 23 = 43 \text{ m}$$



$$23 \quad BC = \sqrt{(a+0)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$$

$$BM = CM = \frac{1}{2}BC = \frac{\sqrt{a^2 + b^2}}{2}$$

$$M \text{ is at } \left(\frac{a}{2}, \frac{b}{2}\right)$$

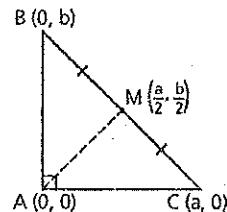
$$MA = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$$MA = \sqrt{\frac{a^2}{4} + \frac{b^2}{4}}$$

$$2MA = \sqrt{a^2 + b^2}$$

$$MA = \frac{\sqrt{a^2 + b^2}}{2}$$

$\therefore M$ is equidistant from A, B, and C.



24 By prop of \square , $B = (a+b, c)$

$$\text{For diagonals, } BD^2 = (a+b)^2 + c^2$$

$$CA^2 = (a-b)^2 + c^2$$

$$BD^2 + CA^2 = a^2 + 2ab + b^2 + c^2 + a^2 - 2ab + b^2 + c^2$$

$$= 2a^2 + 2b^2 + 2c^2$$

$$\text{For sides, } CD^2 = a^2$$

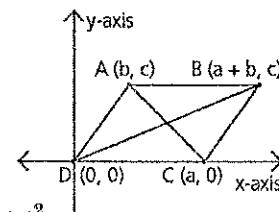
$$AB^2 = a^2$$

$$AD^2 = b^2 + c^2$$

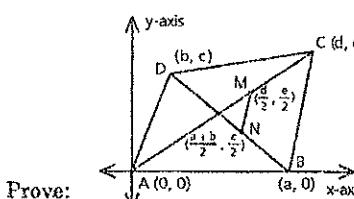
$$BC^2 = b^2 + c^2$$

$$\text{So } 2a^2 + 2b^2 + 2c^2 = 2a^2 + 2b^2 + c^2$$

$$\therefore BD^2 + CA^2 = CD^2 + AB^2 + AD^2 + BC^2$$



25



$$\text{Prove: } AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4MN^2$$

$$AB^2 = a^2$$

$$BC^2 = (a-d)^2 + (e-0)^2$$

$$= a^2 - 2ad + d^2 + e^2$$

$$\begin{aligned}
 CD^2 &= (b-d)^2 + (c-e)^2 \\
 &= b^2 - 2bd + d^2 + c^2 - 2ce + e^2
 \end{aligned}$$

$$DA^2 = b^2 + c^2$$

$$AC^2 = d^2 + e^2$$

$$\begin{aligned}
 BD^2 &= (a-b)^2 + (0-c)^2 \\
 &= a^2 - 2ab + b^2 + c^2
 \end{aligned}$$

$$\begin{aligned}
 MN^2 &= \left(\frac{a+b-d}{2}\right)^2 + \left(\frac{c-e}{2}\right)^2 \\
 &= \frac{a^2 + b^2 + d^2 + 2ab - 2ad - 2bd}{4} + \frac{c^2 - 2ce + e^2}{4}
 \end{aligned}$$

$$4MN^2 = a^2 + b^2 + d^2 + 2ab - 2ad - 2bd + c^2 - 2ce + e^2$$

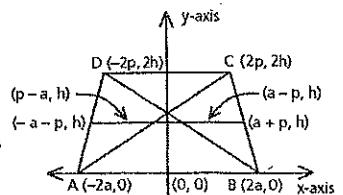
$$AB^2 + BC^2 + CD^2 + DA^2 =$$

$$2a^2 + 2b^2 + 2c^2 + 2d^2 + 2e^2 - 2ad - 2bd - 2ce$$

$$AC^2 + BD^2 + 4MN^2 =$$

$$2a^2 + 2b^2 + 2c^2 + 2d^2 + 2e^2 - 2ad - 2bd - 2ce$$

26



$$a \quad C(2p, 2h), D(-2p, 2h)$$

$$b \quad AB = 4a$$

$$c \quad a + p - (-a - p) = 2a + 2p$$

$$d \quad (p - a) - (a - p) = 2a - 2p$$

27 Equation of \perp bis of given side is $y = -x + 7$. The length of a side is $4\sqrt{2}$, if the remaining vertex is (x, y) :

$$\sqrt{(x-2)^2 + (y-1)^2} = 4\sqrt{2}; y = -x + 7. \text{ Solving, } x = 4 \pm 2\sqrt{3}.$$

Pages 401–404 (Section 9.6)

- 1 a 25 b 36 c 21 d $1\frac{2}{3}$ e 60 2 a 10 b 78 c 36 d 65
 e $6\frac{1}{2}$ 3 a 250 b 48 c 28 d 2.4 e 264 4 a 51
 b 3.4 c 75 d $\frac{4}{5}$ e 80 5 a 12 b $2\sqrt{7}$ c 10 d 0.5
 e 34 f $5\sqrt{7}$ g 72 h 45 i $12\sqrt{7}$

6 (20, 48, ?) belongs to the (5, 12, 13) Family, $4 \times 5 = 20$, $4 \times 12 = 48$, $4 \times 13 = 52$; the diagonal is 52.

7 If the base of the isos Δ is 16, the height forms a rt Δ with sides (8, 15, ?). This is the (8, 15, 17) family Δ so the side of the Δ is 17. Perimeter = $17 + 17 + 16 = 50$ dm.

8 The rt Δ s belong to the (3, 4, 5) family. (? 12, 15) = 3 (3, 4, 5) so the rt Δ is (9, 12, 15). The opp rt Δ (formed by drawing an alt to the base of the trapezoid) is \cong to the first rt Δ . Bases of the Δ s are \cong and = 9. The upper base is $35 - 2(9) = 17$.

9 a Divide by 8.

$$\frac{16}{8} = 2, \frac{8\sqrt{5}}{8} = \sqrt{5}$$

$$2^2 + (\sqrt{5})^2 = x^2$$

$$\sqrt{4+5} = x$$

$$x = 3$$

Multiply by 8,

$$x = 3 \cdot 8 = 24$$

b Divide by 100,

$$\frac{700}{100} = 7, \frac{200}{100} = 2$$

$$2^2 + p^2 = 7^2$$

$$4 + p^2 = 49$$

$$p = \sqrt{49-4}$$

$$p = \sqrt{45} = 3\sqrt{5}$$

Multiply by 100,

$$p = 300\sqrt{5}$$

- 10 UD = 15, using reduced Δ and Pythagorean triple concepts.

$$8^2 + 15^2 = (QD)^2$$

$$64 + 225 = (QD)^2$$

$$289 = (QD)^2$$

$$17 = QD$$

- 11 CB = 11 - 3 = 8, BA = 11 - (-4) = 15
 $CA^2 = 8^2 + 15^2$
 $(8, 15, 17)$ is a Pythagorean triple, so CA = 17.
 $P = 17 + 8 + 15 = 40$
 $A = \frac{1}{2}(8)(15) = 60$

- 12 In a rhombus the diagonals are \perp bis of one another, so $\overline{RP} \cong \overline{PO}$, $RP + PO = 48$ $\overline{HP} \cong \overline{PM}$, $HP + PM = 14$
 $RP = 24$, $PO = 24$ $HP = 7$, $PM = 7$
 $RP^2 + PM^2 = RM^2$ and $RP = 24$, $PM = 7$
 $(7, 24, 25)$ is a Pythagorean triple so $RM = 25$.
All the sides of a rhombus are \cong , so $P = 4(25) = 100$.

