

- 13 $A_{\square} = RCTN = 36 \times 8 = 288$. $A_{\square} = AECT = 288 + 6 = 48$.
Thus the width of $\square AECT = 6$. The coordinates of A must be $(24 - 6, 8)$ or $(18, 8)$.

- 14 $A_{\square} = bh$ If area = 72 and the dimensions are whole numbers, possible dimensions are 1 by 72, 2 by 36, 3 by 24, 4 by 18, 6 by 12, and 8 by 9.

$$\text{Probability} = \frac{\text{#correct}}{\text{possibilities}}$$

$$\frac{4}{6} = \text{first choice}; \frac{3}{6} = \text{second choice}; \frac{4}{6} \cdot \frac{3}{5} = \frac{12}{30} = \frac{2}{5}$$

$$15 \quad 8(5x + 4) = 84 \quad 8(5x + 4) = 124 \\ 40x + 32 = 84 \quad 40x + 32 = 124 \\ x = 1.3 \quad x = 2.3$$

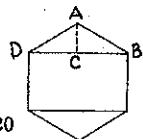
Therefore, $1.3 < x < 2.3$ in order for the rectangle to have an area between 84 and 124 sq mm.

16 $\angle s$ of a polygon = $(n - 2)(180)$

$$\angle s \text{ of hexagon} = (6 - 2)180$$

$$\angle s \text{ of hexagon} = (4)180 = 720$$

$$\text{In a regular hexagon each } \angle = \frac{720}{6} = 120$$



$$\angle ABC = 120 - 90 = 30$$

(Draw $\overline{AC} \perp$ to \overline{BD}), $\angle ACB = 90^\circ$

$\triangle ABC$ is a $30^\circ 60^\circ 90^\circ \Delta$ $\triangle ADC \cong \triangle ABC$ by HL

$$AC = \frac{1}{2}(AB) \quad CB = AC(2\sqrt{3}) \quad DC \cong CB \text{ by CPCTC}$$

$$AC = \frac{1}{2}(12) \quad CB = 6\sqrt{3} \quad DC + CB = DB$$

$$AC = 6 \quad 6\sqrt{3} + 6\sqrt{3} = DB \quad 12\sqrt{3} = DB$$

$$A_{\square} = bh$$

$$A_{\square} = (12\sqrt{3})(12) = 144\sqrt{3} \approx 249.4$$

17 The width of each stripe is $\frac{39}{13} = 3$.

$$\begin{aligned} \text{The area of the red} &= 4 \text{ short stripes} + 3 \text{ long} \\ &= 4(39 \cdot 3) + 3(65 \cdot 3) \\ &= 468 + 585 = 1053 \end{aligned}$$

$$\text{The area of the whole flag is } 39 \cdot 65 = 2535, \text{ so } \frac{1053}{2535} = \frac{27}{65}.$$

$\frac{27}{65}$ of the flag is red.

Pages 519–523 (Section 11.2)

1 a $A_{\Delta} = \frac{1}{2}(22)(18)$ b $A_{\Delta} = \frac{1}{2}(17)(12)$

$$A_{\Delta} = \frac{1}{2} \cdot 396 = A_{\Delta} = (17)(6) =$$

$$198 \text{ sq mm} \quad 102 \text{ sq cm}$$

c $A_{\Delta} = \frac{1}{2}(7)(10)$
 $A_{\Delta} = (7)(5) = 35$

2 $A_{\Delta} = \frac{1}{2}bh$

$$A_{\Delta} = \frac{1}{2}(15) \cdot 16$$

$$A_{\Delta} = (7.5) \cdot 16 = 120$$

3 a $A_{\square} = bh$ $A_{\Delta} = \frac{1}{2}(bh)$
 $A_{\square} = 7 \cdot 3 = 21$ $A_{\Delta} = \frac{1}{2}(7 \cdot 4)$
 $A_{\Delta} = \frac{1}{2}(28) = 14$

$$21 + 14 = 35$$

b $A_{\Delta} = \frac{1}{2}(9 \cdot 12)$ $x^2 = 9^2 + 12^2$

$$A_{\Delta} = \frac{1}{2}(108) = 54$$

$$x^2 = 81 + 144$$

$$x^2 = 225$$

$$x = 15$$

$$A_{\square} = x \cdot 6$$

$$A_{\square} = 15 \cdot 6$$

$$A_{\square} = 90 \quad 54 + 90 = 144$$

4 $21 = \frac{1}{2}(7)(\text{alt})$

$$21 = \frac{7}{2}(\text{alt})$$

$$6 = \text{alt}$$

5 The height = 6 (3-4-5Δ)

$$A_{\Delta} = \frac{1}{2}bh$$

$$A_{\Delta} = \frac{1}{2}(16 \cdot 6)$$

$$A_{\Delta} = 8 \cdot 6 = 48$$

6 $A_{\square} = bh$

$$A_{\square} = 17 \cdot 11 = 187$$

7 $42 = (\text{base})(3) (+ \text{each side by } 3)$

$$14 = \text{base}$$

8 a Since the rt Δ is a $30^\circ 60^\circ 90^\circ \Delta$; h = 8.

$$A_{\Delta} = \frac{1}{2}bh$$

$$A_{\Delta} = \frac{1}{2}(7)(8) = 28$$

b $A_{\Delta} = \frac{1}{2}(10)(12) = 60$

9 a $\triangle ABC$ is a $30^\circ 60^\circ 90^\circ \Delta$, so

$$AB = \frac{1}{2}(AC)$$

$$AB = \frac{1}{2}(12) \quad CB = (AB)\sqrt{3}$$

$$A_{\Delta} = \frac{1}{2}(BH)$$

$$AB = 6$$

$$CB = 6\sqrt{3}$$

$$A_{\Delta} = \frac{1}{2}(6\sqrt{3} \cdot 6)$$

$$A_{\Delta} = \frac{1}{2}(36\sqrt{3})$$

$$A_{\Delta} = 18\sqrt{3}$$

b $\triangle ABC$ is a $45^\circ 45^\circ 90^\circ \Delta$.

$$BC = (AB)\sqrt{2}$$

$$12\sqrt{2} = (AB)\sqrt{2}$$

$$A_{\Delta} = \frac{1}{2}(bh)$$

$$12 = AB$$

$$A_{\Delta} = \frac{1}{2}(12 \cdot 12)$$

$$A_{\Delta} = \frac{1}{2}(144) = 72$$

c $\triangle ABC$ is equilateral and equiangular.

Each $\angle = 60^\circ$.

Draw alt \overline{AD} .

$$A_{\Delta} = \frac{1}{2}(bh)$$

In $\triangle ADB$, shorter

$$A_{\Delta} = \frac{1}{2}(12 \cdot 6\sqrt{3})$$

leg = 6 and alt

$$A_{\Delta} = \frac{1}{2}(72\sqrt{3})$$

is opp $60^\circ \angle$, so alt = $6\sqrt{3}$.

$$A_{\Delta} = 36\sqrt{3}$$

10 a height = $5\sqrt{3}$
 $(30^\circ 60^\circ 90^\circ \Delta)$
 $A_{\square} = bh$
 $= 20 \cdot 5\sqrt{3}$
 $= 100\sqrt{3} \approx 173.2$

b height = $4\sqrt{2}$
 $(45^\circ 45^\circ 90^\circ \Delta)$
 $A_{\square} = bh$
 $= 15 \cdot 4\sqrt{2}$
 $= 60\sqrt{2} \approx 84.9$

11 a $A_{\square} = bh$ $A_{\triangle} = \frac{1}{2}bh$
 $A_{\square} = 10(7) = 70$ $A_{\triangle} = \frac{1}{2}(4)(7) = 14$
 Total area = $70 + 14 = 84$

b $A_{\square} = bh$ $A_{\triangle} = \frac{1}{2}bh$
 $A_{\square} = 13 \cdot 8 = 104$ $A_{\triangle} = \frac{1}{2}(3 \cdot 8) = 12$
 Total area = $104 + 12 + 12 = 128$

12 $A_{\triangle} = \frac{1}{2}bh$
 $A_{\triangle} = \frac{1}{2}(11)(8)$
 $A_{\triangle} = 44$

13 $A_{\square} = A_{\triangle}$ $A_{\triangle} = \frac{1}{2}bh$
 $A_{\square} = bh$ $48 = \frac{1}{2}(8 \cdot h)$
 $A_{\square} = 6 \cdot 8$ $48 = 4h$
 $A_{\square} = 48$ $12 = h$

14 All of the Δ s have the same base and the same height, therefore the same area.

15 $A_{\square} = bh$ $A_{\triangle} = \frac{1}{2}bh$
 $A_{\square} = 17 \cdot 10 = 170$ $A_{\triangle} = \frac{1}{2}(17)(10) = 85$
 Total shaded area = $170 - 85 = 85$

16 Let $3x$ = the base
 $2x$ = the altitude
 $A_{\triangle} = \frac{1}{2}bh$
 $48 = \frac{1}{2}(3x)(2x)$
 $48 = 3x^2$
 $16 = x^2$ The base = 12
 $x = 4$ The altitude = 8

17 $A_{\triangle} = \frac{1}{2}(bh)$ $A_{\square} = bh$ $A_{\triangle} = \frac{1}{2}(bh)$
 $A_{\triangle} = \frac{1}{2}(6 \cdot 8)$ $A_{\square} = 6 \cdot 5$ $A_{\triangle} = \frac{1}{2}(12 \cdot 5)$
 $A_{\triangle} = 24$ $A_{\square} = 30$ $A_{\triangle} = 30$

$24 + 30 + 30 = 84$
 $A_{\text{large}\Delta} = \frac{1}{2}(bh)$
 $A_{\text{large}\Delta} = \frac{1}{2}((8 + 5) \cdot (6 + 12))$
 $A_{\text{large}\Delta} = \frac{1}{2}(13 \cdot 18)$
 $A_{\text{large}\Delta} = \frac{1}{2}(234)$
 $A_{\text{large}\Delta} = 117$

A of shaded region = $117 - 84 = 33$

18 a $A_{\triangle PQR} = \frac{1}{2}(15)(12)$
 $A_{\triangle PQR} = 90$

b $90 = \frac{1}{2}(PS)(RQ)$
 $90 = \frac{1}{2}(10)(RQ)$
 $90 = 5(RQ)$
 $18 = RQ$

19 a Draw alt \overline{AD} , $\triangle ADB \cong \triangle ADC$ (HL)
 $\overline{BD} \cong \overline{DC}$ (CPCTC), $DC = 7$

$A_{\triangle} = \frac{1}{2}bh$

$A_{\triangle} = \frac{1}{2}(14)(24)$

$A_{\triangle} = \frac{1}{2}(336)$

$A_{\triangle} = 168$

b $A_{\triangle} = \frac{1}{2}bh$

$A_{\triangle} = \frac{1}{2}(40)(9)$

$A_{\triangle} = \frac{1}{2}(360)$

$A_{\triangle} = 180$

c $\triangle ABC$ is a $45^\circ 45^\circ 90^\circ \Delta$

$AC = AB\sqrt{2}$

$18 = AB\sqrt{2}$

$9\sqrt{2} = AB$

$A_{\triangle} = \frac{1}{2}bh$

$A_{\triangle} = \frac{1}{2}(9\sqrt{2})(9\sqrt{2})$

$A_{\triangle} = \frac{1}{2}(162) = 81$

20 Each side is 15. By drawing an altitude, a $30^\circ 60^\circ 90^\circ \Delta$ is formed, so the altitude is $\frac{15}{2}\sqrt{3}$.

$A = \frac{1}{2}(15)\frac{15}{2}\sqrt{3}$

$A = \frac{225}{4}\sqrt{3}$ sq meters

21 a Long leg of a $30^\circ 60^\circ 90^\circ \Delta$ is $3\sqrt{3}$.

