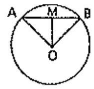
Selected Answers

Unit 8: Circles and Conics

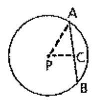
P29

3 Given: $\odot O$
$\overline{OM}$ is a median.
Concl: $\overline{OM}$ an alt of $\triangle BOA$



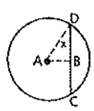
1 $\odot O$	1 Given
2 $\overline{OM}$ is a median.	2 Given
3 $\overline{OM}$ bis $\overline{AB}$.	3 The median bis one side ($\overline{OM}$ is part of a radius).
4 $\overline{OM} \perp \overline{AB}$	4 Radius that bis chord (not diameter) $\perp$ to chord.
5 $\overline{OM}$ an alt of $\triangle BOA$	5 An alt is a seg from the vertex of a $\triangle$ $\perp$ to the opp side.

5 Draw radius $\overline{PA}$ and segment $\overline{PC} \perp$ to $\overline{AB}$. $\overline{PC}$ is $\perp$ to chord $\overline{AB}$ and bis $\overline{AB}$—AC is 6. $\triangle ACP$ is rt $\triangle$ and $\overline{PA}$ is 10—Use Pythagorean Theorem.



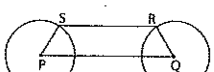
$6^2 + PC^2 = 10^2$
$36 + PC^2 = 100$
$PC^2 = 64$
$PC = \sqrt{64} = 8$ mm

6 AB = 15, AD = 17
$\overline{AB} \perp \overline{CD}$
$\triangle ABD$ is a rt $\triangle$.
Use Pythagorean Theorem.



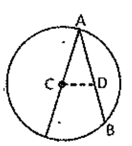
$x^2 + 15^2 = 17^2$
$x^2 + 225 = 289$
$x^2 = 64$
$x = 8 \quad BD = 8$
A radius $\perp$ to chord bis chord, so CD = 16.

7 Given: PQRS is an isos trapezoid with $\overline{SR} \parallel \overline{PQ}$.
Concl: $\odot P \cong \odot Q$



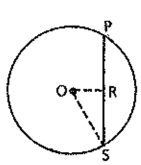
1 PQRS isos trapezoid	1 Given
2 $\overline{SR} \parallel \overline{PQ}$	2 Given
3 $\overline{SP} \cong \overline{RQ}$	3 Legs of isos trapezoid $\cong$.
4 $\odot P \cong \odot Q$	4 If radii of $\odot$s $\cong$, then $\odot$s $\cong$.

11 Draw $\overline{CD} \perp$ to $\overline{AB}$, $\overline{CD}$ bis $\overline{AB}$.
AD = 15. Radius AC = 17.
$\triangle ADC$ is a rt $\triangle$.
Use Pythagorean Theorem.



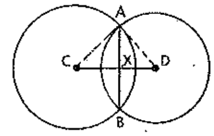
$CD^2 + 15^2 = 17^2$
$CD^2 + 225 = 289$
$CD^2 = 64$
$CD = \sqrt{64} = 8$ m

12 $\overline{RO} \perp \overline{PS}$.
A radius $\perp$ to chord bis chord.
So PR = RS = 12
$\triangle ROS$ is a rt $\triangle$.
Use Pythagorean Theorem.



$OS^2 = 9^2 + 12^2$
$OS^2 = 81 + 144$
$OS = \sqrt{225} = 15$ cm

14 AB = 24 cm, CD = 21 cm
CA = 13. $\overline{CD} \perp \overline{AB}$
$\overline{CX} \perp \overline{AB}$ and bis $\overline{AB}$.
$\overline{CX}$ bis $\overline{AB}$. AX = 12.



$AX^2 + CX^2 = CA^2$
$12^2 + CX^2 = 13^2$
$144 + CX^2 = 169$
$CX^2 = 25$
$CX = \sqrt{25} = 5$

$CD - CX = DX$
$21 - 5 = 16$
$DX = 16$

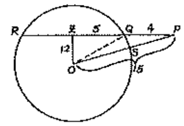
$AD^2 = DX^2 + AX^2$
$AD^2 = 16^2 + 12^2$
$AD^2 = 256 + 144$
$AD = \sqrt{400} = 20$ cm

17 a 13

b $PQ = \sqrt{(19-15)^2 + (16-13)^2}$
$PQ = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

c $(QB)^2 = (PB)^2 - (PQ)^2$
$(QB)^2 = 13^2 - 5^2$
$QB = 12$
$AB = 2QB = 24$

23 $\overline{OT}$ is $\perp$ bis of $\overline{RQ}$.
TQ = 5, TP = 9



$TP^2 + TO^2 = PO^2$
$9^2 + TO^2 = 15^2$
$81 + TO^2 = 225$
$TO = \sqrt{144} = 12$

Draw radius $\overline{QO}$
$QO^2 = TQ^2 + TO^2$
$QO^2 = 5^2 + 12^2$
$QO^2 = 25 + 144$
$QO = \sqrt{169} = 13$

$\overline{QO}$ and $\overline{OS}$ are radii.
$QO = OS = 13$
$PS = PO - OS$
$PS = 15 - 13 = 2$

P30

2 Since $\overline{AB} \cong \overline{CD}$,

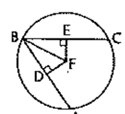
$6x + 14 = 4 - 4x$
$10x + 14 = 4$
$10x = -10$
$x = -1$

$AB = 6x + 14$
$AB = 6(-1) + 14$
$AB = -6 + 14$
$AB = 8$

6 **a** The chord is bisected so a rt $\triangle$ is formed with hypotenuse (radius) 10, side 6, and the distance x from center to fly.
$10^2 = 6^2 + x^2$
$100 = 36 + x^2$
$64 = x^2$
$8 \text{ cm} = x$

b Circle

9 Given: $\odot F$, $\overline{FE} \perp \overline{BC}$
$\overline{FD} \perp \overline{AB}$
$\overline{BF}$ bis $\angle ABC$.
Prove: $\overline{BC} \cong \overline{BA}$



1 $\odot F$, $\overline{FE} \perp \overline{BC}$, $\overline{FD} \perp \overline{AB}$	1 Given
2 $\overline{BF}$ bis $\angle ABC$.	2 Given
3 $\angle BEF$ rt $\angle$	3 $\perp$ lines form rt $\angle$s.
4 $\angle FDB$ rt $\angle$	4 Same as 3
5 $\angle BEF \cong \angle FDB$	5 Rt $\angle$s are $\cong$.
6 $\angle EBF \cong \angle DBF$	6 Bis divides $\angle$ into 2 $\cong$ $\angle$s.
7 $\overline{BF} \cong \overline{BF}$	7 Reflexive prop.

continued above...

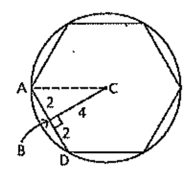
8 $\triangle FBE \cong \triangle FBD$	8 AAS
9 $\overline{EF} \cong \overline{DF}$	9 CPCTC
10 $\overline{BC} \cong \overline{BA}$	10 Chords = dist from center of $\odot$ are $\cong$.

11 The $\perp$ bis of a chord passes through the center of a $\odot$, so $\overline{PB} \cong \overline{PA}$, $\overline{CQ} \cong \overline{QD}$. Chords = dist from the center are $\cong$, so $\overline{AB} \cong \overline{CD}$.