- 13 For Mary, $d = rt$, $d = (10)(\frac{1}{2}) = 15$

For Larry, $d = rt$, $d = (16)(\frac{1}{2}) = 24$

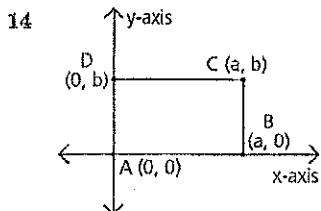
Then divide 15 and 24 by 3

$$\text{and } 5^2 + 8^2 = x^2$$

$$25 + 64 = x^2$$

$$\sqrt{89} = x$$

Now multiply by 3, $3\sqrt{89} \approx 28$ km apart



Given $\square ABCD$, prove $\overline{AC} \cong \overline{BD}$.

$$AC = \sqrt{(0-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$$

$$BD = \sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$$

$$AC = BD, \therefore \overline{AC} \cong \overline{BD}$$

- 15 a Divide 42 and 150 by 6. b Multiply by 8.

The Δ belongs to the

(7, 24, 25) family.

Missing side is 6(24)

or 144.

$$c \sqrt{4^2 + 3^2} = x^2$$

$$4 + 3 = x^2$$

$$x = \sqrt{7}$$

The Δ belongs to the

(3, 4, 5) family.

Missing side is $\frac{3}{8}$

or $\frac{3}{8}$.

- 16 a In the 1st Δ , $15^2 + y^2 = 17^2$

$(8, 15, 17)$ is a Pythagorean triple, so $y = 8$.

In the 2nd Δ ,

$$6^2 + y^2 = z^2$$

$$6^2 + 8^2 = z^2$$

$$(2 \cdot 3)^2 + (2 \cdot 4)^2 = z^2 \text{ and } (5 \cdot 2)^2 + w^2 = (13 \cdot 2)^2$$

$(3, 4, 5)$ is a Pythagorean

$(5, 12, 13)$ is a Pythagorean triple,

In the 4th Δ ,

$$w^2 + x^2 = 25^2$$

$(7, 24, 25)$ is a Pythagorean triple, $x = 7$.

b The 2 lines are \parallel by corr \angle s \cong \parallel lines. If a line is \parallel to 1 side of a Δ and intersects the other 2 sides, it divides them proportionally, so $\frac{12}{3} = \frac{y}{5}$

$$3y = 60, y = 20$$

The large Δ belongs to the $(3, 4, 5)$ family since $(15, ?, 25) = 5(3, 4, 5)$; so the missing leg is 20.

- 17 a $PQ = \sqrt{(-1 - (-4))^2 + 4^2} = \sqrt{9 + 16} = 5$

$$RS = \sqrt{(11 - 8)^2 + (-4)^2} = \sqrt{9 + 16} = 5$$

$$QR = 9, PS = 15; QR \neq PS$$

$\overline{QR} \parallel \overline{PS}$, $PQ = RS$, $\therefore PQRS$ is an isos trap.

- b $h = 4$

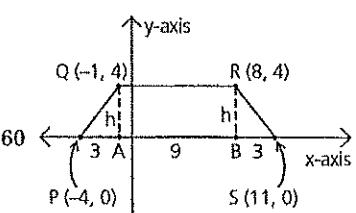
$$A = \frac{h}{2}(b_1 + b_2)$$

$$= \frac{4}{2}(9 + 15) = 48$$

$$c PR^2 = 4^2 + 12^2 = 160$$

$$PR = \sqrt{4^2 + 10}$$

$$= 4\sqrt{10}$$



- 18 Find the hypotenuse of each Δ formed by connecting the path of the submarine.

$$1^2 + 1^2 = c^2$$

$$1 + 1 = c^2$$

$$c = \sqrt{2}$$

There are 20 small Δ s with one km left over. So $20\sqrt{2} + 1$ = dist from starting pt.

$20(1.4142) + 1 \approx 28 + 1$ approximately 29 km

19 a $n^2 + \left(\frac{n^2 - 1}{2}\right)^2 \stackrel{?}{=} \left(\frac{n^2 + 1}{2}\right)^2$
 $n^2 + \left(\frac{n^2 - 1}{2}\right)\left(\frac{n^2 - 1}{2}\right) \stackrel{?}{=} \left(\frac{n^2 + 1}{2}\right)\left(\frac{n^2 + 1}{2}\right)$
 $n^2 + \frac{n^4 - 2n^2 + 1}{4} \stackrel{?}{=} \frac{n^4 + 2n^2 + 1}{4}$
 $\frac{n^4 + 2n^2 + 1}{4} = \frac{n^4 + 2n^2 + 1}{4}$

b $m^2 + \left(\frac{m^2}{4} - 1\right)^2 \stackrel{?}{=} \left(\frac{m^2}{4} + 1\right)^2$
 $m^2 + \left(\frac{m^2}{4} - 1\right)\left(\frac{m^2}{4} - 1\right) \stackrel{?}{=} \left(\frac{m^2}{4} + 1\right)\left(\frac{m^2}{4} + 1\right)$
 $m^2 + \frac{m^4}{16} - \frac{m^2}{2} + 1 \stackrel{?}{=} \frac{m^4}{16} + \frac{m^2}{2} + 1$
 $\frac{m^4}{16} + \frac{m^2}{2} + 1 = \frac{m^4}{16} + \frac{m^2}{2} + 1$

c $\left(\frac{m-n}{2}\right)^2 + (\sqrt{mn})^2 \stackrel{?}{=} \left(\frac{m+n}{2}\right)^2$
 $\left(\frac{m-n}{2}\right)\left(\frac{m-n}{2}\right) + (\sqrt{mn})(\sqrt{mn}) \stackrel{?}{=} \left(\frac{m+n}{2}\right)\left(\frac{m+n}{2}\right)$
 $\frac{m^2 - 2mn + n^2}{4} + mn \stackrel{?}{=} \frac{m^2 + 2mn + n^2}{4}$
 $\frac{m^2 + 2mn + n^2}{4} = \frac{m^2 + 2mn + n^2}{4}$

d $(m^2 - n^2)^2 + (2mn)^2 \stackrel{?}{=} (m^2 + n^2)^2$
 $(m^2 - n^2)(m^2 - n^2) + (2mn)(2mn) \stackrel{?}{=} (m^2 + n^2)(m^2 + n^2)$
 $m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \stackrel{?}{=} m^4 + 2m^2n^2 + n^4$
 $m^4 + 2m^2n^2 + n^4 = m^4 + 2m^2n^2 + n^4$

20 If x = side one, $x + 1$ = side two, $x + 2$ = side

three, then $x^2 + (x+1)^2 = (x+2)^2$

$$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$$

$$x^2 - 2x - 3 = 0$$

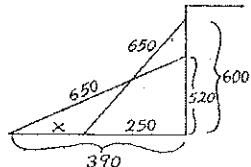
$$(x-3)(x+1) = 0$$

$x = 3$ or $x = -1$ ($x \neq -1$)

The sides can only equal 3, 4, 5.