$A_{\square} = bh$

$A_{\square} = (14)(3\sqrt{3}) = 42\sqrt{3} \approx 72.7$

b The height of a $45^\circ 45^\circ 90^\circ \Delta$ is

$h\sqrt{2} = 10$ $A_{\square} = bh$

$h = \frac{10}{\sqrt{2}}$ $A_{\square} = (17)(5\sqrt{2})$

$h = 5\sqrt{2}$ $A_{\square} = 85\sqrt{2} \approx 120.2$

c $x^2 + 8^2 = 17^2$ $A_{\triangle} = \frac{1}{2}bh$

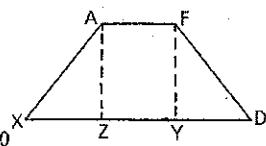
$x^2 + 64 = 289$ $A_{\triangle} = \frac{1}{2}(8)(15)$

$x^2 = 225$ $= 60$ $A_{\square} = A_{\triangle} + A_{\triangle}$

$x = 15$ $A_{\triangle} = \frac{1}{2}(8)(15)$ $A_{\square} = 60 + 60$

$A_{\triangle} = 60$ $A_{\square} = 120$

- 22 a In an isos trap, any upper base \angle supp any lower base \angle .
 $\angle X = \angle D = 180 - 120 = 60^\circ$



Draw alt \overline{FY} . $\triangle FYD$ is a $30^\circ 60^\circ 90^\circ$ \triangle .

$$YD = \frac{1}{2}(FD) \quad FY = (YD)\sqrt{3}$$

$$YD = \frac{1}{2}(6) = 3 \quad FY = 3\sqrt{3}$$

$$A\Delta = \frac{1}{2}(b \cdot h) \quad A\square = b \cdot h$$

$$A\Delta = \frac{1}{2}(3 \cdot 3\sqrt{3}) \quad A\square = 10 \cdot 3\sqrt{3}$$

$$A\Delta = \frac{9\sqrt{3}}{2} \quad A\square = 30\sqrt{3}$$

Areas of $\triangle AZX + \triangle FYD + \square AZYF = A_{\text{trap}}$

$$\frac{9\sqrt{3}}{2} + \frac{9\sqrt{3}}{2} + 30\sqrt{3} = A_{\text{trap}}$$

$$39\sqrt{3} = A_{\text{trap}}$$

- b Draw alt \overline{BE} and \overline{CF}

$$\triangle ABE \cong \triangle DCF$$

$\triangle ABE$ is a $45^\circ 45^\circ 90^\circ$ \triangle .

$$AE + FD = 14 - 8$$

$$AE + FD = 6$$

- AE = 3 $\overline{AE} \cong \overline{EB}$ (if \triangle then \triangle)

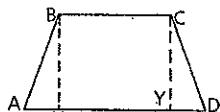
$$A\Delta = \frac{1}{2}(b \cdot h) \quad A\square = b \cdot h$$

$$A\Delta = \frac{1}{2}(3 \cdot 3) = \frac{9}{2} \quad A\square = 8 \cdot 3 = 24$$

Areas of $\triangle ABE + \triangle DCF + \square EBCF = A_{\text{trap}}$

$$\frac{9}{2} + \frac{9}{2} + 24 = A_{\text{trap}}$$

$$33 = A_{\text{trap}}$$



- c Draw alts \overline{BX} and \overline{CY}

$\triangle CYD$ is a $30^\circ 60^\circ 90^\circ$ \triangle .

$$CY = \frac{1}{2}(CD) \quad YD = (CY)\sqrt{3} \quad A_3 = \frac{1}{2}(b \cdot h)$$

$$CY = \frac{1}{2}(16) \quad YD = 8\sqrt{3} \quad A_3 = \frac{1}{2}(8\sqrt{3} \cdot 8)$$

$$CY = 8 \quad A_3 = 32\sqrt{3}$$

$$A_2 = bh$$

$$A_2 = (12)(8) = 96$$

$\triangle BXA$ is a $45^\circ 45^\circ 90^\circ$ \triangle , $\overline{BX} \cong \overline{CY}$, $\overline{BX} = 8$, $\overline{BX} \cong \overline{AX}$

(If \triangle then \triangle)

$$A_1 = \frac{1}{2}(b \cdot h)$$

$$A_1 = \frac{1}{2}(8 \cdot 8) = 32$$

Areas of $\triangle BXA + \triangle CYD + \square BXYC = A_{\text{trap}}$

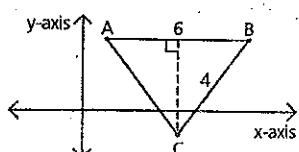
$$32 + 32\sqrt{3} + 96 = A_{\text{trap}}$$

$$128 + 32\sqrt{3} = A_{\text{trap}}$$

23 $A\Delta = \frac{1}{2}bh$

$$A\Delta = \frac{1}{2}(6)(4)$$

$$A\Delta = 12$$



24 a By Pythagorean Theorem, the other leg of the \triangle is 48.
 $A\Delta = \frac{1}{2}b \cdot h$
 $A\Delta = \frac{1}{2}(48)(14)$
 $A\Delta = 336$

b $A\Delta = \frac{1}{2}b \cdot h$
 $336 = \frac{1}{2}(50)h$
 $336 = 25h$
 $h = \frac{336}{25}$ or $13\frac{11}{25}$

25 a $\overline{AZ} = \overline{XY}$ (in a \square opp sides are \cong), so $XY = 8$

$$(4)^2 + (YP)^2 = 8^2$$

$$16 + (YP)^2 = 64$$

$$(YP)^2 = 48$$

$$YP = 4\sqrt{3}$$

Since the sides of the \triangle are 4, $4\sqrt{3}$, 8, it must be a $30^\circ 60^\circ 90^\circ$ \triangle with $\angle Y = 30^\circ$. In a \square opp \angle s are \cong , so $m\angle A = 30^\circ$.

b $A\square = bh$
 $A\square = (8)(6) = 48$
 $A\square = (AX)(4)$
 $48 = (AX)(4)$
 $12 = AX$

26 $A\Delta = \frac{1}{2}(12)(5) = 30$

$$30 \cdot 4 = 120 \quad (4 \cong \Delta s)$$

$$A\square = 120$$

Side of Δ = 13 (Pythagorean triple)

$$13 \cdot 4 = 52$$

$$P\square = 52$$

27 a Let $2x$ = length of side

$$A\Delta = \frac{1}{2}(b \cdot h)$$

$$9\sqrt{3} = \frac{1}{2} \cdot 2x \cdot x\sqrt{3}$$

$$9\sqrt{3} = x^2\sqrt{3}$$

$$9 = x^2$$

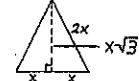
$$x = 3$$

Since each side is $2x$, length is 6.

b Since $A = \frac{1}{2}(b \cdot h)$

$$A = \frac{1}{2}s \cdot \frac{s}{2}\sqrt{3}$$

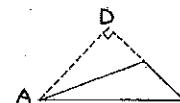
$$A = \frac{s^2}{4}\sqrt{3}$$



28 $\angle CAB = 180 - (120 + 45^\circ)$

$$\angle CAB = 180 - 165^\circ$$

$$\angle CAB = 15^\circ$$



Extend \overline{CB} and drop a \perp from A to form rt $\triangle ABD$.

$$\angle DCA = 180 - 120 = 60^\circ$$

$\triangle CDA$ is a $30^\circ 60^\circ 90^\circ$ \triangle .

$$DC = \frac{1}{2}(AC) \quad DA = (DC)\sqrt{3} \quad \angle DAC + \angle CAB = \angle DAB$$

$$DC = \frac{1}{2}(8) \quad DA = 4\sqrt{3} \quad 30^\circ + 15^\circ = \angle DAB$$

$$DC = 4$$

$$45^\circ = \angle DAB$$

$\triangle DAB$ is a $45^\circ 45^\circ 90^\circ$ \triangle

$\overline{DA} \cong \overline{DB}$ (if Δ then Δ) and $DB = 4\sqrt{3}$

$$CB = DB - DC$$

$$CB = 4\sqrt{3} - 4$$

$$A_{\Delta} = \frac{1}{2}(b \cdot h)$$

$$A_{\Delta} = \frac{1}{2}[4\sqrt{3}(4\sqrt{3} - 4)]$$

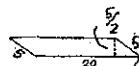
$$A_{\Delta} = \frac{1}{2}(48 - 16\sqrt{3})$$

$$A_{\Delta} = 24 - 8\sqrt{3}$$

29 A \perp forms a $30^\circ 60^\circ 90^\circ \Delta$, so $h = \frac{5}{2}$.

$$A_{\square} = bh$$

$$A_{\square} = 20 \cdot \frac{5}{2} = 50$$

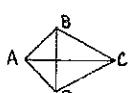


30 Square, because length and width are the same.

31 In a kite one diag is \perp bis of the other.

If $BD = 10$ and $\overline{AC} \perp$ bis of \overline{BD} , then

$$BX = 5, XD = 5.$$



In a kite 2 distinct pairs of consecutive sides are \cong .

$\overline{AB} \cong \overline{AD}, \overline{BC} \cong \overline{DC}$ proving $\Delta ABC \cong \Delta ADC$ by SSS.

$$A_{ABCD} = 2A_{\Delta}$$

$$A_{ABCD} = 2(\frac{1}{2}(b \cdot h))$$

$$A_{ABCD} = 2(\frac{1}{2}(24 \cdot 5))$$

$$A_{ABCD} = 2(\frac{1}{2})(120) = 120$$

32 Perimeter = 154

$$\text{Semiperimeter} = 77$$

Let x = one side

$77 - x$ = other side

$$\text{Area} = b \cdot h$$

10 and 12 are both altitudes.

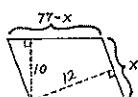
Since area is the same using either altitude or side, set them equal.

$$12x = 10(77 - x) \quad A = b \cdot h$$

$$12x = 770 - 10x \quad A = 12 \cdot 35$$

$$22x = 770 \quad A = 420$$

$$x = 35$$



33 a $A_{\Delta APD} + A_{\Delta BPC}$

$$= \frac{1}{2}xh_1 + \frac{1}{2}xh_2$$

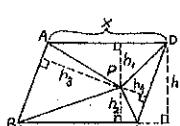
$$= \frac{1}{2}x(h_1 + h_2)$$

$$\frac{1}{2}xh = \frac{1}{2}\text{Area of Rectangle}$$

So $A_{\Delta APB} + A_{\Delta PCD} = \text{area of other } \frac{1}{2} \text{ of rectangle}$

b Yes

c No, because in trapezoid, opp sides are not necessarily \parallel . Also, alt to nonparallel sides are noncollinear.



Pages 525-527 (Section 11.3)

1 a $\frac{8}{2}(11 + 15)$ b $\frac{1}{2}(11 + 15)$

$$4(26) = 104$$

$$\frac{1}{2}(26)$$

$$\text{area} = 104$$

$$\text{median} = 13$$

2 a $A = \frac{h}{2}(b_1 + b_2)$

$$A = \frac{6}{2}(12 + 13)$$

$$A = 3(25)$$

$$A = 75$$

b $A = M \cdot h$

$$A = 12 \cdot 6$$

$$A = 72$$

c $x = 6$ because (3, 4, 5)

$$A = \frac{h}{2}(b_1 + b_2)$$

$$A = \frac{6}{2}(9 + 17)$$

$$A = 3(26) = 78$$

d $A = \frac{h}{2}(b_1 + b_2)$

$$A = \frac{12}{2}(21 + 31)$$

$$A = 6(52) = 312$$

3 a $M = \frac{1}{2}(b_1 + b_2)$

$$M = \frac{1}{2}(6 + 15)$$

$$M = \frac{1}{2}(21) = 10.5$$

b $A_{\text{trap}} = M \cdot h$

$$A_{\text{trap}} = (10.5)(7)$$

$$A_{\text{trap}} = 73.5$$

4 Area $= \frac{h}{2}(8 + 22)$

$$135 = \frac{h}{2}(30)$$

$$270 = h(30)$$

$$h = 9$$

5 $A = \frac{h}{2}(b_1 + b_2)$

$$130 = \frac{10}{2}(15 + b_2)$$

$$130 = 5(15 + b_2)$$

$$130 = 75 + 5b_2$$

$$b_2 = 11$$

6 Middle pole is median.

$$M = \frac{1}{2}(b_1 + b_2)$$

$$M = \frac{1}{2}(30 + 14)$$

$$M = \frac{1}{2}(44) = 22 \text{ feet}$$

7 a $9 \cdot 22 = 198$

$$\frac{1}{2}(22 + 14)11 = (18)11 = 198$$

area of rectangle = 198

total area = 198 + 198 = 396

b area of $\Delta = \frac{1}{2}(8 \cdot 12) = 48$ [Alt is 8 because (3-4-5 Δ)]

$$\text{area of trapezoid} = \frac{1}{2}(8 + 12)3 = 30$$

$$\text{total area} = 48 + 30 = 78$$

8 a Δ is a $30^\circ 60^\circ 90^\circ \Delta$.

$$2x = 6$$

b The top trapezoid:

$$A = \frac{h}{2}(b_1 + b_2)$$

$$A = \frac{5}{2}(10 + 16)$$

$$A = \frac{5}{2}(26) = 65$$

$$b_2 = 7 + 3 + 3 = 13$$

$$A = \frac{h}{2}(b_1 + b_2)$$

$$A = \frac{3\sqrt{3}}{2}(7 + 13)$$

$$A = \frac{3\sqrt{3}(20)}{2}$$

$$A = 30\sqrt{3}$$

The bottom trapezoid:

$$A = \frac{h}{2}(b_1 + b_2)$$

$$A = \frac{5}{2}(16 + 4)$$

$$A = \frac{5}{2}(20) = 50$$

$$\text{Total area} = 65 + 50 = 115$$

$$\begin{aligned} 9 \quad M &= \frac{1}{2}(b_1 + b_2) \\ 17 &= \frac{1}{2}(10 + b_2) \\ 17 &= 5 + \frac{b_2}{2} \\ 12 &= \frac{b_2}{2} \\ 24 &= b \end{aligned}$$

$$\begin{aligned} 10 \quad a \quad 14 &= \frac{1}{2}(18 + PQ) \quad b \quad 25 = \frac{1}{2}(10)(h) \\ 14 &= 9 + \frac{1}{2}PQ \quad 25 = 5 \cdot h \\ 5 &= \frac{1}{2}PQ \quad \text{height} = 5 \\ PQ &= 10 \end{aligned}$$

c height of $\triangle PQS$ = height of trapezoid PQRS
height of trapezoid = 5
d area = $\frac{1}{2}(10 + 18)5 = 70$

11 Find area of square, subtract areas of trapezoids.

$$\begin{aligned} A &= s^2 \quad \text{The height} = 4 \quad (3-4-5\Delta) \\ A &= 13 \cdot 13 \quad \text{So the } A = \frac{h}{2}(b_1 + b_2) \\ A &= 169 \quad A = \frac{4}{2}(7 + 13) = 40 \end{aligned}$$

The area of the shape is $169 - 40 - 40 = 89$.