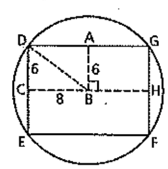
a
$2(3x - 17) = 15 - x$
$6x - 34 = 15 - x$
$7x = 49$
$x = 7$
$AB = 2(3x - 17)$
$AB = 8$

b
$OP^2 + BP^2 = OB^2$
$3^2 + 4^2 = r^2$
$9 + 16 = r^2$
$25 = r^2$
Radius $= 5$

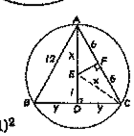
12 $S_{HEX} = \frac{24}{6} = 4$
$\therefore r \odot = 4$
$BC \perp$ bis AD, so AB = 2
$BC^2 = 4^2 - 2^2$
$BC = \sqrt{12} = 2\sqrt{3}$



13 Draw $\overline{AB}$, $\overline{CB}$, $\overline{DB}$, $\overline{CH}$.
$\overline{CB} \perp$ bis $\overline{DE}$, so $\overline{DC} = 6$
$\overline{AB} \perp$ bis $\overline{CH}$, so $\overline{CB} = 8$
$DB^2 = 6^2 + 8^2$
$DB = 10$



15 In $\triangle EDC$, $1 + y^2 = x^2$
$y^2 = x^2 - 1$
In $\triangle ADB$, $y^2 + (x+1)^2 = 12$
$y^2 = 12^2 - (x+1)^2$



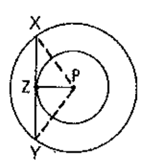
$x^2 - 1 = 144 - x^2 - 2x - 1$
$2x^2 + 2x - 144 = 0$
$x^2 + x - 72 = 0$
$(x+9)(x-8) = 0$
$x = -9, 8 \quad x \neq -9$
Radius $= 8$

$y^2 = x^2 - 1$
$y^2 = 64 - 1$
$y^2 = 63$
$y = \sqrt{63} = 3\sqrt{7}$
$P = 2(12) + 2(3\sqrt{7})$
$P = 24 + 6\sqrt{7}$

P31

2 Draw $\overline{PX}$ and $\overline{PY}$. PX = 10, PZ = 8. $\triangle PZX$ is a (3-4-5) family $\triangle$, so XZ = 6 (6-8-10) family $\triangle$.
XZ = ZY, XY = 12 or

$(PZ)^2 + (XZ)^2 = (XP)^2$
$8^2 + (XZ)^2 = 10^2$
$64 + (XZ)^2 = 100$
$(XZ)^2 = 36$
$XZ = 6$
$XY = 12$



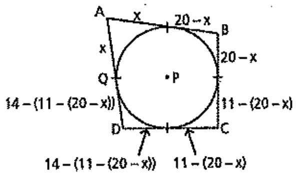
5 **a** $\overline{PO}$ and $\overline{PQ}$ are radii of $\odot P$, so $\overline{PO} \cong \overline{PQ}$.
PO = 8, $\therefore$ PQ = 8, so Q (16, 0)
$\overline{RO}$ and $\overline{RS}$ are radii of $\odot R$, so $\overline{RO} \cong \overline{RS}$.
RO = 19, so S (38, 0)

b QR = RO − QO
QR = 19 − 16 = 3

6 Since $\overline{AB}$ and $\overline{AC}$ are tangent to $\odot O$, $\overline{AB} \cong \overline{AC}$.
$14 + 4x = 19 - 6x$
$5 = 10x$
$\frac{1}{2} = x$
$2.5 = 5x$

10 $AD = 14 - (11 - (20 - x)) + x$

$AD = 23$

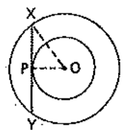


11 a $PQ = \sqrt{(4+2)^2 + (9-5)^2}$

$PQ = \sqrt{52} = 2\sqrt{13}$ or ≈ 7.21

b Slope $\overline{PQ} = \frac{9-5}{4+2}\ \frac{2}{3}$

tangent is $\perp$ to $\overline{PR}$, so slope is $-\frac{3}{2}$.



12 $XP^2 + PO^2 = XO^2$

$XP^2 + 3^2 = 7^2$

$XP^2 + 9 = 49$

$XP^2 = 40$

$XP = 2\sqrt{10}$

$XY = XP + YP$

$XY = 2(2\sqrt{10})$

$XY = 4\sqrt{10}$

13 a $(DE)^2 + (EC)^2 = (DC)^2$

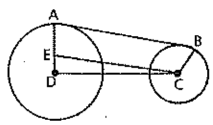
$5^2 + (EC)^2 = 13^2$

$25 + (EC)^2 = 169$

$(EC)^2 = 144$

$EC = 12$

$AB = 12$



b Yes

14 Draw $\overline{FH} \parallel$ to $\overline{GE}$ and extend $\overline{DE}$ to $\overline{DH}$.

$\angle FHD$ is a rt $\angle$.

$ED = 5$ $\quad (HD)^2 + (HF)^2 = (FD)^2$

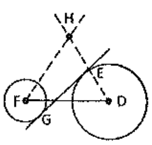
$HE = 3$ $\quad 8^2 + (HF)^2 = 10^2$

$FD = 10$ $\quad 64 + (HF)^2 = 100$

$(HF)^2 = 36$

$HF = 6$

$EG = 6$



16 $BC = 13$

$BC = x + 11 - (8 - x)$

$13 = 2x + 3$

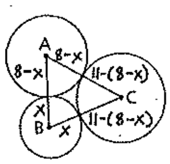
$2x = 10$

$x = 5$

Radius of $\odot A = 3$

Radius of $\odot B = 5$

Radius of $\odot C = 8$



22 $QX = PX = 4$

$WS = WQ = y$ (Two-Tangent Theorem)

$YS = YP = x$ (Two-Tangent Theorem)

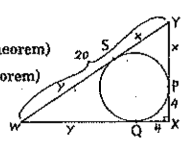
$WY = WS + YS$

$20 = x + y$

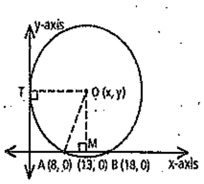
Perimeter of $\triangle WXY = WQ + QX + PX + YP + YS + WS$

$P = 4 + 4 + (y + x) + (x + y)$

$P = 8 + (20) + (20) = 48$



24 $\overline{OM}$ is $\perp$ bis of chord $\overline{AB}$. So coordinates of M are (13, 0). $\therefore$, $x = 13$. $OT = OA = 13$, $AM = 5$, so $\triangle AOM$ is a 5-12-13$\triangle$, and $OM = 12$. Thus, O (13, 12) or O (13, −12), since both circles would satisfy given conditions.



P32

Pages 495 – 498 (Section 10.8)

1 $3x = (5)(9)$

$3x = 45$

$x = 15$

$y^2 = (4)(16)$

$y = \sqrt{64}$

$y = 8$

$3z = (5)(15)$

$3z = 75$

$z = 25$

2 a $8(TR) = (20)(20)$

$8TR = 400$

$TR = 50$

b $PR = PT + TR$

$PR = 8 + 50$

$PR = 58$

4 a $(6)(4) = 3ED$

$24 = 3ED$

$8 = ED$

b $(8)6 = 16EC$

$48 = 16EC$

$3 = EC$

$DC = 19$

c $x^2 = (18)(2)$

$x^2 = 36$

$x = 6$

$AE = \pm 6$ (reject −6)

$AB = 12$

8 a $6^2 = (4 + x)4$

$36 = 16 + 4x$

$4x = 20$

$x = 5$

$(3 + y)3 = 9(4)$

$9 + 3y = 36$

$3y = 27$

$y = 9$

b Since the sides are 6, 13, 12,

obtuse $\triangle \Rightarrow 13^2 > 6^2 + 12^2$ $\quad$ 169 ? 36 + 144

or right $\triangle \Rightarrow 13^2 = 6^2 + 12^2$ $\quad$ 169 ? 180

or acute $\triangle \Rightarrow 13^2 < 6^2 + 12^2$ $\quad$ 169 < 180

The $\triangle$ is acute.