21 $520 + 80 = 600$ cm

The Δ is $(z, 600, 650)$ or
 $(z, 50 \cdot 12, 50 \cdot 13)$
and $(5, 12, 13)$ is a
Pythagorean triple.



So $z = 50 \cdot 5 = 250$

$$650^2 = 520^2 + y^2$$

Divide by 10, $65^2 = 52^2 + \left(\frac{y}{10}\right)^2$

$$4225 = 2704 + \frac{y^2}{100}$$

$$1521 = \frac{y^2}{100}$$

$$39 = \frac{y}{10}, y = 390$$

$$y - z = x$$

$$390 - 250 = x$$

$$x = 140 \text{ cm}$$

22 $x^2 + (3x+y)^2 = (4x-y)^2$
 $x + 9x^2 + 6xy + y^2 = 16x^2 - 8xy + y^2$
 $14xy = 6x^2$
 $14y = 6x$
 $\frac{x}{y} = \frac{14}{6} = \frac{7}{3}$

23 Probability = $\frac{\text{number correct}}{\text{number of possibilities}}$

- a Possibilities
(3, 4) (4, 5) 5, 6 (6, 8) (8, 10)
(3, 5) 4, 6 5, 8 (6, 10)
3, 6 4, 8 5, 10

3, 8 4, 10

3, 10

Probability = $\frac{6}{15} = \frac{2}{5}$

b Possibilities:

- 3, 4 (4, 5) 5, 6 6, 8 (8, 10)
(3, 5) 4, 6 5, 8 (6, 10)
3, 6 4, 8 5, 10

3, 8 4, 10

3, 10

Probability = $\frac{4}{15}$

24 $16^2 + b^2 = c^2$, so $16^2 = c^2 - b^2$

$\therefore 256 = (c+b)(c-b)$

Since b and c are integers, examine integral pairs of factors of 256, 16^2 , until we find the largest pair that works.

$c + b = 256$ Since c will be

$c - b = 1$ a fraction, this

$2c = 257$ pair doesn't work.

$c + b = 128$

$c - b = 2$

$2c = 130$, so $c = 65$

$\therefore 65 + b = 128$ and $b = 63$

The length of the hypotenuse of this 16, 65, 63 triangle is 65.

25 Case I. 20 is a leg.

$$20^2 + b^2 = c^2, \text{ so } 20^2 = c^2 - b^2$$

$$\therefore 400 = (c+b)(c-b)$$

Since b and c are integers, examine integral pairs of factors of 400.

$$c+b=400 \quad c+b=200$$

$$c-b=1 \quad c-b=2$$

$$2c = 401 \quad 2c = 202$$

$$c = 200\frac{1}{2} \quad c = 101$$

Impossible. $\therefore b = 99$

$$c+b=100 \quad c+b=80$$

$$c-b=4 \quad c-b=5$$

$$2c = 104 \quad 2c = 85$$

$$c = 52 \quad c = 42\frac{1}{2}$$

$\therefore b = 48$ Impossible.

$$c+b=50 \quad c+b=40$$

$$c-b=8 \quad c-b=10$$

$$2c = 58 \quad 2c = 50$$

$$c = 29 \quad c = 25$$

$\therefore b = 21$ $\therefore b = 15$

$$c+b=25$$

$$c-b=16$$

$2c = 41$, $c = 20\frac{1}{2}$ Impossible.

Case II. 20 is the hypotenuse.

$$a^2 + b^2 = 20^2, a \leq b$$

$$\therefore a^2 = 400 - b^2$$

Perfect squares between 200 and 400 are 225, 256, 289, 324, and 361. Let b^2 equal each of these perfect squares. Since a^2 must be a perfect square, then $400 - b^2$ must be a perfect square. The only value of b^2 that works is 256; that is, $400 - 256 = 144$ and $\therefore a = 12$. So the triangles are (20, 99, 101); (20, 48, 52); (20, 21, 29); (15, 20, 25); and (12, 16, 20).

Pages 408-412 (Section 9.7)

1 a $7, 7\sqrt{3}$ b $10\sqrt{3}, 20$ c $5, 10$ d $173\sqrt{3}, 346$ e $114, 114\sqrt{3}$

2 a $2\sqrt{3}, 4\sqrt{3}$ b $\frac{15}{2}, \frac{15\sqrt{3}}{2}$ c $5\sqrt{3}, 10\sqrt{3}$ d $\frac{7\sqrt{3}}{3}, \frac{14\sqrt{3}}{3}$

3 a $2\sqrt{3}$ b $14\sqrt{3}$ c $13\sqrt{3}$ 4 a $8\sqrt{2}$ b $4\sqrt{2}$ c $17\sqrt{2}$ d $19\sqrt{2}$

5 Each side of square is 11.

$$11^2 + 11^2 = x^2$$

$$121 + 121 = x^2$$

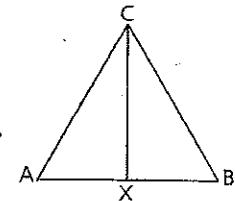
$$242 = x^2$$

$$x = \sqrt{242} = 11\sqrt{2}$$

7 We can prove the Δ s \cong by AAS.

The vertex \angle is bisected making two $30^\circ 60^\circ 90^\circ$ Δ s.

If $AB = 6$ mm, then $BX = \frac{1}{2}(AB)$ or 3 mm. $AX = BX(\sqrt{3}) = 3\sqrt{3}$ mm



8 $2x = 8\sqrt{3}$

$$x = \frac{8\sqrt{3}}{2}$$

$$x = 4\sqrt{3}, \text{ so } CD = 4\sqrt{3}$$

9 In a kite, one of the diags is the \perp bis of the other.

$YW = YT = 5$, $RY = 5$, so $\triangle RYT$ is an isos rt Δ , $45^\circ 45^\circ 90^\circ$ Δ and $TR = 5\sqrt{2}$.

b $\triangle WYX$ is a rt Δ of the (5, 12, 13) family, so $WX = 13$.

$$10 \text{ a } \frac{\text{longer leg}}{\text{hypotenuse}} = \frac{x\sqrt{3}}{2x} \quad \text{b } \frac{\text{leg}}{\text{hypotenuse}} = \frac{x}{x\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2} \quad = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

11 Publisher's note: This question is open-ended, and was intended to generate discussion of proportion and of part-whole relations in regular polygons.