12 If the upper base = x , the lower base equals

$$\begin{aligned} b^2 &= 35 - (8 + 7 + x) \quad A_{\text{trap}} = \frac{h}{2}(b_1 + b_2) \\ &= 35 - (15 + x) \quad A_{\text{trap}} = \frac{5}{2}(20 - x + x) \\ &= 35 - 15 - x \quad A_{\text{trap}} = \frac{5}{2}(20) = 50 \\ &= 20 - x \end{aligned}$$

$$\begin{aligned} 13 \quad 10x + 2x + 5x + 5x &= 44 \\ 22x &= 44 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} \text{height} &= 3x \quad (3-4-5\Delta) \\ \text{area} &= \frac{1}{2}(b_1 + b_2)h \\ \text{area} &= \frac{1}{2}(4 + 20)6 \\ \text{area} &= 12 \cdot 6 = 72 \end{aligned}$$

14 Each height is 6. So the base of the Δ and top rect is 16 (3-4-5 Δ).

$$\begin{aligned} \text{The } A \text{ of the } \Delta \text{ is:} \quad &\text{The area of the rectangle:} \\ A &= \frac{1}{2}b \cdot h \quad A = b \cdot h \\ A &= \frac{1}{2}(16 \cdot 6) \quad A = 16 \cdot 6 \\ A &= 48 \quad A = 96 \end{aligned}$$

Since all sides of trapezoid are \parallel to all sides of Δ and height is the same, nonparallel sides \cong . The area of the trapezoid is:

$$\begin{aligned} A &= \frac{h}{2}(b_1 + b_2) \\ A &= \frac{6}{2}(16 + 32) \\ A &= 3(48) = 144 \end{aligned}$$

The area of the bottom rect is:

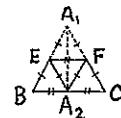
$$\begin{aligned} A &= b \cdot h \\ A &= 32 \cdot 6 = 192 \end{aligned}$$

The total area is: $48 + 96 + 144 + 192 = 480$

15 $\triangle A_1EF \cong \triangle A_2EF$ because it

is the same Δ folded down.

$\overline{A_1E} \cong \overline{A_1F} \cong \overline{A_2F} \cong \overline{A_2E}$ (in an isosceles Δ at least 2 sides are \cong), $\overline{A_1D} \cong \overline{A_2D}$ (alts of the same Δ), so \overline{EF} is $\frac{1}{2}$ the way



between $\overline{A_1A_2}$ and is also the mdpt of \overline{AB} and \overline{AC} .

So, $\overline{EB} \cong \overline{A_1E}$ and $\overline{FC} \cong \overline{A_1F}$. If $\overline{A_1A_2}$ is an alt of $\triangle A_1BC$ and $\overline{AB} \cong \overline{AC}$ because the Δ is isosceles, then $\triangle A_1BA_2 \cong \triangle A_1CA_2$ by HL. $\overline{BA_2} \cong \overline{CA_2}$ by CPCTC. Using Midline Theorem, $\overline{EF} = \frac{1}{2}(\overline{BC})$. $\overline{EF} \cong \overline{BA_2} \cong \overline{CA_2}$.

$\triangle A_2EF \cong \triangle A_1EF \cong \triangle EBA_2 \cong \triangle FCA_2$ by SSS.

Since the area of the trap is 12, and the area of each Δ inside of it is equal, then the 3 Δ s inside— $\triangle A_2EF$, $\triangle EBA_2$, $\triangle FCA_2$ —all have an area $\frac{1}{3}(12) = 4$.

$\triangle A_1EF + \triangle A_2EF + \triangle EBA_2 + \triangle FCA_2 = \text{Area of } \triangle A_1BC$

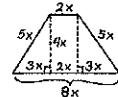
$4 + 4 + 4 + 4 = \text{Area} = 16$

16 Let $4x = \text{altitude}$ (3-4-5 Δ)

$$\begin{aligned} \text{area} &= \frac{1}{2}(b_1 + b_2)h \\ 245 &= \frac{1}{2}(8x + 2x)4x \\ 245 &= 5x \cdot 4x \\ 245 &= 20x^2 \\ x^2 &= \frac{49}{4} \\ x &= \frac{7}{2} \end{aligned}$$

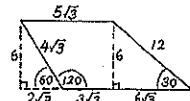
$$\text{altitude} = 4x = 4 \cdot \frac{7}{2} = 14$$

$$\text{perimeter} = 20x = 20 \cdot \frac{7}{2} = 70$$



17 a. If the hyp of the $30^\circ 60^\circ 90^\circ$ Δ is 12, then

$$\begin{aligned} 2x &= 12 \\ x &= 6 \\ x\sqrt{3} &= 6\sqrt{3} \end{aligned}$$



If the long leg of the other $30^\circ 60^\circ 90^\circ$ Δ is 6, then

$$\begin{aligned} x\sqrt{3} &= 6 \\ x &= \frac{6}{\sqrt{3}} \\ x &= 2\sqrt{3} \end{aligned}$$

and the hyp = $2x$ or $4\sqrt{3}$. That makes $3\sqrt{3}$ as the remaining portion of the base.

$$\begin{aligned} A &= \frac{1}{2}(b_1 + b_2) \\ A &= \frac{6}{2}(3\sqrt{3} + 6\sqrt{3} + 5\sqrt{3}) \\ A &= 3(14\sqrt{3}) = 42\sqrt{3} \end{aligned}$$

- b If the hyp of the larger $30^\circ 60^\circ 90^\circ \Delta$ is 12, then

$$2x = 12$$

$$x = 6$$

$$x\sqrt{3} = 6\sqrt{3}$$

If the hyp of smaller $30^\circ 60^\circ 90^\circ \Delta$ is 6, then

$$2x = 6$$

$$x = 3$$

$$x\sqrt{3} = 3\sqrt{3}$$

height = $3\sqrt{3}$ and bases = 12 and 6.

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{3\sqrt{3}}{2}(12 + 6)$$

$$A = \frac{3\sqrt{3}}{2}(18)$$

$$A = \frac{54\sqrt{3}}{2}$$

$$A = 27\sqrt{3}$$

- 18 If X is the mdpt of \overline{AD} and P is the mdpt of \overline{DB} , then using Midline Theorem $\overline{XP} = \frac{1}{2}(b_2)$.

If X is the mdpt of \overline{AD} and Q is the mdpt of \overline{AC} , then using Midline Theorem $\overline{XQ} = \frac{1}{2}(b_1)$.

$$PQ = XQ - XP$$

$$PQ = \frac{1}{2}(b_1) - \frac{1}{2}(b_2)$$

$$PQ = \frac{b_1 - b_2}{2}$$

- 19 a A of $\Delta = \frac{1}{2}b \cdot h$

$$A_{\text{trap}} = A_{\Delta 1} + A_{\Delta 2}$$

$$A_{\text{trap}} = \frac{1}{2}b_1h + \frac{1}{2}b_2h$$

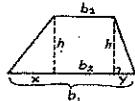
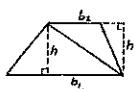
$$A_{\text{trap}} = \frac{1}{2}h(b_1 + b_2)$$

b $A = \frac{1}{2}xh + \frac{1}{2}yh + b_2h$

$$A = \frac{1}{2}h(x + b_2 + b_2 + y)$$

but $b_1 = x + b_2 + y$

$$A = \frac{1}{2}h(b_1 + b_2)$$



20 Slope of $\overline{MN} = \frac{c - e}{d + a - b} = 0$

Slope of $\overline{TR} = \frac{0 - 0}{2a - 0} = 0$

Slope of $\overline{PA} = \frac{2c - 2e}{2d - 2b} = 0$

So, $\overline{MN} \parallel \overline{PA} \parallel \overline{TR}$.

$$MN = d + a - b; PA = 2d - 2b; TR = 2a$$

$$\frac{1}{2}(PA + TR) = d + a - b; \text{ So } MN = \frac{1}{2}(PA + TR).$$

Pages 529–530 (Section 11.4)

1 $A = \frac{1}{2}(d_1)(d_2)$
 $A = \frac{1}{2}(6)(20) = 60$

2 a $A = \frac{1}{2}(8)(14)$

$$A = 56$$

b Rt Δ is (5, 12, 13)

Pythagorean triple.

$$A = \frac{1}{2}(10)(17) = 85$$

c One isos Δ is equilateral so

the remaining side (other diagonal) is 16.

$$A = \frac{1}{2}(16)(20) = 160$$

3 $20 = \frac{1}{2}(8)(d_2)$

$$20 = 4d_2$$

$$5 = d_2$$

4 To find remaining segs: 6, 8, 10 (3-4-5 Δ)

$$8, 15, 17 (8-15-17\Delta) \quad A = \frac{1}{2}(16 \cdot 21) = 168$$

5 a $7^2 + x^2 = 25^2$

$$49 + x^2 = 625$$

$$x^2 = 576$$

$$x = 24$$

$$A = \frac{1}{2}(48 \cdot 14) = 336$$

b A rhombus is a \square .

$$A = (25)(20) = 500$$

6 In a kite one diag is the \perp bis of the other. $\overline{AC} \perp$ bis \overline{BD} ,

$$BD = 10 \text{ so } BX = 5. \Delta ABD \text{ is an isoscles rt } \Delta (45^\circ 45^\circ 90^\circ \Delta).$$

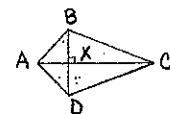
$\angle ABX = 45^\circ$, so ΔABX is also a $45^\circ 45^\circ 90^\circ \Delta$. $\overline{AX} \cong \overline{BX}$,

$AX = 5$. In rt ΔBXC , $BX = 5$, $BC = 13$ and (5, 12, 13) is a Pythagorean triple, so $XC = 12$.

$$A = \frac{1}{2}[(AX + XC) \cdot BD]$$

$$A = \frac{1}{2}[(5 + 12) \cdot 10]$$

$$A = \frac{1}{2}(17 \cdot 10) = 85$$

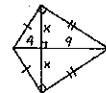


7 Alt to hypotenuse of rt Δ is mean prop between seg of hyp.

$$x^2 = 9 \cdot 4$$

$$x^2 = 36 \quad A = \frac{1}{2}(13 \cdot 12)$$

$$x = 6 \quad A = 78$$



8 In a $30^\circ 60^\circ 90^\circ \Delta$ when $2x = 10$, $x = 5$; $x\sqrt{3} = 5\sqrt{3}$

$$A = \frac{1}{2}(10 \cdot 10\sqrt{3}) = 50\sqrt{3}$$

9 a $A_I = \frac{1}{2}(4)(4) = 8 \quad A_{II} = \frac{1}{2}(8)(7) = 28$

$$A_{III} = \frac{1}{2}(12)(3) = 18$$

b Area of OBD = $(12 \times 7) - (8 + 18 + 28) = 84 - 54 = 30$

10 $AC = 18, BD = 24$

In a rhombus the diagonals are \perp bis of each other, so
 $AL = \frac{1}{2}(AC) = 9$ $LD = \frac{1}{2}(BD) = 12$

$\triangle ALD$ is a rt \triangle (diagonals are \perp). $AL = 3 \cdot 3$

$LD = 3 \cdot 4 (3, 4, 5)$ is a Pythagorean triple,
so $AD = (3)(5) = 15$

$A = b \cdot h$. A rhombus is a parallelogram. So
 $A = (b)(h)$ also, $A = \frac{1}{2}(d_1)(d_2)$

$$A = b \cdot h = \frac{1}{2}(d_1)(d_2)$$

$$A = (AD)(h) = \frac{1}{2}(AC)(BD)$$

$$15h = \frac{1}{2}(18 \cdot 24)$$

$$15h = 216$$

$$h = \frac{216}{15} = \frac{72}{5}$$

11 Find the area of each \triangle formed.

$$A = \frac{1}{2}wy + \frac{1}{2}xy + \frac{1}{2}xz + \frac{1}{2}wz$$

$$A = \frac{1}{2}[y(w+x) + z(w+x)]$$

$$A = \frac{1}{2}(w+z)(y+z)$$

12 Find the area of each \triangle formed— $\triangle PQS$ and $\triangle RQS$.

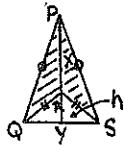
$$A = \text{total - unshaded}$$

$$A = \frac{1}{2}y(x+h) - \frac{1}{2}yh$$

$$A = \frac{1}{2}(x+h-h)$$

$$A = \frac{1}{2}xy$$

Yes, it holds true.