9 $(AE)(EB) = (CE)(ED)$

$(7 - x)(x) = 3(2)$

$7x - x^2 = 6$

$0 = x^2 - 7x + 6$

$0 = (x - 6)(x - 1)$

$x = 6$ or $x = 1$

10 $2^2 = PQ(PQ + 3)$

$4 = (PQ)^2 + 3PQ$

$(PQ)^2 + 3PQ - 4 = 0$

$(PQ + 4)(PQ - 1) = 0$

$PQ = -4$ or 1 (Reject −4)

$PQ = 1$

11 $10(6) = (7 + x)x$

$60 = 7x + x^2$

$x^2 + 7x - 60 = 0$

$(x + 12)(x - 5) = 0$

$x = -12$ or 5 (Reject −12)

$x = 5$

14 $(10)(10) = 5x$

$100 = 5x$

$x = 20$

diameter $= 25$

Radius $= 12\frac{1}{2}$

16 By Chord-Chord Power Theorem,

$(2x - 1)(4) = (x - 2)(x + 7)$

$8x - 4 = x^2 + 5x - 14$

$x^2 - 3x - 10 = 0$

$(x + 2)(x - 5) = 0$

$x = -2$ or $x = 5$

If $x = -2$, then $x - 2 = 0$, $\therefore$ $x = 5$.

17 In smaller circle: $3x = 2y$, so $x = \frac{2}{3}y$

In larger circle:

$(4 + 3)(x + EF) = (3 + 2)(y + CB)$

$7(x + 4) = 5(y + 3)$

$7x + 28 = 5y + 15$

$7x - 5y + 13 = 0$

$7(\frac{2}{3}y) - 5y + 13 = 0$ by substitution

$\frac{14}{3}y - 5y + 13 = 0$

$-\frac{1}{3}y + 13 = 0$

$13 = \frac{1}{3}y$

$39 = y$

$39 = DC$

$x = \frac{2}{3}y$

$x = \frac{2}{3}(39)$

$x = 26$

$DE = 26$

P33

5 $(10)(6) = 5x$

$60 = 5x$

$12 = x$

The diameter $= 12 - 5 = 7$

The radius $= 3.5$

6 $(AP)(PB) = (PD)(CP)$

$(3)(12) = (PD)2$

$36 = (PD)2$

$18 = PD$

$(AQ)(QB) = (EQ)(QD)$

$(8)(7) = (EQ)14$

$56 = (EQ)14$

$4 = EQ$

7 $6^2 = 4(7 + XY)$

$36 = 28 + 4XY$

$8 = 4XY$

$2 = XY$

$3 \times 2 = 1(XT)$

$6 = XT$

10 $2^2 = PQ(PQ + 3)$

$4 = (PQ)^2 + 3PQ$

$(PQ)^2 + 3PQ - 4 = 0$

$(PQ + 4)(PQ - 1) = 0$

$PQ = -4$ or 1 (Reject −4)

$PQ = 1$

11 $10(6) = (7 + x)x$

$60 = 7x + x^2$

$x^2 + 7x - 60 = 0$

$(x + 12)(x - 5) = 0$

$x = -12$ or 5 (Reject −12)

$x = 5$

12 By Tangent-Secant Power Theorem,

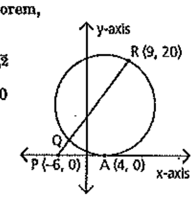
$PQ \times PR = (PA)^2$

$PR = \sqrt{(9 + 6)^2 + (20 + 0)^2}$

$PR = \sqrt{625} = 25$; $PA = 10$

$25PQ = 100$

$PQ = 4$



13 $AC = AB + 4$

$(AC)(BC) = (DC)^2$

$(AB + 4)(4) = (6)^2$

$4AB + 16 = 36$

$4AB = 20$

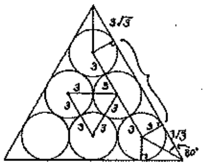
$AB = 5$

$\frac{1}{2}(AB) = $ Radius

$\frac{1}{2}(5) = $ Radius

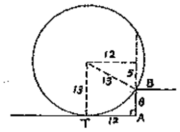
$2.5 = $ Radius

18 a Draw $\overline{PE}$ and $\overline{RF} \perp$ to $\overline{WY}$. Then $\triangle WEP$ and $\triangle YFR$ are 30°60°90° △s, therefore WE and CY are each $3\sqrt{3}$. EF is the sum of 4 radii, or 12. $WY = 12 + 6\sqrt{3}$.



b Perimeter of $\triangle ABC$ is the sum of 6 radii = 18. Perimeter of $\triangle PQR$ is the sum of 12 radii = 36. Perimeter of $\triangle WXY$ is $3(12 + 6\sqrt{3} = 36 + 18\sqrt{3}$. The ratio is $1{:}2{:}2 + \sqrt{3}$.

20 a Using the Tangent-Secant Power Theorem:



$$\begin{aligned}(AT)^2 &= (AX)(AB)\\ (12)^2 &= (AB+XB)(AB)\\ 144 &= (8+XB)(8)\\ 144 &= 64 + 8(XB)\\ 80 &= 8(XB)\\ 10 &= XB\end{aligned}$$

A radius ⊥ to a chord bis that chord, so

YB = 5

ZY = 12 (5-12-13) is a Pythagorean triple

ZB = 13 diameter = 26

b How far is the ⊙ from pt A?

$$\begin{aligned}(AT)^2 + (ZT)^2 &= (AZ)^2\\ (12)^2 + (13)^2 &= (AZ)^2\\ 144 + 169 &= (AZ)^2\\ 313 &= (AZ)^2\\ AZ &= \sqrt{313}\end{aligned}$$

$\sqrt{313} - 13$ is the distance.

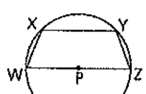
P34

Pages 454–458 (Section 10.3)

1 a 6 b 2 c 5 d 4 e 3 f 7 g 1 **2** a $\widehat{QPR}$ or $\widehat{QRP}$ b $\widehat{BC}$ or $\widehat{AB}$ c 180 d m$\widehat{PQ}$ e No, the arcs must be in the same ⊙ or in ≅ ⊙s. **3** a 90 b 130 c 230 d 180 e 220 **4** m∠A is 25 and m∠B is 25, m∠AQB = 30, so m$\widehat{AB}$ = 130.

5 Given: ⊙P, $\widehat{WY} \cong \widehat{XZ}$

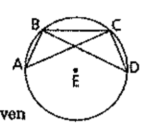
Concl: $\overline{WY} \cong \overline{XZ}$



1 ⊙P, $\widehat{WY} \cong \widehat{XZ}$	1 Given
2 $\widehat{XY} \cong \widehat{XY}$	2 Reflexive prop
3 $\widehat{WX} \cong \widehat{YZ}$	3 Subtraction prop
4 $\overline{WY} \cong \overline{XZ}$	4 ≅ arcs ⇒ ≅ chords.

8 Given: ⊙E, $\overline{AB} \cong \overline{CD}$

Prove: $\overline{BD} \cong \overline{AC}$



1 ⊙E, $\overline{AB} \cong \overline{CD}$	1 Given
2 $\widehat{AB} \cong \widehat{CD}$	2 ≅ chords ⇒ ≅ arcs.
3 $\widehat{BC} \cong \widehat{BC}$	3 Reflexive prop
4 $\widehat{AC} \cong \widehat{BD}$	4 Addition prop
5 $\overline{BD} \cong \overline{AC}$	5 ≅ arcs ⇒ ≅ chords.