12 a $\triangle OAB$ is a $30^\circ 60^\circ 90^\circ$ rt Δ , so $B(1, \sqrt{3})$

$$\text{b slope } \overleftrightarrow{OB} = \frac{\sqrt{3}-0}{1-0} = \sqrt{3}$$

$$\text{c } AB = \sqrt{3}, OA = 1; \frac{AB}{OA} = \sqrt{3}$$

13 a $\triangle OCD$ is a $45^\circ 45^\circ 90^\circ$ rt Δ , so $D(1, 1)$

$$\text{b slope } \overleftrightarrow{OD} = \frac{1-0}{1-0} = 1$$

$$\text{c tangent of } \angle O = \frac{DC}{OC} = \frac{1}{1} = 1$$

14 $DB = x$

$$CD = x\sqrt{3} \quad \frac{DB}{AD} = \frac{x}{3x} = \frac{1}{3}$$

$$AD = (x\sqrt{3})(\sqrt{3})$$

$$= 3x$$

15 To find the \cong legs of the isos trapezoid, drop altitudes. The rt Δ s are $30^\circ 60^\circ 90^\circ$ with a leg of 3, so the hyp is 6.

$$\therefore EF = HG = 6, P = 10 + 6 + 16 + 6 = 38$$

6 In a $30^\circ 60^\circ 90^\circ$ Δ , side opp 90° Δ is twice the shorter leg.

Shorter leg is 8 m so diagonal is 16 m.

16 $\triangle PJK$ and $\triangle OMN$ are $\cong 45^\circ 45^\circ 90^\circ$ Δ s.

$$PJ = OM = 6\sqrt{2} \quad (\text{hyp} = \text{one leg} \cdot \sqrt{2})$$

$$PO = KN = 8, JK = PK = 6 \quad (45^\circ 45^\circ 90^\circ \Delta \text{ is isosceles.})$$

$$NM = JK = 6 \quad (PKON \text{ is a rectangle and opp sides are } \cong.)$$

$$JM = 6 + 8 + 6 = 20$$

The perimeter of the trapezoid is

$$8 + 6\sqrt{2} + 20 + 6\sqrt{2} = 28 + 12\sqrt{2}.$$

17 a $SR = \frac{1}{2}(VR)$

$$SR = \frac{1}{2}(6)$$

$$SR = 3$$

$$VS = (SR)(\sqrt{3})$$

$$VS = 3\sqrt{3}$$

b $ST = (VS)(\sqrt{3})$

$$ST = (3\sqrt{3})(\sqrt{3})$$

$$ST = 3(3)$$

$$ST = 9$$

c $VT = 2(VS)$

$$VT = 2(3\sqrt{3})$$

$$VT = 6\sqrt{3}$$

d $P \triangle VSR = SR + VS + VR$

$$= 3 + 3\sqrt{3} + 6$$

$$= 9 + 3\sqrt{3}$$

$$P \triangle VRT = TR + VR + VT$$

$$= (TS + SR) + VR + VT$$

$$= (9 + 3) + 6 + 6\sqrt{3}$$

$$= 12 + 6 + 6\sqrt{3}$$

$$= 18 + 6\sqrt{3}$$

$$\frac{\triangle VSR}{\triangle VRT} = \frac{9 + 3\sqrt{3}}{18 + 6\sqrt{3}} = \frac{1}{2} = 1:2$$

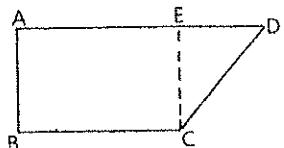
18 Perimeter = 24. Each side = 6. Δ s formed by diagonals are $30^\circ 60^\circ 90^\circ$ Δ s, so side opp 30° \angle is 3 and side opp 60° \angle is $3\sqrt{3}$. The diagonals are 6 and $6\sqrt{3}$.

19 Draw an alt \overline{CE} from the

base of the trapezoid to

the opp side forming a

$30^\circ 60^\circ 90^\circ$ Δ .



$$\text{If } EC = 8 \text{ then } DC = EC(2) \quad AD = AE + ED$$

$$ED = EC(\sqrt{3}) \quad DC = 8(2) \quad AD = 10 + 8\sqrt{3}$$

$$ED = 8\sqrt{3} \quad DC = 16$$

Perimeter of the trapezoid = $AD + DC + BC + AB$

$$P = (10 + 8\sqrt{3}) + 16 + 10 + 8$$

$$P = 44 + 8\sqrt{3} = 57.9$$

20 Draw an alt on the equilateral Δ , and form a $30^\circ 60^\circ 90^\circ$ Δ .

The hyp is 12 and the shorter leg is 6. The side opp the 60° \angle is $6\sqrt{3}$ (shorter leg $\cdot \sqrt{3}$).

$$\text{Alt} = 6\sqrt{3}, \text{Span} = 12\sqrt{3}$$

21 a All the edges = 6, so $P = 8 \times 6 = 48$.

b Since the corner Δ are $45^\circ 45^\circ 90^\circ$ Δ s the hyp is 6, each leg is $6 = x\sqrt{2}$.

$$x = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$$

$$x = 3\sqrt{2}$$

$$\text{The span is } 3\sqrt{2} + 6 + 3\sqrt{2} = 6 + 6\sqrt{2}.$$

22 Drawing an alt \overline{AX} to the base of isos ΔABC forms 2 $\cong \Delta$ s which are $30^\circ 60^\circ 90^\circ$ Δ s. (Vertex \angle of ΔABC is 120° and base \angle s are 30° .)

$$AX = \frac{BX}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 3\sqrt{3}$$

23 Dist from origin to E is $d^2 = 2^2 + 4^2$

$$d = \sqrt{4 + 16}$$

$$d = \sqrt{20} = 2\sqrt{5}$$

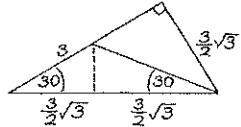
So the new coordinates of E are $(0, 2\sqrt{5})$.

24 Since the Δ is a $30^\circ 60^\circ 90^\circ$, the \angle

bisected is 60° , and each smaller \angle is 30° . Since the 2 \angle s of the lower Δ are \cong , the sides are

$\cong (3)$. If the hyp of the smaller rt Δ is 3, the leg opp the 30° \angle is $\frac{3}{2}$ and $y = \frac{3}{2}\sqrt{3}$. If the shorter leg of the whole Δ (y) is $\frac{3}{2}\sqrt{3}$, then $x = 2(\frac{3}{2}\sqrt{3})$ (x is hyp of larger Δ)

$$x = \frac{6}{2}\sqrt{3} = 3\sqrt{3}$$



25 Draw an alt \overline{DX} to \overline{AB}

forming a $30^\circ 60^\circ 90^\circ \Delta$.

$$AX = \frac{1}{2}(DA)$$

$$AX = \frac{1}{2}(4)$$

$$AX = 2$$

$$\text{So } XB = AB - AX$$

$$XB = 4 - 2$$

$$XB = 2$$

Draw an alt $\overline{EZ} \parallel \overline{DX}$ so $DZ = 2$ because opp sides of a \square are \cong . $DX = AX(\sqrt{3})$

$DX = 2\sqrt{3}$ and $\overline{ZB} \cong \overline{DX}$ because opp sides of a \square are \cong .

So if $ZB = 2\sqrt{3}$, then $ZC = 2\sqrt{3}$ because ΔZBC is a

$45^\circ 45^\circ 90^\circ$ isos rt Δ .

a $DC = DZ + ZC$

b $BC = ZC(\sqrt{2})$

$$DC = 2 + 2\sqrt{3}$$

$$BC = (2\sqrt{3})(\sqrt{2})$$

$$BC = 2\sqrt{6}$$

26 C is at (16, 5), EC = 20, BC = 5.

$$\text{So } A_{\square} = 100$$

$$A_{\Delta} = \frac{1}{2}(CD)(BC) = \frac{1}{2}(CD)(5)$$

$$\text{So } 100 = 8(\frac{1}{2})(5)(CD)$$

$$100 = (20)CD; 5 = CD$$

So D is at (11, 5).

$$\text{Then } d = \sqrt{11^2 + 5^2} = \sqrt{146} \approx 12.08$$

27 Draw $\overline{EF} \perp \overline{BC}$

ΔCEF is a $30^\circ 60^\circ 90^\circ \Delta$

If EF is x , CF is $x\sqrt{3}$,

CE is $2x$, BF is $12 - x\sqrt{3}$.