13 $\overline{AD} \cong \overline{BC}, \overline{AB} \cong \overline{DC}$ (In a \square opp sides are \cong ,) so

$\overline{AP} \cong \overline{BQ}$ (Subtraction) $\overline{AX} \cong \overline{SB} \cong \overline{DY} \cong \overline{YC}$ (x and y are mdpts of \cong sides) $\triangle XQB \cong \triangle QZX, \triangle QZY \cong \triangle YCQ$ (HL), $\triangle XQY \cong \triangle QZX + \triangle QZY; \triangle BCY \cong \triangle XQB + \triangle YCQ + \triangle XQZ + \triangle QZY$

$$A_{\triangle XQY} = \frac{1}{2}A_{\triangle BCY}$$

In the same way:

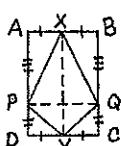
$$A_{\triangle XPY} = \frac{1}{2}A_{\triangle ADY}$$

$$A_{\triangle XQY} = A_{\triangle XQY} + A_{\triangle XPY};$$

$$A_{\text{rect } ABCD} = A_{\triangle BCY} + A_{\triangle ADY}$$

$$A_{\triangle XQY} + A_{\triangle XPY} = \frac{1}{2}(A_{\triangle BCY} + A_{\triangle ADY})$$

$$A_{\triangle XQY} = \frac{1}{2}(A_{\text{rect } ABCD})$$



Pages 533–536 (Section 11.5)

1 $A_{\text{reg poly}} = \frac{1}{2}a \cdot p$

$$A = \frac{1}{2}(3 \cdot 24) = 36$$

2 a $A = \frac{s^2}{4}\sqrt{3}$

$$A = \frac{36}{4}\sqrt{3}$$

$$A = 9\sqrt{3}$$

b $A = \frac{s^2}{4}\sqrt{3}$

$$A = \frac{49\sqrt{3}}{4}$$

c $A = \frac{s^2}{4}\sqrt{3}$

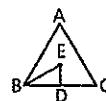
$$A = \frac{64}{4}\sqrt{3}$$

$$A = 16\sqrt{3}$$

d $A = \frac{s^2}{4}\sqrt{3}$

$$A = \frac{(2\sqrt{3})^2\sqrt{3}}{4}$$

$$A = \frac{12\sqrt{3}}{4} = 3\sqrt{3}$$



3 a $\triangle AEDB$ is a $30^\circ 60^\circ 90^\circ \triangle$.

So $BD = 6\sqrt{3}$

and $BC = 2(BD)$

$$A_{\text{eq}\Delta} = \frac{s^2}{4}\sqrt{3}$$

$$BC = 2(6\sqrt{3})$$

$$A_{\text{eq}\Delta} = \frac{(12\sqrt{3})^2}{4}\sqrt{3}$$

$$BC = 12\sqrt{3}$$

$$A_{\text{eq}\Delta} = 108\sqrt{3}$$

b $BD = 4\sqrt{3}$

$$A_{\text{eq}\Delta} = \frac{s^2}{4}\sqrt{3}$$

$$BC = 2(BD)$$

$$A_{\text{eq}\Delta} = \frac{(8\sqrt{3})^2}{4}\sqrt{3}$$

$$BC = 2(4\sqrt{3})$$

$$A_{\text{eq}\Delta} = 48\sqrt{3}$$

$$BC = 8\sqrt{3}$$

$$A_{\text{eq}\Delta} = \frac{s^2}{4}\sqrt{3}$$

c $BD = 3\sqrt{3}$

$$A_{\text{eq}\Delta} = \frac{(6\sqrt{3})^2}{4}\sqrt{3}$$

$$BC = 2(3\sqrt{3})$$

$$A_{\text{eq}\Delta} = 27\sqrt{3}$$

$$BC = 6\sqrt{3}$$

$$A_{\text{eq}\Delta} = \frac{s^2}{4}\sqrt{3}$$

d $BD = 6$

$$A_{\text{eq}\Delta} = \frac{(12)^2}{4}\sqrt{3}$$

$$BC = 2(BD)$$

$$A_{\text{eq}\Delta} = 36\sqrt{3}$$

$$BC = 2(6) = 12$$

$$A_{\text{eq}\Delta} = 36\sqrt{3}$$

4 a $A = \frac{1}{2}ap$

c $A = \frac{1}{2}ap, a = 6$

$$p = 36, a = 3\sqrt{3}$$

$$\frac{1}{12}P = \frac{6}{\sqrt{3}}$$

$$A = \frac{1}{2}(3\sqrt{3})(36)$$

$$p = \frac{72}{\sqrt{3}} = \frac{72\sqrt{3}}{3} = 24\sqrt{3}$$

$$A = 54\sqrt{3} \approx 93.5$$

$$A = \frac{1}{2}(6)(24\sqrt{3}) = 72\sqrt{3} \approx 124.7$$

b $A = \frac{1}{2}ap$

d $A = \frac{1}{2}ap, a = 8$

$$a = 4\sqrt{3}, p = 48$$

$$\frac{1}{12}P = \frac{8}{\sqrt{3}}$$

$$A = \frac{1}{2}(4\sqrt{3})(48)$$

$$p = \frac{96}{\sqrt{3}} = \frac{96\sqrt{3}}{3} = 32\sqrt{3}$$

$$A = 96\sqrt{3} \approx 166.3$$

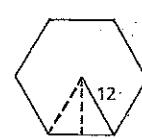
$$A = \frac{1}{2}(8)(32\sqrt{3}) = 128\sqrt{3} \approx 221.7$$

5 a The \triangle formed is a $30^\circ 60^\circ 90^\circ \triangle$,

hyp is 12, side opposite

$30^\circ \angle$ is 6, $2k = 2 \cdot 6 = 12$.

A side is $2x$ or 12.



b If side opp $30^\circ \angle$ is 6, then side opp $60^\circ \angle$ is $6\sqrt{3}$, so the apothem is $6\sqrt{3}$.

c $A = \frac{1}{2}a \cdot p$

$$A = \frac{1}{2}(6\sqrt{3})(72)$$

$$A = \frac{1}{2}(432\sqrt{3}) = 216\sqrt{3}$$

6 a Apothem is 5.

$$\begin{array}{lll} \Delta XBC \text{ is a } 45^\circ 45^\circ 90^\circ \Delta \text{ and } XC = 5, \text{ so } BC = 5. \\ BD = 2(BC) & p = (BD)4 & A = \frac{1}{2}a \cdot p \\ BD = 2(5) & p = (10)4 & A = \frac{1}{2}(5)(40) \\ BD = 10 & p = 40 & A = 100 \end{array}$$

b Apothem is 12.

$$\begin{array}{lll} \Delta XBC \text{ is a } 45^\circ 45^\circ 90^\circ \Delta \text{ and } XC = 12, \text{ so } BC = 12. \\ BD = 2(12) & p = 4(BD) & A = \frac{1}{2}a \cdot p \\ BD = 24 & p = 4(24) & A = \frac{1}{2}(12)(96) \\ & p = 96 & A = 576 \end{array}$$

c Side is 7.

$$A_{\text{square}} = s^2 = 7^2 = 49$$

d In a square, the diagonals \cong and

$$\begin{aligned} A &= \frac{1}{2}(d_1 \cdot d_2) \\ A &= \frac{1}{2}(10 \cdot 10) = 50 \end{aligned}$$

e Radius is half the diagonal. So diagonal = 12

$$\begin{aligned} A &= \frac{1}{2}d_1 \cdot d_2 \\ A &= \frac{1}{2}(12 \cdot 12) = 72 \end{aligned}$$

f perimeter = 12

$$\begin{array}{ll} BD = \frac{\text{per}}{4} & A_{\text{square}} = s^2 \\ BD = \frac{12}{4} = 3 & A_{\text{square}} = 3^2 = 9 \end{array}$$

7 Area of square = $s^2 = 36$, side = 6, p = 24.

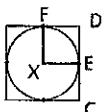
$$\begin{aligned} A &= \frac{1}{2}a \cdot p \\ 36 &= \frac{1}{2}(a \cdot 24) \\ 36 &= 12a \\ a &= 3 \text{ mm} \end{aligned}$$

$$\begin{aligned} 8 \quad A &= \frac{s^2}{4}\sqrt{3} \\ 9\sqrt{3} \text{ km}^2 &= \frac{s^2}{4}\sqrt{3} \\ 36\sqrt{3} \text{ km}^2 &= s^2\sqrt{3} \\ 36 \text{ km}^2 &= s^2, s = 6 \text{ km} \end{aligned}$$

9 $XE = \text{radius of } \odot = 9$

$XE = \text{apothem} = 9$

FXED is a square.



$\overline{DE} \cong \overline{XE}; DE = 9$

$$\begin{array}{lll} 2(DE) = DC & 4(DC) = p & A_{\text{reg poly}} = \frac{1}{2}a \cdot p \\ 2(9) = DC & 4(18) = p & A_{\text{reg poly}} = \frac{1}{2}(9)(72) \\ 18 = DC & 72 = p & A_{\text{reg poly}} = 324 \end{array}$$

10 A $30^\circ 60^\circ 90^\circ \Delta$ is formed. Radius (6) is the side opp $30^\circ \angle$, so side opp $60^\circ \angle$ is $3\sqrt{3}$. Side of equilateral Δ is $2(3\sqrt{3})$ or $6\sqrt{3}$.

$$A_{\Delta} = \frac{s^2\sqrt{3}}{4} = \frac{(6\sqrt{3})^2}{4}\sqrt{3} = 27\sqrt{3}$$

11 The radius is the apothem. It forms a $30^\circ 60^\circ 90^\circ \Delta$ and if

$$\begin{aligned} x\sqrt{3} &= 12 \\ x &= \frac{12}{\sqrt{3}} \\ x &= \left(\frac{12}{\sqrt{3}}\right) \cdot \frac{\sqrt{3}}{\sqrt{3}} = 4\sqrt{3} \end{aligned}$$

Each side of regular hexagon is $2x$ or $8\sqrt{3}$. The perimeter is $6 \cdot 8\sqrt{3}$ or $48\sqrt{3}$.

$$\begin{array}{ll} A = \frac{1}{2}a \cdot p & A = \frac{1}{2}(12 \cdot 48\sqrt{3}) \\ A = 6 \cdot 48\sqrt{3} & A = 288\sqrt{3} \end{array}$$

12 a $A = \frac{s^2}{4}\sqrt{3}$

$$A = \frac{(9)^2}{4}\sqrt{3} = \frac{81\sqrt{3}}{4}$$

b $XE = 7\frac{1}{2}$

FXED is a square.

$$\begin{array}{lll} DE = 7\frac{1}{2} & 4(DC) = p & A = \frac{1}{2}a \cdot p \\ DC = 2(DE) & 4(15) = p & A = \frac{1}{2}(7\frac{1}{2})(60) \\ DC = 2(7\frac{1}{2}) & 60 = p & A = 225 \end{array}$$

DC = 15

c A reg hexagon can be divided up into 6 equilateral Δ s.

$$\begin{aligned} A &= 6\left(\frac{s^2}{4}\sqrt{3}\right) \\ A &= 6\left(\frac{(7\frac{1}{2})^2}{4}\sqrt{3}\right) \\ A &= 6\left(\frac{49\sqrt{3}}{4}\right) = \frac{147\sqrt{3}}{2} \end{aligned}$$

$$\begin{array}{lll} 13 \quad \text{a} \quad A = s^2 & \text{b} \quad A = \frac{s^2}{4}\sqrt{3} & \text{c} \quad \text{side} = \frac{24}{6} \\ 121 = s^2 & 36\sqrt{3} = \frac{s^2}{4}\sqrt{3} & \text{side} = 4 \text{ cm} \\ \text{side} = 11 & 36 = \frac{s^2}{4} & A 30^\circ 60^\circ 90^\circ \Delta \\ A = \frac{1}{2}a \cdot p & 144 = s^2 & \text{is formed so} \\ 121 = \frac{1}{2}a \cdot 44 & s = 12 \text{ m} & a = 2\sqrt{3} \text{ cm.} \\ 121 = 22a & A = \frac{1}{2}a \cdot p & \\ a = 5\frac{1}{2} & 36\sqrt{3} = \frac{1}{2}a \cdot 36 & \\ & \frac{1}{2}a = \sqrt{3} & \\ & a = 2\sqrt{3} \text{ m} & \end{array}$$

14 $A = \frac{1}{2}a \cdot p$
 $64 = \frac{1}{2}(4 \cdot p)$
 $128 = 4 \cdot p, p = 32$

- 15 a. $\triangle BOD$ is a $30^\circ 60^\circ 90^\circ \Delta$.

$$OD = 6 \quad A = \frac{s^2}{4} \sqrt{3}$$

$$BD = 6\sqrt{3} \quad A = \frac{(12\sqrt{3})^2}{4} \sqrt{3}$$

$$BC = 2(BD) \quad A = 108\sqrt{3}$$

$$BC = 2(6\sqrt{3}) = 12\sqrt{3}$$

- b $\triangle BEX$ is a $45^\circ 45^\circ 90^\circ \Delta$.

$$BE \approx EX = 6\sqrt{2}$$

$$BC = 2(BE) \quad p = 4(BC) \quad A = \frac{1}{2}a \cdot p$$

$$BC = 2(6\sqrt{2}) \quad p = 4(12\sqrt{2}) \quad A = \frac{1}{2}(6\sqrt{2})(48\sqrt{2})$$

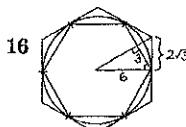
$$BC = 12\sqrt{2} \quad p = 48\sqrt{2} \quad A = 288$$

- c A hexagon can be divided into 6 equilateral Δ s.

$$A = 6\left(\frac{s^2}{4}\sqrt{3}\right)$$

$$A = 6\left(\frac{(12)^2}{4}\sqrt{3}\right)$$

$$A = 6(36\sqrt{3}) = 216\sqrt{3}$$



$$16 \text{ a of reg polygon} = \frac{1}{2}a \cdot p; a = 6, \text{ side} = 4\sqrt{3}, p = 24\sqrt{3}$$

$$\text{a of large hexagon} = \frac{1}{2}(6 \cdot 24\sqrt{3}); 72\sqrt{3};$$

$$a = 3\sqrt{3}, \text{ side} = 6, p = 36$$

$$\text{a of small hexagon} = \frac{1}{2}(3\sqrt{3} \cdot 36)$$

$$\text{a of small hexagon} = 54\sqrt{3}$$

$$\text{Ratio of areas of 2 polygons} = \frac{54\sqrt{3}}{72\sqrt{3}} = \frac{3}{4}$$

- 17 a $A_{\text{shaded region}} = A_{\text{square}} - A_{\Delta}$

$$A_{\text{sq}} = s^2$$

$$A_{\text{sq}} = 6 \cdot 6 = 36$$

$$A_{\Delta} = \frac{s^2}{4}\sqrt{3}$$

$$A_{\Delta} = \frac{36}{4}\sqrt{3} = 9\sqrt{3}$$

$$A_{\text{shaded region}} = 36 - 9\sqrt{3}$$

- b $A_{\text{shaded region}} = A_{\text{polygon}} - A_{\Delta}$

In a $30^\circ 60^\circ 90^\circ \Delta$ if $x = 3, x\sqrt{3} = 3\sqrt{3}$

$$A_{\text{poly}} = \frac{1}{2}a \cdot p$$

$$A_{\text{poly}} = \frac{1}{2}(3\sqrt{3})(36) = 54\sqrt{3}$$

$$A_{\Delta} = \frac{1}{2}b \cdot h, h = 3, b = 6\sqrt{3} \quad (30^\circ 60^\circ 90^\circ \Delta)$$

$$A_{\Delta} = \frac{1}{2}(3)(6\sqrt{3}) = 9\sqrt{3}$$

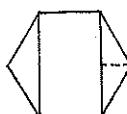
$$3A_{\Delta} = 27\sqrt{3}$$

- c The Δ formed is a $30^\circ 60^\circ 90^\circ \Delta$. If $2x = 6, x = 3$ and $x\sqrt{3} = 3\sqrt{3}$.