10 a $\frac{3}{5}(360) = 216$ b $\frac{5}{9}(360) = 200$ c $.7(360) = 252$

11

$$\begin{aligned}\widehat{AB} + \widehat{BC} + \widehat{CD} &= 180\\ 9x + 30 + 54 - x + 54 - x &= 180\\ 7x + 138 &= 180\\ 7x &= 42\\ x &= 6\end{aligned}$$

m$\widehat{AB}$ = 9(6) + 30 m$\widehat{BC}$ = m$\widehat{CD}$ = 54 − 6

m$\widehat{AB}$ = 84 m$\widehat{BC}$ = 48

m∠AEC = m$\widehat{AB}$ + m$\widehat{BC}$

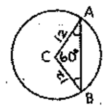
m∠AEC = 84 + 48 = 132

12 m∠ACB = 60

$\overline{CA} \cong \overline{CB}$ so ∠A ≅ ∠B

m∠A = m∠B = 60

△CAB is equilateral, so AB = 12.



13 a $C = 24\pi$; Arc $= 24\pi\left(\frac{5}{8}\right) = 15\pi$

b $\frac{270}{360} = \frac{3}{4}$; $24\pi\left(\frac{3}{4}\right) = 18\pi$

P35 Circle Worksheet

P36

Pages 473–478 (Section 10.5)

1 62 **2** $\frac{1}{2}(130) = 65$ **3** a $\frac{1}{2}(70) = 35$ b $\frac{1}{2}(150) = 75$ **4** a $\frac{120-36}{2} = 42$ b $\frac{160-60}{2} = 50$ c $\frac{220-140}{2} = 40$

9

$$\begin{aligned}m\angle AEB &= \tfrac{1}{2}(\widehat{AB} + \widehat{CD})\\ 30 &= \tfrac{1}{2}(50 + \widehat{CD})\\ 30 &= 25 + \tfrac{1}{2}(\widehat{CD})\\ 5 &= \tfrac{1}{2}(\widehat{CD}),\ \widehat{CD} = 10\end{aligned}$$

11

$$\begin{aligned}22 &= \tfrac{1}{2}(125 - x)\\ 44 &= 125 - x\\ x &= 81,\ \widehat{XZ} = 81°\end{aligned}$$

12. 40°

13

$$\begin{aligned}m\angle BEA &= \tfrac{1}{2}(\widehat{AB} + \widehat{CD}) &\\ m\angle BEA &= \tfrac{1}{2}(85 + 25) & 180 - m\angle BEA &= m\angle AED\\ m\angle BEA &= \tfrac{1}{2}(110) & 180 - 55 &= m\angle AED\\ m\angle BEA &= 55 & 125 &= m\angle AED\end{aligned}$$

18 a $\angle R = \frac{1}{2}(115°)$, $\angle R = 57.5°$ b $\angle PQV = \frac{1}{2}(115°)$, $\angle PQV = 57.5°$

c $\angle P = \frac{1}{2}(\widehat{SR})$, $36° = \frac{1}{2}(\widehat{SR})$, $72° = \widehat{SR}$ d $180° - \widehat{SR} = \widehat{QR}$, $180° - 72° = \widehat{QR}$, $108° = \widehat{QR}$

e $\angle QPR = \frac{1}{2}(\widehat{QR})$, $\angle QPR = \frac{1}{2}(108°)$, $\angle QPR = 54°$ f $\angle QPS = \angle RPS + \angle QPR$, $\angle QPS = 36° + 54°$, $\angle QPS = 90°$

g $\angle QPR = \frac{1}{2}(115° + 72°)$, $\angle QPR = \frac{1}{2}(187°)$, $\angle QTP = 93.5°$ h $180° - \angle STR = \angle QTR$ ($\angle QTP \cong \angle STR$), $180° - 93.5 = \angle QTR$, $86.5° = \angle QTR$

i $\widehat{PRQ} = 360° - \widehat{PQ}$, $\widehat{PRQ} = 360° - 115°$, $\widehat{PRQ} = 245°$ j $\widehat{PRQ} - \widehat{QR} = \widehat{RSP}$, $245° - 108° = \widehat{RSP}$, $137° = \widehat{RSP}$

k $\angle VQS = 90°$ l $\angle QOP = m\widehat{PQ} = 115°$

20

$$\begin{aligned}m\widehat{BC} &= x\\ m\widehat{BDC} &= \tfrac{5}{3}x\\ x + \tfrac{5}{3}x &= 360\\ \tfrac{8}{3}x &= 360\\ x &= 135\\ m\widehat{BC} &= 135 & m\angle A &= \tfrac{1}{2}(225 - 135)\\ m\widehat{BDC} &= 225 & m\angle A &= \tfrac{1}{2}(90) = 45\end{aligned}$$

P37

5 a $180 - 40 = 140$ b $\frac{1}{2}[360 - (120 + 80)] = 80$ c $\frac{1}{2}[360 - (160 + 160)] = 20$ d $\frac{1}{2}(81 + 82) = 81\frac{1}{2}$ e $180 - 40 - 40 = 100$ **6** a 104 b 68 c 90 d 33

10

$$\begin{aligned}17 &= \tfrac{1}{2}(\widehat{SR} - 42)\\ 17 &= \tfrac{1}{2}\widehat{SR} - 21\\ 38 &= \tfrac{1}{2}\widehat{SR},\ \widehat{SR} = 76°\end{aligned}$$

14

$$\begin{aligned}120 &= \tfrac{1}{2}(130 - x)\\ 240 &= 130 + x\\ x &= 110,\ \widehat{WZ} = 110°\end{aligned}$$

21 a

$$\begin{aligned}90 &= \tfrac{1}{2}(x + 48)\\ 90 &= \tfrac{1}{2}x + 24\\ 66 &= \tfrac{1}{2}x\\ 132 &= x\end{aligned}$$

b In △ABC, m∠C = 20, m∠ABC = 80, m∠BAC = 80. In △ABD, m∠D = 40, m∠Y = m∠ABC − m∠ABD = 80 − 70 = 10

c $m\angle V = \frac{1}{2}(100 - 60) = 20$ d $m\angle W = \frac{1}{2}(60) = 30$

22

$m\angle A = \frac{1}{2}(84) = 42$

$m\angle DOE = 180 - (90 + 42) = 180 - 132$, $m\angle DOE = 48$, $m\widehat{DE} = 48$

$m\widehat{DF} = 180 - m\widehat{DE}$, $m\widehat{DF} = 180 - 48 = 132$

23

$$\begin{aligned}m\angle DEC &= \tfrac{1}{2}m(\widehat{AB} + \widehat{DC}) & 360 &= 92 + 72 + m\widehat{AD} + m\widehat{BC}\\ 82 &= \tfrac{1}{2}(92 + m\widehat{DC}) & 360 &= 164 + 2\,m\widehat{AD}\\ 82 &= 46 + \tfrac{1}{2}m\widehat{DC} & 196 &= 2\,m\widehat{AD}\\ 36 &= \tfrac{1}{2}m\widehat{DC} & 98 &= m\widehat{AD}\\ 72 &= m\widehat{DC} &&\end{aligned}$$

25

$$\begin{aligned}m\angle DXC &= \tfrac{1}{2}(m\widehat{DC} + m\widehat{AB})\\ 56 &= \tfrac{1}{2}(87 + m\widehat{AB})\\ 56 &= 43\tfrac{1}{2} + \tfrac{1}{2}m\widehat{AB}\\ 12\tfrac{1}{2} &= \tfrac{1}{2}m\widehat{AB}\\ 25° &= m\widehat{AB}\end{aligned}$$

problems 26 and up in this section are great (challenging)…hope to do some as warm ups

P38

3 An ∠inscribed in a semi ⊙ is a rt ∠, so △ABC is a rt ∠.

AC = 12 = 4 × 3 3-4-5 is a Pythagorean triple, so

BA = 16 = 4 × 4 radius = $\frac{1}{2}$(BC)

BC = 4 × 5 = 20 radius = $\frac{1}{2}(20) = 10$

4 a $m\angle P + m\widehat{QR} = 180$, $m\angle P + 163 = 180$, $m\angle P = 17$ b $m\angle PQR = \frac{1}{2}(m\widehat{QR}) = \frac{1}{2}(163) = 81\frac{1}{2}$

7 $\angle = \frac{1}{2}(126 - 40)$

$\angle = \frac{1}{2}(86)$

$43 + \widehat{PQ} = 180$

$\widehat{PQ} = 137°$

11 a $\widehat{WA}$ is a diameter, so WA = 12.