$\Delta BEF \sim \Delta BAC$ so

$$\frac{EF}{AC} = \frac{BF}{BC}$$

$$\frac{x}{5} = \frac{12 - x\sqrt{3}}{12}$$

$$12x = 60 - 5x\sqrt{3}$$

$$12x + 5x\sqrt{3} = 60$$

$$x(12 + 5\sqrt{3}) = 60$$

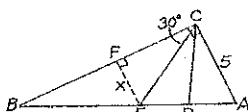
$$x = \frac{60}{12 + 5\sqrt{3}}$$

$$x = \frac{60}{12 + 5\sqrt{3}} \cdot \frac{12 - 5\sqrt{3}}{12 - 5\sqrt{3}}$$

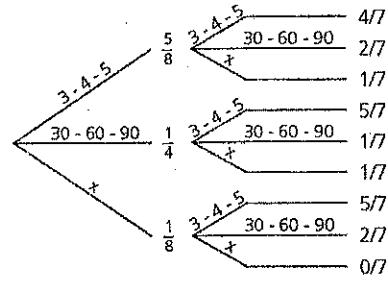
$$x = \frac{60(12 - 5\sqrt{3})}{144 - 75}$$

$$x = \frac{20(12 - 5\sqrt{3})}{23}$$

$$CE = 2x, CE = \frac{40(12 - 5\sqrt{3})}{23}$$



28 a $\frac{5}{8} \cdot \frac{4}{7} = \frac{5}{14}$



b $(\frac{5}{8} \cdot \frac{4}{7}) + (\frac{5}{8} \cdot \frac{2}{7}) + (\frac{5}{8} \cdot \frac{1}{7}) + (\frac{2}{8} \cdot \frac{5}{7}) + (\frac{1}{8} \cdot \frac{5}{7})$

$$\frac{10}{28} + \frac{5}{28} + \frac{5}{56} + \frac{5}{56} = \frac{20}{28} + \frac{10}{56} = \frac{20}{28} + \frac{5}{28} = \frac{25}{28}$$

c $(\frac{5}{8} \cdot \frac{2}{7}) + (\frac{2}{8} \cdot \frac{5}{7}) = \frac{5}{28} + \frac{5}{28} = \frac{10}{28} = \frac{5}{14}$

Pages 414-417 (Section 9.8)

1 a $(BY)^2 + (OB)^2 = (YO)^2$ b $(EY)^2 + (YO)^2 = (EO)^2$

$$9 + 16 = (YO)^2$$

$$144 + 25 = (EO)^2$$

$$25 = (YO)^2$$

$$169 = (EO)^2$$

$$YO = 5$$

$$EO = 13$$

2 $3^2 + 5^2 = x^2$

$$9 + 25 = x^2$$

$$34 = x^2 \quad x = \sqrt{34}$$

$$4^2 + (\sqrt{34})^2 = d^2$$

$$16 + 34 = d^2$$

$$50 = d^2, d = \sqrt{50}; \text{ diagonal} = 5\sqrt{2}$$

3 From Sample Problem 2, the slant height of a regular pyramid is the \perp bis of \overline{EB} and $FG = \frac{1}{2}BC$. $FG = 5$ and ΔAGF is $30^\circ 60^\circ 90^\circ \Delta$, so $AG = 5\sqrt{3}$ and $AF = 10$.

4 a Draw DB, ΔADB a rt Δ ΔHDB is a rt Δ , $HD = 8$

$$DA = 9 \text{ and } AB = 12 \quad (HD)^2 + (DB)^2 = (HB)^2$$

$$(DA)^2 + (AB)^2 = (DB)^2 \quad (8)^2 + (15)^2 = (HB)^2$$

$$9^2 + 12^2 = (DB)^2 \quad 64 + 225 = (HB)^2$$

$$81 + 144 = (DB)^2 \quad 289 = (HB)^2$$

$$225 = (DB)^2 \quad HB = 17$$

$$15 = DB$$

b Because the dimensions are all the same, $AG = 17$.

5 a $AD = 14$ (Base is a square.)

b $YR = 7$ ($YR = \frac{1}{2}ID$)

d AMID is a square.

$$P = 4(14) = 56$$

c $(PY)^2 + (YR)^2 = (PR)^2$

$$24^2 + 7^2 = (PR)^2$$

$$576 + 49 = (PR)^2$$

e The diagonal of

a square ($s = 14$)

is $14\sqrt{2}$.

$$625 = (PR)^2$$

$$25 = PR$$

20 Let the sides be $3x$, $4x$ and $5x$.

$$\text{From #19, } HB = \sqrt{a^2 + b^2 + c^2}$$

$$200\sqrt{2} = \sqrt{(3x)^2 + (4x)^2 + (5x)^2}$$

$$200\sqrt{2} = \sqrt{9x^2 + 16x^2 + 25x^2}$$

$$200\sqrt{2} = \sqrt{50x^2}$$

$$200\sqrt{2} = 5x\sqrt{2} \quad 3x = 120$$

$$200 = 5x$$

$$4x = 160$$

$$x = 40$$

$$5x = 200$$

21 The problem is impossible because if the face diagonals are 2, 3, and 6, they should form a Δ . But a Δ cannot be formed--- $2 + 3 < 6$.

22 Since $\triangle PDA$ is equilateral with sides 6 cm, $DR = 3$.

$$\text{If } DR = 3, PR = 3\sqrt{3} \text{ (slant height)}$$

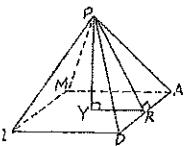
$$\text{In } \triangle PYR, PR = 3\sqrt{3} \text{ and } YR = 3.$$

$$PY^2 + 3^2 = (3\sqrt{3})^2$$

$$PY^2 + 9 = 27$$

$$PY = \sqrt{18} = 3\sqrt{2}$$

(altitude)



23 1 In $\triangle AFB$, $x^2 + y^2 = a^2$

2 In $\triangle CEB$, $x^2 + 4y^2 = b^2$

3 In $\triangle CDB$, $a^2 + b^2 = 9y^2$

From (1) and (2), $a^2 + b^2 = 2x^2 + 5y^2$

Substituting, $9y^2 = 2x^2 + 5y^2$

$$4y^2 = 2x^2$$

$$2y^2 = x^2$$

Substituting in 1, $2y^2 + y^2 = a^2$

$$3y^2 = a^2$$

Substituting in 2, $2y^2 + 4y^2 = b^2$

$$6y^2 = b^2$$

$$\frac{b^2}{a^2} = \frac{6y^2}{3y^2} = \frac{2}{1}, \frac{b}{a} = \frac{\sqrt{2}}{1}$$