Since the alt is the bis of the larger Δ , the length of the rectangle is $6\sqrt{3}$. So

$$A_{\text{rectangle}} = b \cdot h$$

$$A = 6 \cdot 6\sqrt{3} = 36\sqrt{3}$$



- 18 If the figures are numbered consecutively from 1 to 6,

(1—scalene Δ , 2—equilateral Δ , 3—kite, 4—square, 5—regular polygon, and 6—regular hexagon):

possible combinations

1, 2 1, 3 2, 3

1, 4 2, 4 3, 4

1, 5 2, 5 3, 5 4, 5

1, 6 2, 6 3, 6 4, 6 5, 6

combinations that "work"

2, 4 2, 5 4, 5

2, 6 4, 6 5, 6

$$P = \frac{6}{15} = \frac{2}{5}$$

- 19 a $A = \frac{1}{2}ap$

$$A = \frac{1}{2}(15)(60\sqrt{3})$$

$$A = 450\sqrt{3}$$

b $A = \frac{1}{2}ap$

$$32\sqrt{3} = \frac{1}{2}(x\sqrt{3})12x$$

$$32\sqrt{3} = 6x^2\sqrt{3}$$

$$\frac{16}{3} = x^2$$

$$x = \frac{4}{\sqrt{3}}$$

$$x\sqrt{3} = 4$$

$$\text{span} = 2(x\sqrt{3}) = 8$$

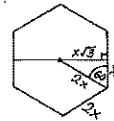
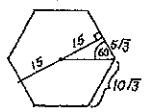
c $x\sqrt{3} = \frac{s}{2}$ (apothem)

$$x = \frac{s\sqrt{3}}{6}$$

$$p = \frac{12s\sqrt{3}}{6}$$

$$A = \frac{1}{2}ap$$

$$A = \frac{1}{2}\left(\frac{s}{2}\right)\left(\frac{12s\sqrt{3}}{6}\right) = \frac{s^2\sqrt{3}}{2}$$

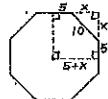


- 20 a In the shaded Δ ($45^\circ 45^\circ 90^\circ \Delta$)

$$10 = x\sqrt{2}$$

$$\frac{10}{\sqrt{2}} = x$$

$$x = 5\sqrt{2}$$



Since the apothem is $5 + x$ and $x = 5\sqrt{2}$, it is $5 + 5\sqrt{2}$.

- b $A_{\text{polygon}} = \frac{1}{2}a \cdot p$

$$A = \frac{1}{2}[(5 + 5\sqrt{2}) \cdot 80]$$

$$A = \frac{1}{2}(400 + 400\sqrt{2}) = 200 + 200\sqrt{2}$$

- 21 a Problem 20a shows that the

apothem of the octagon is

$$5 + 5\sqrt{2}$$

$$\text{diag of square} = 2(5 + 5\sqrt{2})$$

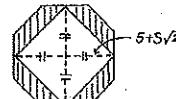
$$\text{diag of square} = 10 + 10\sqrt{2}$$

(The diagonals are \cong and are \perp bis of each other.)

$$A_{\text{sq}} = \frac{1}{2}d_1 \cdot d_2$$

$$A_{\text{sq}} = \frac{1}{2}(10 + 10\sqrt{2})^2$$

$$A_{\text{sq}} = \frac{1}{2}(100 + 200\sqrt{2} + 200) = 150 + 100\sqrt{2}$$



b Problem 20b shows that the area of the reg octagon is $200 + 200\sqrt{2}$.

$$A_{\text{shaded region}} = A_{\text{octagon}} - A_{\text{square}}$$

$$A = (200 + 200\sqrt{2}) - (150 + 100\sqrt{2})$$

$$A = 50 + 100\sqrt{2}$$

22 Since half of the total area of the hexagon is shaded, $A_{\text{shaded region}} = \frac{1}{2}A_{\text{hex}}$. The radius of the hexagon is 4. A $30^\circ 60^\circ 90^\circ \Delta$ is formed with hyp = 4. Apothem is $2\sqrt{3}$ and side is 4. Perimeter is 24.

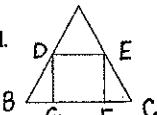
$$\begin{aligned} A_{\text{reg polygon}} &= \frac{1}{2}a \cdot p \\ &= \frac{1}{2}(2\sqrt{3})(24) = 24\sqrt{3} \end{aligned}$$

$$A_{\text{shaded region}} = \frac{1}{2}A_{\text{reg polygon}}$$

$$A_{\text{shaded region}} = \frac{1}{2}(24\sqrt{3}) = 12\sqrt{3}$$

23 $\angle C = 60^\circ$ because $\triangle ABC$ is equilateral. $\triangle EFC$ is a $30^\circ 60^\circ 90^\circ \Delta$.

$$\text{If } EF = x, FC = \frac{x}{\sqrt{3}} = \frac{x\sqrt{3}}{3}$$



$\triangle EFC \cong \triangle DGB$ so,

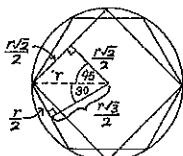
$$\begin{aligned} FC &\cong GB, GB = \frac{12-x}{2} & \frac{x\sqrt{3}}{3} &= \frac{12-x}{2} \\ x &= \frac{36}{3+2\sqrt{3}} \end{aligned}$$

$$\begin{aligned} A_{\text{shaded}} &= A_{\triangle} - A_{\text{square}} \\ &= \frac{s^2}{4}\sqrt{3} - (x)^2 \\ &= \frac{(12)^2}{4}\sqrt{3} - \left(\frac{36}{3+2\sqrt{3}}\right)^2 \\ &= 36\sqrt{3} - \frac{1,296}{21+12\sqrt{3}} \\ &= 1,764\sqrt{3} - 3,024 \end{aligned}$$

24 a side of square = $r\sqrt{2}$

side of hexagon = r

$$\text{ratio} = \frac{r\sqrt{2}}{r} = \frac{\sqrt{2}}{1}$$



$$\begin{aligned} b \frac{A_{\text{square}}}{A_{\text{hexagon}}} &= \frac{(r\sqrt{2})^2}{\frac{1}{2}(r\sqrt{3})(6r)} = \frac{2r^2}{\frac{3r^2\sqrt{3}}{2}} = \frac{4\sqrt{3}}{9} \\ \text{ratio} &= \frac{4\sqrt{3}}{9} \end{aligned}$$

$$25 \text{ a } \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$$

$$\text{b } \frac{1}{2}(a+b)(a+b)$$

$$\text{c } \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2 = \frac{1}{2}(a+b)(a+b)$$

$$ab + \frac{1}{2}c^2 = \frac{1}{2}(a^2 + 2ab + b^2)$$

$$ab + \frac{1}{2}c^2 = \frac{1}{2}a^2 + ab + \frac{1}{2}b^2$$

$$\frac{1}{2}c^2 = \frac{1}{2}a^2 + \frac{1}{2}b^2$$

$$c^2 = a^2 + b^2$$

$$26 \text{ a}_{\text{rectangle}} = 6 \cdot 8 = 48$$

$$\text{Area I} = \frac{1}{2}(8)(2) = 8$$

$$\text{Area II} = \frac{1}{2}(4)(6) = 12$$

$$\text{Area III} = \frac{1}{2}(2)(6) = 6$$

$$\therefore A_{\triangle ABC} = 48 - (8 + 12 + 6) = 22$$

Pages 539–542 (Section 11.6)

$$1 \text{ a } A_{\odot} = \pi r^2 \quad \text{b } A_{\odot} = \pi r^2 \quad \text{c } A_{\odot} = \pi r^2$$

$$\text{r} = 1 \quad \text{r} = 8 \quad \text{r} = 15$$

$$A = \pi 1^2 = \pi \quad A = 64\pi \quad A = 225\pi$$

$$C = 2\pi r \quad C = 2\pi r \quad C = 2\pi r$$

$$C = 2\pi \quad C = 16\pi \quad C = 30\pi$$

$$2 \text{ a } A_{\odot} = \pi r^2 \quad \text{b } A_{\odot} = \pi r^2$$

$$16\pi = \pi r^2 \quad 169\pi = \pi r^2$$

$$16 = r^2 \quad 169 = r^2$$

$$r = 4 \quad r = 13$$

$$3 \quad A_{\odot} = \pi r^2 \quad C = 2\pi r$$

$$100\pi = \pi r^2 \quad C = 2\pi(10)$$

$$100 = r^2 \quad C = 20\pi \text{ cm}$$

$$10 = r$$

$$4 \quad C = \pi d \quad A_{\odot} = \pi r^2$$

$$18\pi = \pi d \quad A = \pi r^2$$

$$d = 18, r = 9 \quad A = 81\pi \text{ sq dm}$$

$$5 \text{ a } A_{\text{sector}} = \left(\frac{m \text{ arc}}{360}\right)\pi r^2 \quad \text{b } A = \left(\frac{180}{360}\right)\pi(8)^2$$

$$A = \left(\frac{90}{360}\right)\pi(2)^2 \quad A = \frac{1}{2}\pi \cdot 64$$

$$A = \frac{1}{4}\pi 4^2 \quad A = 32\pi$$

$$A = \pi$$

$$\text{c } A = \left(\frac{45}{360}\right)\pi(4)^2 \quad \text{d } A = \frac{120}{360}\pi(9)^2$$

$$A = \frac{1}{8}\pi 16 \quad A = \frac{1}{3}\pi 81$$

$$A = 2\pi$$

$$\text{e } A = \frac{270}{360}\pi(10)^2 \quad \text{f } A = \left(\frac{1}{4}\pi(6)^2\right)$$

$$A = \frac{3}{4}\pi 100 \quad A = \frac{1}{4}\pi 36$$

$$A = 75\pi$$

$$A = 9\pi$$

$$6 \text{ a } A_{\odot} = \pi r^2 \quad \odot X + \odot O = \text{Total Area Watered}$$

$$A_{\odot} = \pi(3)^2 \quad 9\pi + 9\pi = \text{Total Area Watered}$$

$$A_{\odot} = 9\pi \quad 18\pi = 57 \text{ sq m} = \text{Total Area Watered}$$

b Total lawn area

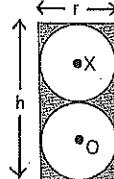
$$h = r + r + r + r$$

$$h = 3 + 3 + 3 + 3 \quad b = r + r$$

$$h = 12 \quad b = 3 + 3$$

$$A = b \cdot h$$

$$A = 6 \cdot 12 = 72 \text{ sq m}$$



c the area of the lawn not watered (shaded)

$$\text{Shaded area} = \text{Area of rect} - \text{Area watered}$$

$$= 72 - 18\pi \approx 15 \text{ sq m}$$

$$\begin{aligned} 7 \quad \text{Area rect} &= bh & \text{Area}_{\odot} &= \pi r^2 \\ &= (15)(10) & &= \pi(5)^2 \\ &= 150 & &= 25\pi \end{aligned}$$

$$\text{Total Area} = 150 + 25\pi$$

$$\begin{aligned} 8 \quad a \quad A &= \pi r^2 & b \quad A &= \pi r^2 \\ 24\pi &= \pi r^2 & 36 &= \pi r^2 \\ 24 &= r^2 & r^2 &= \frac{36}{\pi} \\ r &= \sqrt{24} & r &= \frac{\sqrt{36}}{\sqrt{\pi}} \\ r &= 2\sqrt{6} \approx 4.9 & r &= \frac{6}{\sqrt{\pi}} \text{ or } \frac{6\sqrt{\pi}}{\pi} \approx 3.4 \end{aligned}$$

$$\begin{aligned} 9 \quad a \quad A_{\text{sector}} &= \left(\frac{m \text{ arc}}{360}\right)\pi r^2 & b \quad A &= \left(\frac{80}{360}\right)(12^2)\pi \\ &= \left(\frac{60}{360}\right)(12)^2(\pi) & &= \frac{1}{12}(144\pi) \\ &= \frac{1}{6}(144\pi) & &= 12\pi \\ &= 24\pi \end{aligned}$$

$$\begin{aligned} c \quad \text{Inscribed } \angle &= \frac{1}{2} \text{ arc} & A &= \left(\frac{m \text{ arc}}{360}\right)\pi r^2 \\ 10^\circ &= \frac{1}{2} \text{ arc} & A &= \left(\frac{20}{360}\right)(12)^2\pi \\ 20^\circ &= \text{ arc} & A &= \left(\frac{1}{18}\right)(144\pi) \\ & & A &= 8\pi \end{aligned}$$

$$\begin{aligned} 10 \quad A_{\text{sector}} &= \left(\frac{m \text{ arc}}{360}\right)\pi r^2 \\ 24\pi &= \frac{m \text{ arc}}{360} \cdot 60\pi \\ \frac{2}{5} &= \frac{m \text{ arc}}{360} \\ 5 \text{ m arc} &= 720 \\ \text{m arc} &= 144 \end{aligned}$$