$AX = \frac{1}{2}WA$ because $\widehat{WX} = 120°$.

$\widehat{XA} = 180° - 120° = 60°$

$AX = \frac{1}{2}(12) = 6$

b $\angle X$ is a rt $\angle$.

$\angle A = \frac{1}{2}(\widehat{WX})$

$\angle A = \frac{1}{2}(120°) = 60°$

$\triangle$ WAX is a 30°60°90° $\triangle$, so $WX = 6\sqrt{3}$.

$\text{Perimeter} = 12 + 6 + 6\sqrt{3} = 18 + 6\sqrt{3}$

12 $\widehat{AM} \cong \widehat{BM}$, because M is mdpt of $\widehat{AB}$. $\therefore \angle D \cong \angle C$.

$3x - 31 = x + 7$ $\quad$ $m\widehat{CD} = 2(m\angle DMC)$

$2x = 38$ $\quad$ $m\widehat{CD} = 2(4(19) - 15)$

$x = 19$ $\quad$ $m\widehat{CD} = 122°$

16 $\widehat{BD} = 360° - \widehat{DFB}$

$\widehat{BD} = 360° - 223° = 137°$

The sum of a tangent-tangent $\angle$ and its minor arc is 180°.

$\angle C = 180 - \widehat{BD}$ $\quad$ $\frac{1}{2}(\widehat{AE}) = \angle C$ (an inscribed $\angle$)

$\angle C = 180 - 137°$ $\quad$ $\frac{1}{2}(\widehat{AE}) = 43°$

$\angle C = 43°$ $\quad$ $\widehat{AE} = 86°$

17 a $88 = \frac{1}{2}(104 + \widehat{QR})$ $\quad$ $\widehat{RS} = 90$

$88 = 62 + \frac{1}{2}\widehat{QR}$ $\quad$ $\angle P = \frac{1}{2}(\widehat{SQ} - \widehat{TQ})$

$72° = \widehat{QR}$ $\quad$ $\angle P = \frac{1}{2}(162 - 104)$

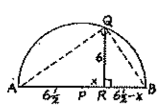
$\angle P = \frac{1}{2}(58) = 29°$

b $\angle STQ = \frac{1}{2}(90 + 72) = \frac{1}{2}(162) = 81°$

23 $\triangle AQB$ is a rt $\triangle$, $\overline{QR}$ is an alt.

Radius AP = 6.5, PR = x

The alt is the mean proportional, so



$\frac{6.5 + x}{6} = \frac{6}{6.5 - x}$

$6^2 = (6.5 + x)(6.5 - x)$

$36 = 42.25 - x^2$

$x^2 = 6.25$ $\quad$ $RB = 6.5 - x$

$x = \sqrt{6.25} = 2.5$ $\quad$ $RB = 6.5 - 2.5 = 4$

P39

Pages 489–492 (Section 10.7)

1 $180 - 67 = 113$ $\quad$ $180 - 104 = 76$

$\angle D = 113°$ $\quad$ $\angle C = 76°$

2 $m\angle R = \frac{1}{2}(m\widehat{QPS})$ $\quad$ $m\widehat{QRS} = 360 - 210 = 150$

$m\angle R = \frac{1}{2}(210)$ $\quad$ $m\angle P = \frac{1}{2}(m\widehat{QRS})$

$m\angle R = 105$ $\quad$ $m\angle P = \frac{1}{2}(150) = 75$

3 a If a quad is inscribed in a $\odot$, opp $\angle$s are supp.

$\angle C = 180° - \angle A$

$\angle C = 180° - 110° = 70°$

b Since $\overline{BC} \cong \overline{CD}$, $\widehat{BC} \cong \widehat{CD}$ (If 2 chords of a $\odot$ are $\cong$, then corr arcs are $\cong$.)

$\angle A = \frac{1}{2}(\widehat{BCD})$ $\quad$ $\widehat{BC} + \widehat{CD} = \widehat{BCD}$

$110° = \frac{1}{2}(\widehat{BCD})$ $\quad$ $2\widehat{BC} = 220°$

$220° = \widehat{BCD}$ $\quad$ $\widehat{BC} = 110°$

c $\angle B = 180° - \angle D$

$\angle B = 180° - 95° = 85°$

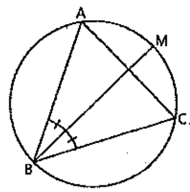
d $\angle D = \frac{1}{2}(\widehat{ABC})$ $\quad$ $\widehat{ABC} - \widehat{BC} = \widehat{AB}$

$95° = \frac{1}{2}(\widehat{ABC})$ $\quad$ $190° - 110° = \widehat{AB}$

$190° = \widehat{ABC}$ $\quad$ $80° = \widehat{AB}$

7 a It must be a square, since a rhombus with a rt $\angle$ is a square. b It must be an isos trapezoid.

8 Since $\overline{BM}$ bis $\angle ABC$, $\angle ABM \cong \angle CBM$.



$\frac{m\widehat{AM}}{2} = m\angle ABM$

$\frac{m\widehat{CM}}{2} = m\angle CBM$

By substitution,

$\frac{m\widehat{AM}}{2} = \frac{m\widehat{CM}}{2}$

$m\widehat{AM} = m\widehat{CM}$

$\therefore \overline{BM}$ bis $\widehat{AC}$.

9 a $\angle ADC = 180° - \angle B$

$\angle ADC = 180° - 115° = 65°$

b $\angle ADC = \frac{1}{2}(\widehat{ABC})$

$65° = \frac{1}{2}(\widehat{ABC})$

$130° = \widehat{ABC}$

$\widehat{DAC} = \widehat{AD} + \widehat{ABC}$

$\widehat{DAC} = 60° + 130° = 190°$

$\widehat{CD} = 360° - \widehat{DAC}$

$\widehat{CD} = 360° - 190° = 170°$

$\angle CDF = \frac{1}{2}(170°) = 85°$

c $\angle CDF = \angle C$

$\angle C = 85°$

d $\angle A = 180° - \angle C$

$\angle A = 180° - 85$

$= 95°$

10 $RP = 25$

$\angle S = 90$

$(RS)^2 + (SP)^2 = (RP)^2$

$7^2 + SP^2 = 25^2$

$49 + SP^2 = 625$

$SP^2 = 576$

$SP = \sqrt{576} = 24$

11 Label the points of tangency M, N, P, and Q. By the Two Tangent Theorem, $\overline{XM} \cong \overline{XQ}$, $\overline{MW} \cong \overline{WN}$, $\overline{NZ} \cong \overline{PZ}$, $\overline{QY} \cong \overline{YP}$. By the Addition prop, XW + YZ = XY + WZ. XW + YZ = 16 + 7 = 23. So the perimeter is 23 + 23 = 46.

15 Since opp $\angle$s of inscribed quad are supp, $\angle Q$ supp $\angle A$.

$(100 - 2x) + x^2 = 180$

$x^2 - 2x - 80 = 0$

$x = \frac{2 \pm \sqrt{4 - 4(-80)}}{2} = \frac{2 \pm \sqrt{324}}{2} = \frac{2 \pm 18}{2}$

$x = 10$ or $x = -8$

$m\angle Q = 100 - 2(10) = 80$ or

$m\angle Q = 100 - 2(-8) = 116$

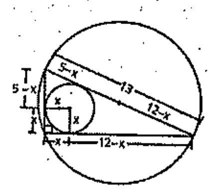
24 By walk-around analysis,

$(12 - x) + (5 - x) = 13$, so $x = 2$.