Pages 420-422 (Section 9.9)

1 a $\frac{8}{17}$ b $\frac{15}{17}$ c $\frac{8}{15}$ d $\frac{15}{17}$ e $\frac{8}{17}$ f $\frac{15}{8}$ 2 a $\frac{1}{2}$ b $\frac{\sqrt{3}}{2}$ c $\frac{\sqrt{3}}{3}$ d $\frac{\sqrt{3}}{2}$

e $\frac{1}{2}$ f $\sqrt{3}$ 3 a $\frac{\sqrt{2}}{2}$ b $\frac{\sqrt{2}}{2}$ c 1 4 a $\frac{39}{89}$ b $\frac{39}{80}$ 5 $\frac{4}{5}$

6 a $\tan \angle A$ b $\cos \angle A$ c $\sin \angle A$ 7 a $2\sqrt{6}$

b $\frac{2\sqrt{6}}{7}$ c $\frac{5\sqrt{6}}{2\sqrt{6} \cdot \sqrt{6}} = \frac{5\sqrt{6}}{12}$

8 a $\frac{12}{13}$ b $RE = 10$ so $\cos \angle RET = \frac{5}{13}$

9 a Draw alt from C to base. In an isos Δ , the alt bis base, so $\cos \angle A = \frac{7}{25}$ b $DG = 8$ (3, 4, 5 family), so $\sin \angle E = \frac{8}{17}$.

c $\sin \angle DFG = \frac{8}{10} = \frac{4}{5}$

10 $\sin 40^\circ = \frac{h}{200}$

$$.6428 = \frac{h}{200}$$

$$200(.6428) = h$$

$$129 \text{ m} = h$$

11 a $\tan = \frac{\text{opp leg}}{\text{adj leg}}$, so $\tan = \frac{1}{1}, \frac{2}{1}$ etc. The Δ is an isos rt Δ so

$$m\angle A = 45^\circ$$

b $\sin = \frac{\text{opp leg}}{\text{hyp}}$. If $\sin \angle P = \frac{1}{2}$, the Δ is a $30^\circ 60^\circ 90^\circ \Delta$,
 $m\angle P = 30^\circ$.

12 $\sin \angle P = \frac{3}{5}$, $PQ = 10$. Since 10 is the hypotenuse, $\frac{3}{5} = \frac{6}{10}$ and

$\triangle PQR$ is a member of the 3-4-5 family, $PR = 8$.

$$\cos \angle P = \frac{8}{10} = \frac{4}{5}$$

13 Since the Δ s are \sim , the sides are proportional.

a $\frac{DC}{DB} = \frac{AD}{CD}$ b $6^2 + 9^2 = BC^2$

$$\frac{6}{9} = \frac{AD}{6} \quad 36 + 81 = BC^2$$

$$36 = 9AD \quad \sqrt{117} = BC$$

$$4 = AD \quad 3\sqrt{13} = BC$$

$$AB = 4 + 9 = 13$$

$$\tan \angle ACD = \frac{AD}{CD} \quad \sin \angle A = \frac{BC}{AB}$$

$$= \frac{4}{6} \quad = \frac{3\sqrt{13}}{13}$$

14 Diagonals of a rhombus are \perp bis of each other,

$$BO = RB = 9, MB = HB = 12$$

a $\triangle BRM$ is a rt Δ .

$$9^2 + 12^2 = RM^2$$

b In $\triangle BHO$

$$\tan \angle BHO = \frac{9}{12} = \frac{3}{4}$$

$$81 + 144 = RM^2$$

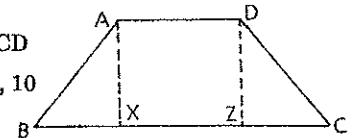
$$\sqrt{225} = RM$$

$$15 = RM$$

$$\cos \angle BRM = \frac{9}{15} = \frac{3}{5}$$

15 Given: Trapezoid ABCD

sides 5, 10, 17, 10



We know trap ABCD is an isos trap because the legs are \cong . Draw alt AX and DZ.

$$BX \cong ZC \text{ and } XZ = 5 \text{ then}$$

$$BX + ZC = BC - XZ$$

Now find AX.

$$BX + ZC = 12$$

$$AB = 10 = 5(2)$$

$$BX = 6, ZC = 6$$

$$BX = 6 = 3(2) \text{ so}$$

$$AX = 4(2) = 8$$

(3, 4, 5 is a Pythagorean triple)

$$\sin \angle B = \frac{AX}{AB} = \frac{8}{10} = \frac{4}{5}$$

$$9 \quad \cos \angle x = \frac{4}{5} = .8$$

$$\angle x = 37^\circ$$

$$\cos \angle x = \frac{3}{5} = .6$$

$$\angle x = 53^\circ$$

(3, 4, 5) is a Pythagorean triple
making the \triangle a rt \triangle ,
so $m\angle y = 90$.

$$11 \text{ a } \cos \angle B = \frac{7}{18}$$

$$\cos \angle B = .3888$$

$$\angle B \approx 67^\circ$$

$$\text{b } 7^2 + AD^2 = 18^2$$

$$AD = \sqrt{324 - 49}$$

$$AD = \sqrt{275} = 5\sqrt{11}$$

$$12 \text{ In a rhombus the diags are } \perp \text{ bis of each other, so}$$

$$\sin \angle BAC = \frac{BE}{AB}$$

$$\sin 27^\circ = \frac{BE}{6.2}$$

$$.4540 = \frac{BE}{6.2}$$

$$BE = 2.8148$$

$$2(BE) = 2(2.8148)$$

$$BD \approx 5.6$$

$$\cos \angle BAC = \frac{AE}{AB}$$

$$6.2(\cos 27^\circ) = AE$$

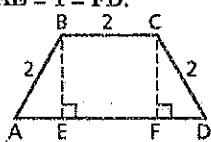
$$6.2(.8910) = AE$$

$$AE = 5.5242$$

$$AC = 2(5.5242) \approx 11.0$$

$$13 \text{ Trapezoid ABCD is isos. If } \cos \angle A = \frac{1}{2}, AE = 1 = FD.$$

$$\text{Perimeter} = 2 + 2 + 2 + (1 + 2 + 1) = 10$$



14 PENTA is a reg pentagon, so each side is 10 cm.

An apothem is the \perp bis of a side, so AY is 5.

$$\angle AXP \text{ is } 72^\circ \left(\frac{360}{n} \right), \text{ so } \angle YXA \text{ is } 36^\circ.$$

$$\tan 36^\circ = \frac{5}{XY}$$

$$.7265 = \frac{5}{XY}$$

$$XY = 6.88$$

$$15 \quad \tan 30^\circ = \frac{x}{100}$$

$$.5774 = \frac{x}{100}$$

$$x = 57.74$$

$$\tan 25^\circ = \frac{y}{100}$$

$$.4663 = \frac{y}{100}$$

$$y = 46.63$$

$$RB = 57.74 + 46.63 = 104.37$$

$$RB \approx 104 \text{ dm}$$

$$16 \quad \tan 47^\circ = \frac{1,000}{x}$$

$$x = \frac{1,000}{\tan 47^\circ}$$

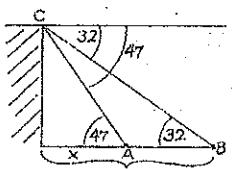
$$x \approx 932.48$$

$$\tan 32^\circ = \frac{1,000}{y}$$

$$y = \frac{1,000}{\tan 32^\circ}$$

$$y \approx 1,600.26$$

$$1,600.26 - 932.48 \approx 668 \text{ dm}$$



$y - x$ = distance between
ships.