11 For this problem, $A_{\text{segment}} = A_{\text{sector}} - A_{\Delta}$

$$A = \left(\frac{m \text{ arc}}{360}\right)\pi r^2 - \frac{1}{2}bh$$

a Δ is $45^\circ 45^\circ 90^\circ$, Base = $8\sqrt{2}$, h = $4\sqrt{2}$

$$\begin{aligned} A &= \left(\frac{90}{360}\right)\pi 8^2 - \frac{1}{2}(8\sqrt{2})(4\sqrt{2}) \\ A &= \frac{1}{4}\pi(64) - \frac{1}{2}(32)(2) \\ A &= 16\pi - 32 \end{aligned}$$

b Δ is equilateral so $b = 8$, $h = 4\sqrt{3}$

$$A = \left(\frac{60}{360}\right)\pi 8^2 - \frac{1}{2}(8)(4\sqrt{3})$$

$$A = \frac{1}{6}\pi(64) - \frac{1}{2}(18\sqrt{3})$$

$$A = \frac{32}{3}\pi - 16\sqrt{3}$$

c Δ forms two $30^\circ 60^\circ 90^\circ$ Δ s, $h = 3$, $b = 3\sqrt{3}$.

$$A = \left(\frac{120}{360}\right)\pi 6^2 - \frac{1}{2}(6\sqrt{3})(3)$$

$$A = \frac{1}{5}\pi(36) - \frac{1}{2}(18\sqrt{3})$$

$$A = 12\pi - 9\sqrt{3}$$

$$\begin{aligned} 12 \quad a \quad A_{\text{washer}} &= A_{\text{outer}} - A_{\text{inner}} \\ &= \pi(5)^2 - \pi(3)^2 \\ &= 25\pi - 9\pi \\ &= 16\pi \end{aligned} \quad \begin{aligned} b \quad A &= \pi(R^2 - r^2) \\ &= \pi R^2 - \pi r^2 \\ &= (R^2 - r^2)\pi \end{aligned}$$

13 Shaded area = $A_{\text{large}} - 2A_{\text{sem}}$

$$\begin{aligned} a \quad A &= \pi 6^2 - 2(\pi 3^2) & A &= \pi 10^2 - 2(\pi 5^2) \\ A &= 36\pi - 18\pi & A &= 100\pi - 50\pi \\ A &= 18\pi & A &= 50\pi \\ A &= \pi 7^2 - 2\pi \left(\frac{7}{2}\right)^2 \\ A &= 49\pi - \frac{49}{2}\pi \\ A &= \frac{49}{2}\pi \text{ or } 24\frac{1}{2}\pi \end{aligned}$$

b Shaded area is half the entire area.

$$\begin{aligned} 14 \quad a \quad r &= \frac{1}{2}(10) = 5 \\ A_{\odot} &= \pi r^2 & A_{\text{square}} &= s^2 \\ A_{\odot} &= \pi(5)^2 & A_{\text{square}} &= (10)^2 \\ A_{\odot} &= 25\pi & A_{\text{square}} &= 100 \\ A_{\text{shaded region}} &= A_{\text{square}} - A_{\odot} \\ A_{\text{shaded region}} &= 100 - 25\pi \\ b \quad A_{\Delta} &= \frac{s^2}{4}\sqrt{3} & \Delta \text{BOD is a } 30^\circ 60^\circ 90^\circ. \\ A_{\Delta} &= \frac{(10)^2}{4}\sqrt{3} & BD = 5 \text{ so, } OD = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3} \end{aligned}$$

$$A_{\Delta} = 25\sqrt{3}$$

$$A_{\odot} = \pi r^2 \quad A_{\text{shaded region}} = A_{\Delta} - A_{\odot}$$

$$A_{\odot} = \pi \left(\frac{5\sqrt{3}}{3}\right)^2 \quad A = 25\sqrt{3} - \frac{25}{3}\pi$$

$$A_{\odot} = \frac{25}{3}\pi$$

$$c \ A_{\text{square}} = s^2 \quad R = \frac{1}{2}(\text{side})$$

$$A_{\text{square}} = (10)^2 = 100 \quad r = \frac{1}{2}(10) = 5$$

All of the sectors are \cong .

$$A_{\text{sector}} = \left(\frac{m \text{ arc}}{360}\right)\pi r^2 \quad A_{\text{all sectors}} = 4\left(\frac{25\pi}{4}\right)$$

$$A_{\text{sector}} = \frac{90}{360}(5)^2\pi \quad A_{\text{all sectors}} = 25\pi$$

$$A_{\text{sector}} = \frac{1}{4}(25)\pi = \frac{25}{4}\pi$$

$$A_{\text{shaded area}} = A_{\text{square}} - A_{\text{sectors}}$$

$$A_{\text{shaded area}} = 100 - 25\pi$$

$$d \quad A_{\Delta} = \frac{s^2}{4}\sqrt{3} \quad \text{All the sectors are } \cong,$$

$$A_{\Delta} = \frac{(10)^2}{4}\sqrt{3} \quad \text{the radius of each sector}$$

$$A_{\Delta} = 25\sqrt{3} \quad \text{is } 5 \text{ (Two Tangent Theorem).}$$

$$A_{\text{sector}} = \left(\frac{m \text{ arc}}{360}\right)\pi r^2 \quad \text{There are 3 sectors,}$$

$$A_{\text{sector}} = \frac{60}{360}\pi(5)^2 \quad \text{so } \frac{3(25\pi)}{2} = \frac{25\pi}{2}$$

$$A_{\text{sector}} = \frac{1}{6}(25\pi) = \frac{25\pi}{6}$$

$$A_{\text{shaded figure}} = A_{\Delta} - A_{\text{sectors}}$$

$$A_{\text{shaded figure}} = 25\sqrt{3} - \frac{25\pi}{2}$$

$$e \ A_{\text{square}} = s^2 \quad \text{diag} = \text{side } \sqrt{2}$$

$$A_{\text{square}} = (10)^2 = 100 \quad \text{diag} = 10\sqrt{2}$$

$$\text{diameter} = \text{diag} = 10\sqrt{2}$$

$$\text{radius} = \frac{1}{2}(\text{diameter}) \quad A_{\odot} = \pi r^2$$

$$\text{radius} = \frac{1}{2}(10\sqrt{2}) = 5\sqrt{2} \quad A_{\odot} = \pi(5\sqrt{2})^2$$

$$A_{\odot} = 50\pi$$

$$A_{\text{shaded}} = A_{\odot} - A_{\text{square}}$$

$$A_{\text{shaded}} = 50\pi - 100$$

f A hexagon can be divided into 6 equilateral Δ s.

$$A_{\text{hex}} = 6\left(\frac{s^2}{4}\sqrt{3}\right)$$

$$A_{\text{hex}} = 6\left(\frac{(10)^2}{4}\sqrt{3}\right)$$

$$A_{\text{hex}} = 6(25\sqrt{3}) = 150\sqrt{3}$$

radius (circle) = apothem hexagon = alt Δ

The alt divides the equilateral Δ into two $30^\circ 60^\circ 90^\circ$ Δ s.

$$\text{The base} = \frac{1}{2}(10) = 5 \quad A_{\odot} = \pi r^2$$

$$\text{alt} = \text{base } (\sqrt{3}) = 5\sqrt{3} \quad A_{\odot} = \pi(5\sqrt{3})^2$$

$$\text{alt} = \text{radius} = 5\sqrt{3} \quad A_{\odot} = 75\pi$$

$$A_{\text{shaded}} = A_{\text{hex}} - A_{\odot}$$

$$A_{\text{shaded}} = 150\sqrt{3} - 75\pi$$

$$15 \text{ Area of shaded region} = \frac{1}{2}(A_{\text{large } \odot}) - 3\left(\frac{1}{2}A_{\text{semi } \odot}\right)$$

$$A_{\text{large } \odot} = \pi r^2 \quad 3\left(\frac{1}{2}A_{\text{semi } \odot}\right) = 3\left(\frac{1}{2}\pi 2^2\right) \\ = \pi 6^2 \quad = \frac{3}{2}\pi 4 \\ = 36\pi \quad = 6\pi$$

$$A_{\text{semi } \odot} = 18\pi$$

$$\text{Area of shaded region} = 18\pi - 6\pi = 12\pi$$

16 To find the areas of different bands, use the formula for an ANNULUS, as derived in problem 12b:

$$A_{\text{ANNULUS}} = (R^2 - r^2)\pi$$

The radius of the entire figure is 25 cm. Each subsequent band has a radius 5 cm less than the previous band. The areas are as follows.

$$a \quad \text{Large dark band} = (25^2 - 20^2)\pi \approx 706.9 \text{ sq cm}$$

$$\text{Medium dark band} = (15^2 - 10^2)\pi \approx 392.7 \text{ sq cm}$$

$$\text{Bulls-eye} = (5^2)\pi \approx 78.5 \text{ sq cm}$$

$$\text{Total shaded area} \approx 1,178 \text{ sq cm}$$

$$b \quad \text{Large light band} = (20^2 - 15^2)\pi \approx 549.8 \text{ sq cm}$$

$$\text{Small light band} = (10^2 - 5^2)\pi \approx 235.6 \text{ sq cm}$$

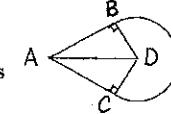
$$\text{Total unshaded area} \approx 785 \text{ sq cm}$$

$$c \quad \text{Probability} = \frac{A_{\text{bulls-eye}}}{A_{\text{target}}} = \frac{25\pi}{625\pi} = \frac{1}{25}$$

17 The pieces fit together to make 8 solid squares. The A of one square is $4(A_{\text{sq}} = b \cdot h)$, so the area of 8 squares is 32 sq cm.

18 a $\angle B$ and $\angle C$ are rt \angle s because

a tangent line is \perp to the radius drawn to the pt of contact.



$$\angle CDA \cong \angle BDA = 60^\circ; \Delta ABD \text{ and } \Delta ACD \text{ are}$$

$$30^\circ 60^\circ 90^\circ \Delta \text{s}, AB = 12, BD = 4\sqrt{3}$$

$$A_{\text{sector}} = \left(\frac{m \text{ arc}}{360}\right)\pi r^2 \quad A_{\Delta} = \frac{1}{2}b \cdot h$$

$$A_{\text{sector}} = \frac{120}{360}(4\sqrt{3})^2\pi \quad A_{\Delta} = \frac{1}{2}(12 \cdot 4\sqrt{3})$$

$$A_{\text{sector}} = \frac{1}{3}(48)\pi = 16\pi$$

$$ABDC = 2(\Delta ABD)$$

$$ABDC = 2(24\sqrt{3}) = 48\sqrt{3}$$

$$A_{\text{shaded}} = A_{ABDC} - A_{\text{sector}} = 48\sqrt{3} - 16\pi$$

b Shaded region is 2 sectors + 2 equil Δ s.

$$A_{\Delta} = \frac{s^2\sqrt{3}}{4} \quad A_{\text{sector}} = \frac{m \text{ arc}}{360} \cdot \pi r^2$$

$$A_{\Delta} = \frac{6^2\sqrt{3}}{4} \quad A_{\text{sector}} = \frac{120}{360} \cdot \pi(6)^2$$

$$A_{\Delta} = 9\sqrt{3} \quad A_{\text{sector}} \approx 12\pi$$

$$2A_{\Delta} = 18\sqrt{3} \quad 2A_{\text{sector}} = 24\pi$$

$$A_{\text{shaded region}} = 18\sqrt{3} + 24\pi$$



$$\begin{aligned}
 19 \text{ a } A_{\text{sector}} &= \frac{60}{360} \cdot 10^2 \pi \\
 &= \frac{1}{6} \cdot 100 \cdot \pi \\
 &= \frac{50}{3} \pi \\
 &= \frac{100}{4} \sqrt{3} = 25\sqrt{3}
 \end{aligned}$$

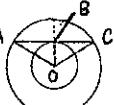
$$\begin{aligned}
 A_{\text{segment}} &= \frac{50}{3} \pi - 25\sqrt{3} \\
 &= \text{Total area } 3(\frac{50}{3} \pi - 25\sqrt{3}) + A_{\text{triangle}} \\
 &= 50\pi - 75\sqrt{3} + 25\sqrt{3} \\
 &= 50\pi - 50\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{b } C &= \pi d \\
 &= \frac{1}{6} \cdot 20\pi \quad \text{There are 3 segs, so} \\
 P &= 3(\frac{10}{3}\pi) = 10\pi
 \end{aligned}$$

20 a $A_{\text{shaded}} = A_{\text{large circle}} - (A_{\text{small circle}} + A_{\text{seg}})$ where

$$A_{\text{seg}} = A_{\text{sector}} - A_{\Delta AOC}$$

$$\begin{aligned}
 A_{\text{large circle}} &= \pi r^2 & A_{\text{small circle}} &= \pi r^2 \\
 A_{\text{large circle}} &= (12^2)\pi & A_{\text{small circle}} &= (6^2)\pi \\
 A_{\text{large circle}} &= 144\pi & A_{\text{small circle}} &= 36\pi
 \end{aligned}$$



Triangle ABO is a 30°60°90° triangle.