$\therefore$ Radius of inscribed $\odot = 2$

Area of inscribed $\odot = 4\pi$

Radius of circumscribed $\odot = \frac{13}{2}$



Area of circumscribed $\odot = \frac{169\pi}{4}$

$\frac{A_{inscr}}{A_{circum}} = \frac{4\pi}{\frac{169\pi}{4}} = \frac{16}{169}$

P40

2 a $56\pi = \pi d$

$56 = d$

$\text{radius} = 28$

b $314 = \pi d$

$314 = 3.14d$

$100 \approx d$

$\text{radius} \approx 50$

c $17\pi = \pi d$

$17 = d$

$\text{radius} = 8.5$

d $88 \approx \frac{22}{7}d$

$\frac{616}{22} \approx d$

$28 \approx d$

$\text{radius} \approx 14$

3 a $\frac{\text{arc}}{360}(2\pi r) = \text{length}$

$\frac{72}{360}(2\pi 10) = \text{length}$

$\frac{1}{5}(20\pi) = \text{length}$

$4\pi = \text{length}$

c $\frac{\text{arc}}{360}(2\pi r) = \text{length}$

$\frac{60}{360}(2\pi 10) = \text{length}$

$\frac{1}{6}(20\pi) = \text{length}$

$\frac{10}{3}\pi = \text{length}$

5 a The four corners form a circle. That part of the perimeter is 6π. The four sides are 40, so the total is $40 + 6\pi$.

b The perimeter (or circumference) of the semicircle is $\frac{1}{2}(8\pi) = 4\pi$. The other 3 sides are $8 \times 3 = 24$. The total is $24 + 4\pi$.

c The semicircle is $\frac{1}{2}(6\pi) = 3\pi$. The other two sides are 12. The total is $12 + 3\pi$.

d The larger semicircle is $\frac{1}{2}(10\pi) = 5\pi$. The smaller semicircle is $\frac{1}{2}(4\pi) = 2\pi$. The other two edges are 6, so the total is $6 + 7\pi$.

7 The semicircle at each end is $\frac{1}{2}(12\pi) = 6\pi$. The other two sides are 100. The total is $100 + 12\pi$, or approximately 138 meters.

10 $4(3.14) \approx 12.57$

$12.57(100) \approx 1257$ cm

11 The circumference of the loop is 30π which ≈ 94.2 m, so the distance he must ride is ≈ 214 m.

12 The 2 small semicircles form 1 $\odot$.

The 2 large semicircles form 1 $\odot$.

Small $\odot$: $C = d\pi$ $\quad$ Large $\odot$: $C = d\pi$

$C = 4\pi$ $\quad$ $C = 9\pi$

$\text{Perimeter} = 4\pi + 9\pi = 13\pi$

13 $D = 2(20) + \pi(18)$ $\quad$ $D = 2(20) + \pi(10)$

$40 + (18)3.14 \approx 96.52$ $\quad$ $40 + (10)3.14 \approx 71.4$

P41

Pages 635–637 (Section 13.6)

1 a $x^2 + y^2 = 16$

b $(x + 2)^2 + (y - 1)^2 = 25$

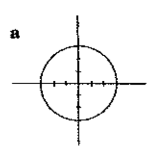
c $x^2 + (y + 2)^2 = (2\sqrt{3})^2$

$x^2 + (y + 2)^2 = 12$

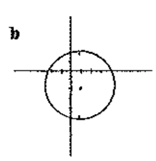
d $(x + 6)^2 + y^2 = (\frac{1}{2})^2$

$(x + 6)^2 + y^2 = \frac{1}{4}$

2 a



b



3 a center = (0, 0); $r = \sqrt{36} = 6$; $d = 2r = 12$;

$C = \pi d = 12\pi$; $A = \pi r^2 = 36$

b center = (–5, 0); $r = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$; $d = 2r = 3$;

$C = \pi d = 3\pi$; $A = \pi r^2 = \frac{9}{4}\pi$

c center = (3, –6); $r = \sqrt{100} = 10$; $d = 2r = 20$

$C = \pi d = 20\pi$; $A = \pi r^2 = 100\pi$

d $\frac{(x + 5)^2}{3} + \frac{(y - 2)^2}{3} = 27$

$(x + 5)^2 + (y - 2)^2 = 81$

center = (–5, 2); $r = \sqrt{81} = 9$; $d = 2r = 18$;

$C = \pi d = 18\pi$; $A = \pi r^2 = 81\pi$

4 a $r = 5$, $x^2 + y^2 = 25$ b $r = 4$, $(x + 4)^2 + y^2 = 16$

c $r = 6$, $(x - 5)^2 + (y - 6)^2 = 36$

5 a $(4 - 3)^2 + (2 + 2)^2 = 1 + 16 = 17$

It is on the circle.

b $(3 - 3)^2 + (-2 + 2)^2 \neq 17$

It isn't on the circle.

6 **a** A point circle. **b** An imaginary circle.

7 $A = (1 - 7, 3)$ or $(-6, 3)$;
$B = (1 + 7, 3)$ or $(8, 3)$;
$C = (1, 3 + 7)$ or $(1, 10)$;
$D = (1, 3 - 7)$ or $(1, -4)$

8 **a** $r = 5$, so $x^2 + y^2 = 25$

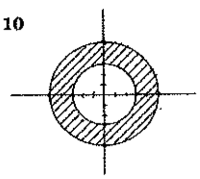
b center = mdpt = $(3, 13)$;
$r = \sqrt{(8-3)^2 + (25-13)^2} = 13$
$(x-3)^2 + (y-13)^2 = 13^2$,
$(x-3)^2 + (y-13)^2 = 169$

c $r = \sqrt{(-1-0)^2 + (7-0)^2} = \sqrt{50}$,
so $(x+1)^2 + (y-7)^2 = 50$

d $r = \sqrt{(3-2)^2 + (0-(-3))^2} = \sqrt{10}$
so $(x-2)^2 + (y+3)^2 = 10$

9 **a** $2^2 + 5^2 = 29$, so it is on the circle.
b $3^2 + 0^2 = 9 < 100$, so it is inside.
c $(0-2)^2 + (0+5)^2 = 29 > 16$, so it is outside.
d $(-2)^2 + (1+6)^2 = 53 > 23$, so it is outside.

10



11 **a**
$$\begin{aligned} x^2 + y^2 - 8y &= 9 \\ x^2 + y^2 - 8y + 16 &= 9 + 16 \\ x^2 + (y-4)^2 &= 25 \end{aligned}$$
center = $(0, 4)$; $r = 5$

b
$$\begin{aligned} (x+7)^2 + y^2 + 6y &= 27 \\ (x+7)^2 + y^2 + 6y + 9 &= 27 + 9 \\ (x+7)^2 + (y+3)^2 &= 36 \end{aligned}$$
center = $(-7, -3)$; $r = 6$

c
$$\begin{aligned} x^2 + 10x + y^2 - 12y &= -10 \\ x^2 + 10x + 25 + y^2 - 12y + 36 &= -10 + 25 + 36 \\ (x+5)^2 + (y-6)^2 &= 51 \end{aligned}$$
center = $(-5, 6)$; $r = \sqrt{51}$

d
$$\begin{aligned} x^2 + y^2 &= 8x - 14y + 35 \\ x^2 - 8x + y^2 + 14y &= 35 \\ x^2 - 8x + 16 + y^2 + 14y + 49 &= 35 + 16 + 49 \\ (x-4)^2 + (y+7)^2 &= 100 \end{aligned}$$
center = $(4, -7)$; $r = 10$