$$17 \text{ a } FT = 10, FT^2 = 35^2 + 10^2$$

$$PT = \sqrt{1,225 + 100}$$

$$PT = \sqrt{1,325} \approx 36.4$$

$$\text{b } PB^2 = 36.4^2 + 10^2$$

$$PB = \sqrt{1,325 + 100}$$

$$PB \approx 37.75$$

$$\text{c } \tan \angle PTF = \frac{35}{10}$$

$$\angle PTF \approx 74^\circ$$

$$\text{d } \sin \angle PBF = \frac{35}{37.75}$$

$$\sin \angle PBF = .9271$$

$$\angle PBF \approx 68^\circ$$

$$18 \quad AC = 1, CB = x$$

$$\tan 30^\circ = \frac{PB}{x+1}$$

$$.5774 = \frac{PB}{x+1}$$

$$.5774x + .5774 = PB$$

$$\tan 35^\circ = \frac{PB}{x}$$

$$.7002 = \frac{PB}{x}$$

$$.7002x = PB$$

Substituting,

$$.5774x + .5774 = .7002x \quad PB = .7002x$$

$$.5774 = .1228x \quad PB = (.7002)(4.7)$$

$$4.7 = x \quad PB \approx 3.29 \text{ km}$$

$$19 \text{ a } \text{slope } h = \frac{4-0}{11-5} = \frac{2}{6} = \frac{1}{3}$$

$$\text{b } \tan \angle 1 = \frac{4}{6} = \frac{2}{3}$$

$$\tan \angle 1 \approx .6667$$

By trig table, $\angle 1 \approx 34^\circ$

$$20 \text{ Let: } \overline{DC} = x$$

$$\overline{AD} = b - x$$

$$\overline{BD} = y$$

So: In $\triangle BDA$

$$y^2 + (b-x)^2 = c^2$$

$$y^2 = c^2 - (b-x)^2$$

$$c^2 - (b-x)^2 = a^2 - x^2$$

$$c^2 - (b^2 - 2bx + x^2) = a^2 - x^2$$

$$c^2 - b^2 + 2bx - x^2 = a^2 - x^2$$

$$c^2 - b^2 + 2bx = a^2$$

$$c^2 = b^2 - 2bx + a^2$$

But $\cos \angle C = \frac{x}{a}$, $x = a \cos C$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$21 \text{ a } \sin 70^\circ = \frac{x}{10}$$

$$(.9397)(10) = x$$

$$9.397 = x$$

$$\sin \angle R = \frac{9.397}{12} = .7831$$

$$\angle R \approx 52^\circ$$

- b In $\triangle APSQ$, $QS \approx 3.4$ ($PS^2 + QS^2 = PQ^2$)
 In $\triangle APSR$, $SR \approx 7.5$ ($PS^2 + SR^2 = PR^2$)
 $QR = QS + SR$
 $QR \approx 3.4 + 7.5 \approx 11$
- c If $\frac{PR}{\sin \angle Q} = \frac{PQ}{\sin \angle R}$, $.9397 \cdot \frac{12}{.9397} = \frac{10}{.9397}$ and $\frac{12}{.9397} = \frac{12}{.9397}$
- d $\sin \angle Q = \frac{x}{PQ}$ $\sin \angle R = \frac{x}{PR}$
 $(PQ)(\sin \angle Q) = x$, $(PR)(\sin \angle R) = x$.
 Then $(PQ)(\sin \angle Q) = (PR)(\sin \angle R)$
 and $\frac{PR}{\sin \angle Q} = \frac{PQ}{\sin \angle R}$

Pages 429-433 Chapter 9 Review Problems

1 a $\frac{HG}{EG} = \frac{EG}{FG}$ b $\frac{GH}{EH} = \frac{EH}{HF}$
 $\frac{4}{6} = \frac{6}{x}$ $\frac{4}{x} = \frac{x}{16}$
 $4x = 36$ $x^2 = 64$
 $x = 9$ $x = 8$

c $\frac{GF}{EF} = \frac{EF}{FH}$ d $EH^2 + EF^2 = HF^2$
 $\frac{4}{2\sqrt{5}} = \frac{2\sqrt{5}}{x}$ $2^2 + 3^2 = HF^2$
 $4x = 20$ $13 = HF^2$
 $x = 5$ $\sqrt{13} = HF$

- 2 a $30^\circ 60^\circ 90^\circ$ b 3-4-5 c 5-12-13 d 8-15-17 e $45^\circ 45^\circ 90^\circ$
 3 a $30^\circ 60^\circ 90^\circ$ family) b $5, 5\sqrt{3}$ ($30^\circ 60^\circ 90^\circ$ Δ)
 c 7 (7, 24, 25 family) d 15 (3-4-5 family) e $4\sqrt{5}$ (3, 4, 5 family)
 f 9 (9-40-41 family) g $5\sqrt{3}, 10\sqrt{3}$ ($30^\circ 60^\circ 90^\circ$ Δ)
 h $\frac{25}{2}$ (7-24-25 family) i 26 (5-12-13 family)
 j $4\sqrt{2}, 4\sqrt{2}$ ($45^\circ 45^\circ 90^\circ$ Δ)
 4 $6^2 + 8^2 = AB^2$, $100 = AB^2$, $AB = 10$, perimeter = 40

- 5 The Δ shown is equilateral



so each \angle is 60° . The alt
forms a $30^\circ 60^\circ 90^\circ$ Δ and is

opp the $60^\circ \angle$. (The side opp the $60^\circ \angle$ is $\sqrt{3}$ times the shorter leg.) Shorter leg is 3. Alt is $3\sqrt{3}$.

6 $3^2 + 4^2 = d^2$ 7 $24^2 + x^2 = 25^2$
 $9 + 16 = d^2$ $576 + x^2 = 625$
 $25 = d^2$ $x^2 = 49$
 $d = 5$ km $x = \sqrt{49} = 7$
 The foot of the ladder is 7 feet from the wall.

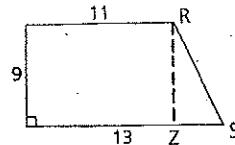
- 8 The altitude to the base of an isos Δ bis the base and forms 2 rt Δ s.
 So $8^2 = 3^2 + (\text{alt})^2$
 $64 = 9 + (\text{alt})^2$
 $\sqrt{64 - 9} = \text{alt}$
 $\sqrt{55} = \text{alt}$
 $7.4 \approx \text{alt}$

- 9 If the alt of the equilateral Δ is $8\sqrt{3}$, the shorter leg is 8 and the hyp (side) is 16. The perimeter is 48.

10 $2^2 + 5^2 = \text{diagonal}^2$
 $4 + 25 = \text{diagonal}^2$
 $29 = \text{diagonal}^2$
 $\sqrt{29} = \text{diagonal}$

- 11 Draw alt RZ.