Therefore AB = $6\sqrt{3}$ and AC = $12\sqrt{3}$, $\angle AOC = 120^\circ$.

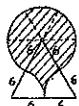
$$\begin{aligned}
 A_{\text{seg}} &= \left(\frac{\text{m arc}}{360}\right)\pi r^2 - \frac{1}{2}b \cdot h \\
 &= \frac{120}{360}(\pi)(12^2) - \frac{1}{2}(12\sqrt{3})(6) \\
 &= \frac{144\pi}{3} - 36\sqrt{3} = 48\pi - 36\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 A_{\text{shaded}} &= 144\pi - (36\pi + 48\pi - 36\sqrt{3}) \\
 &= 144\pi - 84\pi + 36\sqrt{3}
 \end{aligned}$$

$$A_{\text{shaded}} = 60\pi + 36\sqrt{3}$$

$$\begin{aligned}
 \text{b } A_{\text{semi circle}} &= \frac{1}{2}\pi r^2 & \text{The shaded} \\
 A_{\text{shaded region}} &= 2[\frac{1}{2}\pi(R^2 - r^2)] & \text{figure becomes} \\
 &= 2[\frac{1}{2}\pi(6^2 - 3^2)] & \text{a circle with} \\
 &= 2[\frac{1}{2}\pi \cdot 27] & \text{radius } 3\sqrt{3}. \\
 &= 27\pi
 \end{aligned}$$

c The shaded area is formed by subtracting 2 sectors from the Δ and then adding 5 sectors in the \odot .



$$\begin{aligned}
 A_{\text{sector}} &= \frac{\text{m arc}}{360} \cdot \pi r^2 & A_{\Delta} &= \frac{s^2\sqrt{3}}{4} \\
 &= \frac{60}{360} \cdot \pi \cdot 6^2 & &= \frac{12^2\sqrt{3}}{4} \\
 &= \frac{1}{6} \cdot 36\pi = 6\pi & &= \frac{144}{4}\sqrt{3} = 36\sqrt{3}
 \end{aligned}$$

$$A_{\Delta} - 2 \text{ sectors} = 36\sqrt{3} - 2(6\pi)$$

$$A_{\text{shaded}} = A_{\Delta} - 2A_{\text{sectors}} + 5A_{\text{sectors}}$$

$$A_{\text{shaded}} = 36\sqrt{3} - 12\pi + 5(6\pi) = 36\sqrt{3} + 18\pi$$

21 a The shaded area is formed by adding the Δ and the 2 smaller semicircles then subtracting the larger semicircle.

$$r = \frac{1}{2}(10) = 5$$

$$\begin{aligned}
 A_{\Delta} &= \frac{1}{2}b \cdot h & A_{\text{sector}} &= \left(\frac{\text{m arc}}{360}\right)\pi r^2 \\
 A_{\Delta} &= \frac{1}{2}(24 \cdot 10) & A_{\text{sector}} &= \frac{180}{360}\pi(5)^2 \\
 A_{\Delta} &= 120 & A_{\text{sector}} &= \frac{1}{2}(25)\pi \\
 & & A_{\text{sector}} &= \frac{25\pi}{2}
 \end{aligned}$$

$$r = \frac{1}{2}(24) = 12$$

$$\begin{aligned}
 A_{\text{sector}} &= \frac{120}{360}\pi r^2 & A_{\text{sector}} &= \frac{1}{2}(144\pi) \\
 A_{\text{sector}} &= \frac{180}{360}\pi(12)^2 & A_{\text{sector}} &= \frac{144\pi}{2}
 \end{aligned}$$

Δ is a Pythagorean triple (10, 24, 26)

$$r = \frac{1}{2}(26) = 13$$

$$\begin{aligned}
 A_{\text{lg semi circle}} &= \frac{1}{2}\pi r^2 \\
 A_{\text{lg semi circle}} &= \frac{1}{2}(13^2\pi) \\
 A_{\text{lg semi circle}} &= \frac{1}{2}(169\pi)
 \end{aligned}$$

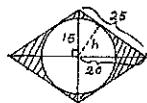
$$\begin{aligned}
 \text{Shaded Area} &= A_{\Delta} + A_{\text{sector}} + A_{\text{sector}} - A_{\text{lg semi circle}} \\
 \text{Shaded Area} &= 120 + \frac{144\pi}{2} + \frac{25\pi}{2} - \frac{169\pi}{2} = 120
 \end{aligned}$$

b $A_{\Delta} = \frac{1}{2} b \cdot h$
 $A = \frac{1}{2}(10 \cdot 24) = 120$

22 $A_{\text{rhomb}} - A_{\odot} = A_{\text{shaded region}}$
 $A_{\text{rhomb}} = \frac{1}{2}(d_1 \cdot d_2)$
 $A_{\text{rhomb}} = \frac{1}{2}(30 \cdot 40) = 600$

Base (the hyp) of Δ formed = 25
Since: $\frac{1}{2} \cdot 15 \cdot 20 = \frac{1}{2} \cdot 25 \cdot h$
 $150 = \frac{h}{2}(25)$
 $\frac{h}{2} = 6$
 $h = 12$

$A_{\odot} = \pi r^2$
 $A_{\odot} = 12^2\pi = 144\pi$
 $A_{\text{shaded region}} = 600 - 144\pi$



23 Area of smaller circle = $\pi(4)^2 = 16\pi$

Area of larger circle = $\pi(7)^2 = 49\pi$

The area of the shaded region is $49\pi - 16\pi$, or 33π . Thus, the probability = $\frac{33\pi}{49\pi}$, or $\frac{33}{49}$.

Pages 546–549 (Section 11.7)

1 a $A_{\text{rect}} = bh$ $A_{\text{par}} = bh$
= $(10)(9) = 90$ = $(10)(9) = 90$

Ratio is 1:1.

b $A = \frac{1}{2}bh$ $A = \frac{1}{2}bh$
= $\frac{1}{2}(8)(5) = 20$ = $\frac{1}{2}(16)(5) = 40$

Ratio is 20:40 or 1:2.

c $A = \frac{1}{2}(10)(6)$ $A = \frac{1}{2}(4)(8)$
 $A = 30$ $A = 16$

Ratio is 30:16 or 15:8.

d $A = \frac{1}{2}(2)(5)$ $A = \frac{1}{2}(3)(4)$
 $A = 5$ $A = 6$

Ratio is 5:6.

2 a $\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$ c $\frac{A_1}{A_2} = \frac{8^2}{4^2} = \frac{64}{16} = \frac{4}{1}$
 $\frac{A_1}{A_2} = \frac{5^2}{2^2} = \frac{25}{4}$ Ratio is 4:1.

Ratio is 25:4.

b $\frac{A_1}{A_2} = \frac{6^2}{9^2} = \frac{36}{81} = \frac{4}{9}$ d $\frac{A_1}{A_2} = \frac{2^2}{6^2} = \frac{4}{36} = \frac{1}{9}$
Ratio is 4:9. Ratio is 1:9.

- 3 a $A_{\Delta PQM}: A_{\Delta PRM}$
A median of a Δ divides the Δ into 2 Δ s with equal areas. $A_{\Delta PQM} = A_{\Delta PRM}$, ratio is 1:1.
b $A_{\Delta PQM}: A_{\Delta PQR}$ c QR:MR $\overline{QM} \cong \overline{MR}$, so $A_{\Delta PQM}: 2(A_{\Delta PQR})$
Ratio is 1:2. QR = QM + MR
 QR = MR + MR
 MR + MR: MR = 2:1

4 $\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2 = \left(\frac{4}{9}\right)^2$

Ratio of areas is 16:81.

5 $\frac{9}{16} = \frac{a_1}{a_2}$

$\frac{9}{16} = \left(\frac{s_1}{s_2}\right)^2$ so $\frac{s_1}{s_2} = \frac{3}{4}$

Ratio of alt is 3:4.

6 Area of a square garden is s^2 . If side is doubled, $(2s)^2 = 4s^2$, so size is multiplied by 4.

7 $\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2 = \left(\frac{2}{8}\right)^2 = \frac{4}{64}$

Ratio of areas is 1:16.

8 In the large $30^\circ 60^\circ 90^\circ \Delta$, the side opp 30° is $\frac{1}{2}(14) = 7$

$\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2 = \frac{7^2}{4^2}$

Ratio is 49:16.

9 a $\frac{A_I}{A_{II}} = \left(\frac{s_I}{s_{II}}\right)^2 = \left(\frac{8}{15}\right)^2$ b $\frac{A_I}{A_{II}} = \frac{\frac{1}{2}b \cdot h}{b \cdot h} = \frac{\frac{1}{2}(10)(14)}{(10)(14)}$

Ratio is 64:225. Ratio is 1:2.

c Even though the large Δ is trisected, $A_I = A_{II}$ because the bases are \cong and the heights are the same. The ratio is 1:1.

d Using \parallel lines \Rightarrow alt. int. \angle s \cong and vert \angle s are \cong , $\Delta I \sim \Delta II$ by AA.

$\frac{A_I}{A_{II}} = \left(\frac{s_I}{s_{II}}\right)^2 = \left(\frac{12}{18}\right)^2 = \left(\frac{2}{3}\right)^2$

Ratio is 4:9.

10 a $\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$ b $A_1 = \frac{1}{2} \cdot 2h = h$
= $\left(\frac{2}{9}\right)^2$ $A_2 = \frac{1}{2} \cdot 9h = \frac{9}{2}h$

Ratio of areas = 4:81. $\frac{A_1}{A_2} = \frac{h}{\frac{9}{2}h} = \frac{2}{9}$

Ratio of areas = 2:9.

c $A_1 = \frac{1}{2} \cdot 1b$ d $\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$
= $\frac{1}{2} \cdot b$ = $\left(\frac{12}{18}\right)^2$

$A_2 = \frac{1}{2} \cdot 3b$ = $\frac{144}{324} = \frac{4}{9}$
= $\frac{3}{2}b$ Ratio of areas = 4:9.

$\frac{A_1}{A_2} = \frac{\frac{1}{2}b}{\frac{3}{2}b} = \frac{1}{3}$

Ratio of areas = 1:3.

11 $\Delta 1$ is 5-12-13 Δ , $\Delta 2$ is 3-4-5 Δ , so sides are 6, 8, 10.

$A_{\Delta 1} = \frac{1}{2}b \cdot h$ $A_{\Delta 2} = \frac{1}{2}bh$
= $\frac{1}{2}(12)(5)$ = $\frac{1}{2}(6)(8)$

= 30 = 24

$\frac{A_{\Delta 1}}{A_{\Delta 2}} = \frac{30}{24}$; ratio is 5:4.

12 a $\left(\frac{s_1}{s_2}\right)^2 = \frac{A_1}{A_2} = \frac{8}{18} = \frac{4}{9}$

$$\frac{s_1}{s_2} = \frac{2}{3}$$

Ratio is 2:3.

b The ratio of the perimeters of 2 ~ figures = the ratio of any pair of corr sides. 2:3

13 $\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$

$$\frac{40}{A_2} = \left(\frac{2}{5}\right)^2$$

$$\frac{40}{A_2} = \frac{4}{25}$$

$$4A_2 = 1,000$$

$$A_2 = 250$$

14 $A_{\Delta} = \frac{1}{2}(b \cdot h)$ $A = \frac{1}{2}(b \cdot h)$

The Δ forms two 5-12-13 Δ s so $h = 12$.

$$A = \frac{1}{2}(10 \cdot 12)$$

$$A = \frac{1}{2}(12)(16)$$

$$A = 60$$
 $A = 96$

$$\frac{A}{A} = \frac{60}{96}$$

$$\frac{A}{A} = \frac{5}{8}; \text{ ratio is } 60:96 \text{ or } 5:8.$$

15 $A_{\odot} = \pi r^2$ $A_{\odot} = \pi r^2$

$$A_{\odot} = \pi(4)^2$$

$$A_{\odot} = 16\pi$$

\odot s are always similar so, $\frac{A_1}{A_2} = \frac{16\pi}{81\pi}$; ratio is 16:81.

16 $\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$ $\frac{A_1}{A_2} = \left(\frac{6}{8}\right)^2$ $\frac{A_1}{A_2} = \frac{36}{64}$; ratio is 9:16.