12 **a**
$$\begin{aligned} x^2 + y^2 &= 25 \\ x &= 3 \\ 3^2 + y^2 &= 25 \\ y^2 &= 16, y = \pm 4 \end{aligned}$$
$\{(3, 4), (3, -4)\}$

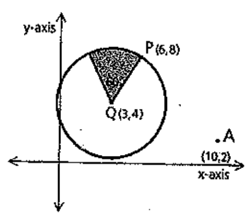
b
$$\begin{aligned} x^2 + y^2 &= 25 \\ \underline{x^2 - y^2} &\underline{= 7} \\ 2x^2 &= 32 \\ x^2 &= 16 \\ x &= \pm 4 \\ x^2 + y^2 &= 25 \\ 16 + y^2 &= 25 \\ y^2 &= 9, y = \pm 3 \end{aligned}$$
$\{(4, 3), (-4, 3), (-4, -3), (4, -3)\}$

c
$$\begin{aligned} x^2 + y^2 &= 34 \\ x + y &= 8 \\ y &= 8 - x \\ x^2 + (8-x)^2 &= 34 \\ x^2 + 64 - 16x + x^2 &= 34 \\ 2x^2 - 16x &= -30 \\ x^2 - 8x &= -15 \\ x^2 - 8x + 15 &= 0 \\ (x-5)(x-3) &= 0 \\ x = 5, \quad x &= 3 \end{aligned}$$
$y = 8 - 5, y = 3; y = 8 - 3, y = 5$
$\{(5, 3), (3, 5)\}$

d
$$\begin{aligned} |y| &= 6 \\ x^2 + y^2 &= 100 \\ y &= \pm 6 \\ x^2 + (\pm 6)^2 &= 100 \\ x^2 + 36 &= 100 \\ x^2 &= 64 \\ x &= \pm 8 \end{aligned}$$
$\{(8, 6), (8, -6), (-8, 6), (-8, -6)\}$

13
$$\begin{aligned} x^2 + y &= 17 \\ y &= 3 - x \\ x^2 + (3-x)^2 &= 17 \\ x^2 + 9 - 6x + x^2 &= 17 \\ 2x^2 - 6x &= 8 \\ x^2 - 3x &= 4 \\ x^2 - 3x - 4 &= 0 \\ x = 4, \quad x &= -1 \end{aligned}$$
$y = 3 - 4 = -1, \quad y = 3 - (-1) = 4$
$(4, -1), (-1, 4)$ are the intersection pts.
$d = \sqrt{(4-(-1))^2 + (-1-4)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

14 **a** $m(\overline{PQ}) = \frac{8-4}{6-3} = \frac{4}{3}$
m(tangent) $= -\frac{3}{4}$
$y = -\frac{3}{4}x + b$
$8 = -\frac{3}{4}(6) + b$
$\frac{25}{2} = b$
$y = -\frac{3}{4}x + \frac{25}{2}$



b $r = \sqrt{(6-3)^2 + (8-4)^2} = 5$; $C = 2\pi r = 10\pi$
c $AQ = \sqrt{(10-3)^2 + (2-4)^2} = \sqrt{49+4} = \sqrt{53}$
d Distance from A to circle = $AQ - r = \sqrt{53} - 5 \approx 2.3$
e A of sector = $A_{\odot}\left(\frac{60°}{360°}\right) = \pi r^2 \left(\frac{1}{6}\right) = \frac{25\pi}{6} \approx 13.1$

15 Slope of line from center to point $= \frac{-8-(-3)}{8-2} = -\frac{5}{6}$
Slope of tangent $= \frac{6}{5}$
$y + 8 = \frac{6}{5}(x - 8)$

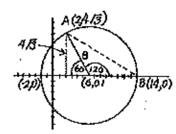
16
$$\begin{aligned} 3x^2 + 12x + 3y^2 - 5y &= 2 \\ x^2 + 4x + y^2 - \tfrac{5}{3}y &= \tfrac{2}{3} \\ x^2 + 4x + 4 + y^2 - \tfrac{5}{3}y + \tfrac{25}{36} &= \tfrac{2}{3} + 4 + \tfrac{25}{36} \\ (x+2)^2 + (y - \tfrac{5}{6})^2 &= \tfrac{193}{36} \end{aligned}$$
center = $(-2, \frac{5}{6})$; $r = \sqrt{\frac{193}{36}} = \frac{\sqrt{193}}{6}$

17 The center of the circle is equidistant from $(0, 0)$ and $(8, 0)$, so it lies on the ⊥ bis, $x = 4$. The center $(4, y)$ is also equidistant from $(0, 0)$ and $(5, 3)$ so
$$\begin{aligned} \sqrt{(4-5)^2 + (y-3)^2} &= \sqrt{4^2 + y^2} \\ (4-5)^2 + (y-3)^2 &= 4^2 + y^2 \\ 1 + y^2 - 6y + 9 &= 16 + y^2 \\ -6 &= 6y \\ -1 &= y \end{aligned}$$
$r = \sqrt{17}$ so $A = 17\pi$

18
$$\begin{aligned} \sqrt{(x-3)^2 + (y-0)^2} &= 2\sqrt{(x+3)^2 + (y-0)^2} \\ (x-3)^2 + y^2 &= 4((x+3)^2 + y^2) \\ x^2 - 6x + 9 + y^2 &= 4x^2 + 24x + 36 + 4y^2 \\ 0 &= 3x^2 + 30x + 3y^2 + 27 \\ 0 &= x^2 + 10x + y^2 + 9 \\ 0 + 16 &= x^2 + 10x + 16 + 9 + y^2 \\ 16 &= x^2 + 10x + 25 + y^2 \\ 16 &= (x+5)^2 + y^2 \end{aligned}$$

19
$$\begin{aligned} x^2 - 12x + y^2 &= 28 \\ x^2 - 12x + 36 + y^2 &= 28 + 36 \\ (x-6)^2 + y^2 &= 64 \end{aligned}$$

a In the △ drawn, since the hypotenuse is twice as long as one leg, we know that the ∠ between them is 60°. ∴ the marble traveled 120°.
$r = 8$, $C = 2\pi r = 16\pi$
It traveled $16\pi\left(\frac{120°}{360°}\right) = \frac{16\pi}{3}$



b Its straight line path from $(2, 4\sqrt{3})$ to $(14, 0)$ has a
distance $= \sqrt{(14-2)^2 + (4\sqrt{3})^2} = \sqrt{144+48} = \sqrt{192} = 8\sqrt{3}$
distance saved $= \frac{16\pi}{3} - 8\sqrt{3} \approx 2.90$

P42

Pages 505–509 Chapter 10 Review Problems

1 **a** $180 - 86 = 94°$ **b** $180 - 86 = 94°$ **c** $\frac{1}{2} \times 86 = 43°$

2 $\angle y = \frac{1}{2}(98 - 34) = 32°$
$\angle x = \frac{1}{2}(98 + 34) = 66°$

3 **a** Using Chord-Chord Power Theorem
$$\begin{aligned} AE \times EC &= DE \times EB & \qquad EB + DE &= BD \\ 6 \times 8 &= 4 \times EB & 12 + 4 &= BD \\ 48 &= 4 \times EB & 16 &= BD \\ 12 &= EB \end{aligned}$$

b Using Tangent-Secant Power Theorem
$$\begin{aligned} PR \times PQ &= (PT)^2 \\ 16 \times 4 &= (PT)^2 \\ 64 &= (PT)^2 \\ 8 &= PT \end{aligned}$$

c Using Secant-Secant Power Theorem

$SZ \times SY = SX \times SW$

$15 \times 3 = 5(5 + WX)$

$45 = 25 + 5(WX)$

$20 = 5(WX)$

$4 = WX$

4 a Draw a rt △ using seg 5 and $\frac{1}{2}$ of the chord, 12; 5-12-13 is a Pythagorean triple, so the radius is 13.

b Using Pythagorean triple 8-15-17, diameter = 17, so radius is $8\frac{1}{2}$.

c 7-24-25 is Pythagorean triple, so radius is 7.