$RZ \approx 9$, $ZS = 2$.
 $(RS)^2 = (ZS)^2 + (RZ)^2$
 $(RS)^2 = 2^2 + 9^2$
 $RS = 4 + 81$
 $RS = \sqrt{85}$



- 12 The side opp the $60^\circ \angle$ is $\sqrt{3}$ times the shorter leg. $VZ = 2$

$2 = (\sqrt{3})(TZ)$
 $\frac{2}{\sqrt{3}} = TZ$
 $\frac{2\sqrt{3}}{3} = TZ$
 $TV = 2\left(\frac{2\sqrt{3}}{3}\right)$
 $TV = \frac{4\sqrt{3}}{3}$

- 13 Find HC.

$$(HC)^2 = (HB)^2 + (BC)^2$$

$$(HC)^2 = 3^2 + 12^2$$

$$(HC)^2 = 9 + 144$$

$$HC = \sqrt{153}$$

$$HC = 3\sqrt{17}$$

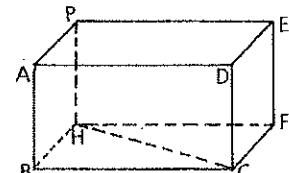
Find PC, the diagonal.

$$(PC)^2 = (PH)^2 + (HC)^2$$

$$(PC)^2 = 4^2 + (3\sqrt{17})^2$$

$$(PC)^2 = 16 + 9(17)$$

$$PC = \sqrt{16 + 153} = \sqrt{169} = 13$$



14 If $\overline{PR} = 20$ and $\overline{PS} = 25$, then draw \overline{RS} and

$$20^2 + RS^2 = 25^2$$

$$400 + RS^2 = 625$$

$$RS^2 = 225$$

$$RS = \sqrt{225} = 15$$

$RS = \frac{1}{2}MK$ so $MK = 30$ Perimeter = $4(30) = 120$

15 In $\triangle HEG$, draw \overline{EG} . In $\triangle AEG$,

$$(EG)^2 = (HE)^2 + (HG)^2 \quad (AG)^2 = (AE)^2 + (EG)^2$$

$$(EG)^2 = 4^2 + 12^2 \quad (AG)^2 = 7^2 + (4\sqrt{10})^2$$

$$EG = \sqrt{16 + 144}$$

$$(AG)^2 = 49 + 160$$

$$EG = \sqrt{160} = 4\sqrt{10}$$

$$AG = \sqrt{209} \approx 14.5$$

16 a $\triangle ABC$ is a member of the 8-15-17 family, so $CB = 8$.

b $\frac{AD}{DE} = \frac{AC}{CB}$ d $AB - AE = EB$

$$\frac{AD}{4} = \frac{15}{8}$$

$$17 - 8\frac{1}{2} = EB$$

$$8(AD) = 60$$

$$8\frac{1}{2} = EB$$

$$AD = 7\frac{1}{2}$$

c $\frac{AE}{DE} = \frac{AB}{CB}$ e $AC - AD = DC$

$$\frac{AE}{4} = \frac{17}{8}$$

$$15 - 7\frac{1}{2} = DC$$

$$8(AE) = 68$$

$$7\frac{1}{2} = DC$$

$$AE = 8\frac{1}{2}$$

17 $AB = \sqrt{(4-1)^2 + (15-11)^2}$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{25} = 5$$

18 $m\angle M = \frac{1}{2}(90 - 40) = \frac{1}{2}(50) = 25$

19 a $m\widehat{AD} = 180 - m\widehat{BE} - m\widehat{DE}$

$$= 180 - 30 - 30 = 70$$

$$m\widehat{AF} = 360 - m\widehat{BF} - m\widehat{BE} - m\widehat{DE} - m\widehat{AD}$$

$$= 360 - 60 - 80 - 30 - 70 = 120$$

b $m\angle C = \frac{1}{2}(m\widehat{AF} - m\widehat{DE})$

$$= \frac{1}{2}(120 - 30) = \frac{1}{2}(90) = 45$$

c $m\angle BAD = \frac{1}{2}(m\widehat{BD})$

$$= \frac{1}{2}(110)^\circ = 55^\circ$$

20 a $m\widehat{RTC} = 2m\angle REC = 2(90) = 180$

b $\ell \widehat{RTC} = \frac{180}{360}(10\pi) = 5\pi$

c $A_c = \pi(5)^2 = 25\pi \approx 78.5$

$$A_R = 8 \cdot 6 = 48$$

$$A_C - A_R \approx 78.5 - 48 \approx 30.5$$

21 a $m\angle DEF = \frac{1}{2}(m\widehat{DF}) = \frac{1}{2}(180) = 90$

b $m\widehat{DEF} = 180$

c $r = 10$

$$\ell \widehat{DEF} = \frac{180}{360}(20\pi) = 10\pi$$

22 a $m\widehat{BC} = 2(m\angle CAB) = 2(30) = 60$

b $m\widehat{AC} = 180 - m\widehat{BC} = 180 - 60 = 120$

c $\ell \widehat{BC} = \frac{60}{360}(12\pi) = 2\pi \approx 6.28$

d $A = \frac{60}{360}(\pi(6)^2) = 6\pi \approx 18.85$

23 The boats traveled for 3 hours each, so Boat A traveled 60 km, Boat B 45 km. $60 = (15)(4)$, $45 = (15)(3)$ and the distance between is the hypotenuse of a rt \triangle in the 3-4-5 family. The hypotenuse is $(5)(15)$ or 75 km.

24 a $\frac{8}{10} = \frac{10}{8+x}$

b $\frac{y}{6} = \frac{6}{y+9}$

$$100 = 64 + 8x$$

$$36 = y^2 + 9y$$

$$36 = 8x$$

$$y^2 + 9y - 36 = 0$$

$$4\frac{1}{2} = x$$

$$(y+12)(y-3) = 0$$

$$y = -12 \text{ or } y = 3$$

Length cannot be negative.

$$y = 3$$

25 To swim directly:

$$1^2 + 3^2 = x^2$$

$$1 + 9 = x^2$$

$$\sqrt{10} = x$$

$$\sqrt{10} \text{ miles at 2 mph}$$

$$\text{would be } \frac{\sqrt{10}}{2} \text{ or } \frac{3}{4} \text{ hr. So } \frac{1}{2} + \frac{3}{4} = 1\frac{1}{4} \text{ hr.}$$

approximately $1\frac{1}{2}$ hrs.

It is quicker to swim across and then walk.

26 Assume that the dock is \perp to the water. Two rt \triangle s are formed,

$$15^2 + y^2 = 25^2 \quad 15^2 + z^2 = 17^2$$

$$225 + y^2 = 625 \quad 225 + z^2 = 289$$

$$y^2 = 400$$

$$z^2 = 64$$

$$y = 20$$

$$z = 8$$

$$y - z = x$$

$$20 - 8 = x$$

$$12 = x$$

27 (5, 12, 13) is a Pythagorean triple, so $z = 5$. (3, 4, 5) is a Pythagorean triple, so $y = 4$. $8\frac{1}{2} = \frac{17}{2}$, $4 = \frac{8}{2}$ and (8, 15, 17) is a Pythagorean triple, so $x = \frac{16}{2}$ or 7.5.