5 Since ∠P is inscribed in the larger ⊙, m∠P = 55. ∠P is a tangent-tangent ∠ of the smaller ⊙.

$m\overset{\frown}{WX} = x$

$m\overset{\frown}{WEX} = 360 - x$

$m\angle P = \frac{1}{2}[(360 - x) - x]$

$55 = \frac{1}{2}[360 - 2x]$

$110 = 360 - 2x$

$2x = 250$

$x = 125, m\overset{\frown}{WX} = 125$

6 △ABC is isos, $\overline{BC} \cong \overline{CA}$ so $\overset{\frown}{BC} \cong \overset{\frown}{CA}$. Since $m\overset{\frown}{BC} = 160$, $m\overset{\frown}{AC} = 160$. $m\overset{\frown}{AB} = 360 - 2(160) = 40$, ∠DAC = 70° and is an inscribed ∠ so $m\overset{\frown}{CD} = 140$. $m\overset{\frown}{AD} = m\overset{\frown}{AC} - m\overset{\frown}{CD}$, $m\overset{\frown}{AD} = 160 - 140 = 20$.

7 a $\left(\frac{45}{360}\right)(-12\pi) = \frac{3}{2}\pi$ b $P = 2(6) + \frac{3}{2}\pi = 12 + \frac{3}{2}\pi$

8 a Since radii of the same ⊙ are ≅,

$x + 3 = 11 - x$

$2x = 8$

$x = 4$

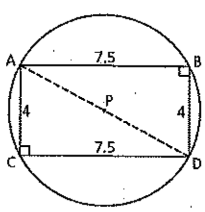
The radius of ⊙A = 4

b The radius of ⊙B = 7 − 4 = 3

The radius of ⊙C = 4 + 3 = 7

⊙C is the largest.

12 Since ▱ is inscribed in ⊙, it must be a ▭.



$(AD)^2 = (AB)^2 + (BD)^2$

$(AD)^2 = (7.5)^2 + (4)^2 = 72.25$

$AD = 8.5$

Radius ⊙ $P = \frac{AD}{2} = \frac{8.5}{2} = 4.25$

13 $8^2 = 6(6 + RQ)$

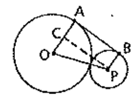
$64 = 36 + 6RQ$

$28 = 6RQ$

$RQ = 4\frac{2}{3}$

14 Draw $\overline{CP} \perp \overline{OA}$ forming ▭ ABPC.

OA = 8, AC = 2, so OC = 6. OP = 10. In rt △OCP,



$(OC)^2 + (CP)^2 = (OP)^2$

$6^2 + (CP)^2 = 10^2$

$36 + (CP)^2 = 100$

$(CP)^2 = 64$

$CP = 8 = AB$

15 a $m\overset{\frown}{BA} = 2m\angle ATB = 2(40) = 80$

$\frac{80}{360} = \frac{2}{9}$

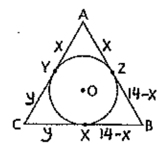
b $m\overset{\frown}{TUB} = 360 - m\overset{\frown}{TA} - m\overset{\frown}{BA}$

$m\overset{\frown}{TUB} = 360 - 2m\angle ETA - m\overset{\frown}{BA}$

$m\overset{\frown}{TUB} = 360 - 70 - 80 = 210$

$\frac{210}{360} = \frac{7}{12}$

16 a Since the perimeter = 38,



$2y + 2x + 2(14 - x) = 38$

$2y + 2x + 28 - 2x = 38$

$2y + 28 = 38$

$2y = 10$

$y = 5, XC = 5$

b If $\overline{AB}$ is the base, then $\overline{AC} \cong \overline{BC}$, AC = BC = 12, and BX = 12 − 5 = 7.

If CB is the base, then AC = AB = 14, and CB = 10.

Then BX = 10 − 5 = 5.

If AC is the base, then AB = CB = 14 and BX = 14 − 5 = 9.

17 $x + 2x + 5x + 4x = 360$

$12x = 360$

$x = 30$

$\angle BCP = \frac{1}{2}(150 + 60) = 105°$

$\angle CPE = \frac{1}{2}(30 + 60) = 45°$

$\angle PEB = \frac{1}{2}(30 + 120) = 75°$

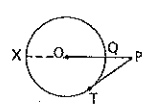
$\angle EBC = \frac{1}{2}(120 + 150) = 135°$

18 a $m\angle X = \frac{1}{2}(320 - 40) = 140$

b $m\angle Y = \frac{1}{2}(90 - 30) = 30$

c $m\angle Z = \frac{1}{2}(70 - 50) = 10$

19 Extend PO to X.



$(TP)^2 = (PQ)(PX)$

$(15)^2 = 5(5 + QX)$

$225 = 25 + 5(QX)$

$200 = 5(QX)$

$40 = QX, 20 = \text{radius}$

20 $m\angle P = \frac{1}{2}(\overset{\frown}{AB} - \overset{\frown}{CD})$

$30 = \frac{1}{2}(\overset{\frown}{AB} - \overset{\frown}{CD})$

$60 = \overset{\frown}{AB} - \overset{\frown}{CD}$

$\overset{\frown}{AB} - \overset{\frown}{CD} = 160$

$\overset{\frown}{AB} - \overset{\frown}{CD} = 60$

$2\overset{\frown}{AB} = 220$

$\overset{\frown}{AB} = 110, \overset{\frown}{CD} = 50$

22 a $\overline{WZ} \cong \overline{YZ}$ by Two-Tangent Theorem. WXYZ is a rhombus because a ▱ with 2 consecutive sides ≅ is a rhombus.

b ∠X ≅ ∠WYZ because if 2 inscribed or tangent chord ∠s intercept the same arc, they are ≅. ∠X ≅ ∠Z (In a ▱ opp ∠s are ≅.)

∠Z ≅ ∠WYZ by Transitive prop

$\overline{WY} \cong \overline{WZ}$ because if △ then △

$\overline{WZ} \cong \overline{YZ}$ (see a)

△WYZ is equilateral and equiangular, so m∠Z = 60.

c From part b, △WYZ is equilateral, so WZ = 15. From part a, WXYZ is a rhombus. So the perimeter = 4 × 15 = 60.

d In a rhombus diags are ⊥ bis of each other making ∠WVZ = 90°. Because △WYZ is equiangular, ∠VWZ = 60°, making △WVZ a 30°60°90° △.

$WV = \frac{1}{2}(WZ)$ $VZ = WV(\sqrt{3})$ $2(VZ) = XZ$

$WV = \frac{1}{2}(15)$ $VZ = \frac{15}{2}(\sqrt{3})$ $2\left(\frac{15\sqrt{3}}{2}\right) = XZ$

$WV = \frac{15}{2}$ $VZ = \left(\frac{15\sqrt{3}}{2}\right)$ $15\sqrt{3} = XZ$

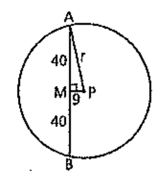
23 $10^2 = 5(5 + x + 9)$ $9 \times 6 = 3y$

$100 = 70 + 5x$ $54 = 3y$

$30 = 5x$ $y = 18$

$x = 6$

25 $\overline{MP}$ bis $\overline{AB}$, so AM = 40



$40^2 + 9^2 = r^2$

$1600 + 81 = r^2$

$41 = r$

$d = 2r = 82$

$C = \pi d = 82\pi \approx 258$ cm