17 $\frac{AD}{6} = \frac{6}{9}$ (alt is mean prop between segs of hyp)

$$9AD = 36$$

$$AD = 4$$

$$\begin{aligned} A_{\Delta ACD} &= \frac{1}{2}(bh) & A_{\Delta ABCD} &= \frac{1}{2}(bh) \\ &= \frac{1}{2}(4 \cdot 6) & &= \frac{1}{2}(9 \cdot 6) \\ &= \frac{1}{2}(24) = 12 & &= \frac{1}{2}(54) = 27 \end{aligned}$$

$$\frac{A_{\Delta ACD}}{A_{\Delta BCD}} = \frac{12}{27}; \text{ ratio is } 4:9.$$

18 a ΔWYZ to ΔXYZ

The base and height in both of these Δ s are the same, so ratio is 1:1.

b ΔWXZ to ΔWXY

The height and base in both of these are the same, as in part a, so ratio is 1:1.

c ΔWPZ to ΔXPY

$$\Delta WPZ = \Delta WXZ - \Delta WPX, \Delta XPY = \Delta WXY - \Delta WPX.$$

Since ΔWXY and ΔWXZ are a ratio of 1:1, the Δ s left after subtracting ΔWPX from each are also in a ratio of 1:1.

d Δ s are similar by AA.

$$\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$$
 $\frac{A_1}{A_2} = \left(\frac{12}{16}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$

Ratio is 9:16.

e ΔWPX to ΔXPY (see part d) $\frac{WP}{PY} = \frac{3}{4}$

ΔWPX and ΔXPY have = heights.

$$\frac{\Delta WPX}{\Delta XPY} = \frac{\frac{1}{2}(b \cdot h)}{\frac{1}{2}(b \cdot h)} = \frac{\frac{1}{2}(3 \cdot h)}{\frac{1}{2}(4 \cdot h)} = \frac{3}{4}; \text{ ratio is } 3:4$$

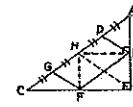
19 a All 8 small Δ s have the

same area, so the ratio of

$$\Delta ABC$$
 to $\Delta EBF = 8:2 = 4:1$

b 8:1 c 1:1 d 4:8 = 1:2

e $GD = 10$ and \overline{DE} median to hypotenuse of ΔHEA , $DE = 5$, perimeter = $2 \cdot 10 + 2 \cdot 5 = 30$



20 $\Delta ABH \cong \Delta XBH$

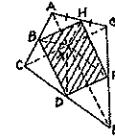
$$\Delta GHF \cong \Delta XHF$$

$$\Delta BCD \cong \Delta XBD$$

$$\Delta EDF \cong \Delta XDF$$

So the unshaded portion is = to the shaded portion.

$$\frac{\text{Area of the large quadrilateral}}{\text{Area of the small quadrilateral}} = \frac{2}{1} \text{ or } 2:1$$



21 Because the bases of the trap are ||, the height of both Δ s is the same.

$$A_{\Delta} = \frac{1}{2}b \cdot h$$

$$A_{\Delta I} = \frac{1}{2}(15 \cdot h)$$

$$\frac{A_{\Delta I}}{A_{\Delta II}} = \frac{\frac{1}{2}(15h)}{\frac{1}{2}(18h)} = \frac{15}{18} = \frac{5}{6}$$

Ratio is 5:6.

22 a Areas of II + III = 30

$$I + IV = 30$$

$$I + II = 15$$

$$III + IV = 45$$

$$\Delta I \sim \Delta III \text{ so } \frac{I}{III} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$4(I) = III \text{ or } \frac{III}{4} = I$$

Substitute and solve simultaneously.

$$I + II = 15 \text{ becomes } \frac{III}{4} + II = 15$$

$$\text{and } II + III = 30$$

$$III + 4II = 60$$

$$III + II = 30$$

$$3II = 30$$

$$II = 10, \text{ so } I = 5, III = 20, IV = 25$$

b $\frac{QT}{TR} = \frac{x}{y}$ $\frac{I}{PQRS} = ?$

Area I = $\frac{1}{2}(wx)$

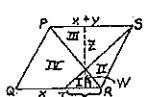
Area PQRS = $(x+y)(w+z)$

III - I so $\frac{w}{z} = \frac{y}{x+y}$ and $\frac{w}{w+z} = \frac{y}{x+2y}$

$\frac{A_I}{A_{PQRS}} = \frac{\frac{1}{2}(wx)}{(x+y)(w+z)}$ or (substituting)

$$\frac{\frac{1}{2}y \cdot y}{(x+y)(x+2y)}$$

$A_I = \frac{\frac{1}{2}y^2}{x^2 + 3xy + 2y^2} = \frac{y^2}{2x^2 + 6xy + 4y^2}$



Pages 551–552 (Section 11.8)

1 a $A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{6(6-3)(6-4)(6-5)}$

$A = \sqrt{6(3)(2)(1)} = \sqrt{36} = 6$

b $A_\Delta = \sqrt{5(5-3)(5-3)(5-4)}$

$A = \sqrt{5(2)(2)(1)}$

$A = \sqrt{4 \cdot 5} = 2\sqrt{5}$

c $A_\Delta = \sqrt{10(10-5)(10-6)(10-9)}$

$A = \sqrt{10(5)(4)(1)}$

$A = \sqrt{200} = 10\sqrt{2}$

d $A_\Delta = \sqrt{9(9-3)(9-7)(9-8)}$

$A = \sqrt{9(6)(2)(1)}$

$A = \sqrt{108} = 6\sqrt{3}$

e $A_\Delta = \sqrt{20(20-8)(20-15)(20-17)}$

$A = \sqrt{20(12)(5)(3)}$

$A = \sqrt{3600} = 60$

f $A_\Delta = \sqrt{21(21-13)(21-14)(21-15)}$

$A = \sqrt{21(8)(7)(6)}$

$A = \sqrt{7056} = 84$

2 $s = \frac{a+b+c}{2}$ $A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{8+8+8}{2}$ $A = \sqrt{12(12-8)(12-8)(12-8)}$

$s = \frac{24}{2} = 12$ $A = \sqrt{12(4)(4)(4)}$

$A = \sqrt{768} = 16\sqrt{3}$

3 a $A = \sqrt{(13-5)(13-7)(13-4)(13-10)}$

$A = \sqrt{(8)(6)(9)(3)} = \sqrt{1,296} = 36$

b $A = \sqrt{(10-2)(10-4)(10-5)(10-9)}$

$A = \sqrt{(8)(6)(5)(1)} = \sqrt{240} = 4\sqrt{15}$

c $A = \sqrt{(11-3)(11-5)(11-9)(11-5)}$

$A = \sqrt{(8)(6)(2)(6)} = \sqrt{576} = 24$

d $A = \sqrt{(13-1)(13-5)(13-9)(13-11)}$

$A = \sqrt{(12)(8)(4)(2)} = \sqrt{768} = 16\sqrt{3}$

4 a 2,5,7 $s = \frac{2+5+7}{2}, s = 7$

$A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$A_\Delta = \sqrt{7(7-2)(7-5)(7-7)}$

$A_\Delta = \sqrt{7(5)(2)(0)} = \sqrt{0} = 0$

This will not be a triangle. The semiperimeter is the same as one of the sides.

b 4,6,12 $s = \frac{4+6+12}{2}, s = 11$

$A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$A_\Delta = \sqrt{11(11-4)(11-6)(11-12)}$

$A_\Delta = \sqrt{11(7)(5)(-1)} = \sqrt{-385}$

This will not be a triangle. The semiperimeter is less than one of the sides.

5 $A_{quad} = 2A_\Delta$
 $s = \frac{5+7+10}{2} = 11$

$A_\Delta = \sqrt{11(11-10)(11-7)(11-5)}$

$= \sqrt{11(1)(4)(6)}$

$= \sqrt{66 \cdot 4} = 2\sqrt{66}$

$A_{quad} = 4\sqrt{66} \approx 32.5$

6 a $s = \frac{3+5+6}{2}, s = 7$

$A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$A_\Delta = \sqrt{7(7-3)(7-5)(7-6)}$

$A_\Delta = \sqrt{7(4)(2)(1)} = \sqrt{56} = 2\sqrt{14}$

b $s = \frac{7+23+20}{2}, s = 25$

$A_\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$A_\Delta = \sqrt{25(25-7)(25-23)(25-20)}$

$A_\Delta = \sqrt{25(18)(2)(5)} = \sqrt{4,500} = 30\sqrt{5}$

7 $\triangle I$ is a rt \triangle in

(3, 4, 5) triple family, so
the hypotenuse (or diagonal)
of quad is 15.

$$A_{\triangle I} = \frac{1}{2}(9)(12) = 54$$

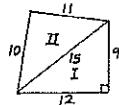
$$\text{For } \triangle II, s = \frac{15+11+10}{2} = 18$$

$$A_{\triangle II} = \sqrt{18(18-15)(18-11)(18-10)}$$

$$A = \sqrt{18(3)(7)(8)}$$

$$A = \sqrt{21(144)} = 12\sqrt{21}$$

Total area is $12\sqrt{21} + 54$.



8 First find the area of the \triangle .

$$s = \frac{13+14+15}{2}, s = 21$$

$$A_{\triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A_{\triangle} = \sqrt{21(21-13)(21-14)(21-15)}$$

$$A_{\triangle} = \sqrt{21(8)(7)(6)} = \sqrt{7,056} = 84$$

Then use the area and the formula $A = \frac{1}{2}bh$ to find altitudes.

$$A = \frac{1}{2}bh$$

$$84 = \frac{1}{2}bh$$

$$84 = \frac{1}{2}(13)h$$

$$84 = \frac{13}{2}h$$

$$\frac{168}{13} = h$$

$$A = \frac{1}{2}bh$$

$$84 = \frac{1}{2}bh$$

$$84 = \frac{1}{2}(14)h$$

$$84 = 7h$$

$$12 = h$$

$$A = \frac{1}{2}bh$$

$$84 = \frac{1}{2}bh$$

$$84 = \frac{1}{2}(15)h$$

$$84 = \frac{15}{2}h$$

$$\frac{168}{15} = h$$

$$\frac{56}{5} = h$$

9 a $A_{\text{inscribed quad}} = \sqrt{(5-4)(5-3)(5-2)(5-1)}$

$$= \sqrt{(1)(2)(3)(4)} = 2\sqrt{6}$$

10 From the distance formula, the lengths of the sides of the triangle are 9, 18.38, and 17.12. The semiperimeter is 22.25. Thus,

$$A_{\triangle} = \sqrt{22.25(22.25-9)(22.25-18.38)(22.25-17.12)}$$

$$= \sqrt{22.25(13.25)(3.87)(5.13)}$$

$$= \sqrt{5852.94} = 76.5$$

11 a It approaches a \triangle . b It becomes Hero's formula.

12 Form quad, find area using Brahmagupta's formula.

Form \triangle , find area using Hero's formula.

$$A_{\text{quad}} = \sqrt{\left(\frac{25}{2} \cdot 5\right)^2 \left(\frac{25}{2} - 10\right)} = \frac{73\sqrt{3}}{4}$$

$$A_{\triangle} = \sqrt{12(12-6)(12-8)(12-10)} = 24$$

$$A_{\text{pentagon}} = \frac{73\sqrt{3}}{4} + 24 \approx 56.5$$

13 a E, the midpoint of \overline{CD} , is (6, 8). $\triangle CEO$ is a $30^\circ 60^\circ 90^\circ$ \triangle
with $CE = 3$, $EO = 3\sqrt{3}$. So, $O = (6, 8 - 3\sqrt{3})$.

b $\triangle COD$ is equilateral and $CO = 6$. So, $C = 12\pi \approx 37.7$

Pages 554–559 Chapter 11 Review Problems

1 a $(12)(7) = 84$ d $\frac{1}{2}(3+10)8 = 52$

b $\frac{1}{2}(12)(7) = 42$ e $\frac{1}{2}(5)(8) = \frac{1}{2} \cdot 40 = 20$

c $(15)(5) = 75$ f $(2)(4) = 8$

2 a $A_{\text{rhombus}} = bh$ b $A_{\text{rhombus}} = \frac{1}{2}d_1d_2$
 $= (9)(7) = 63$ $= \frac{1}{2}(6)(11) = 33$

3 a Total Area = $A_{\text{lg rect}} - A_{\text{sm rect}}$

$$A_{\text{lg rect}} = (13)(7) = 91$$

$$A_{\text{sm rect}} = (7)(3) = 21$$

$$\text{Shaded Area} = 91 - 21 = 70$$

b $A_{\text{trap}} = \frac{1}{2}(b_1+b_2)h$ c $A_{\triangle} = \frac{s^2}{4}\sqrt{3}$

$$= \frac{1}{2}(5+7)4$$

$$A_{\triangle} = \frac{8^2}{4}\sqrt{3}$$

$$= 24$$

$$A_{\triangle} = \frac{64}{4}\sqrt{3} = 16\sqrt{3}$$

4 $A_{\text{rect}} = bh$

$$A_{\text{rect}} = (4)(20) = 80 \text{ sq m}$$

Total cost = A driveway times amt per sq meter

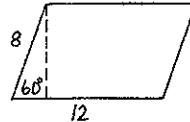
Total cost = $80 \text{ sq m} \cdot (\$15 \text{ per sq m})$

Total cost = \$1,200.00

5 Draw the altitude.

A $30^\circ 60^\circ 90^\circ$ \triangle is formed.

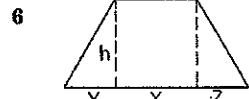
Side opp $60^\circ \angle$ is



$$\frac{1}{2}\text{hyp} \cdot \sqrt{3} = 4\sqrt{3}$$

$$A = bh$$

$$A = (12)(4\sqrt{3}) = 48\sqrt{3}$$



To find h , draw 2 alts of the trap. The middle figure formed is a rect, $x = 8$. y and z are \cong because it is an isosceles trap.

$$40 - x = y + z$$

$$40 - 8 = y + z$$

$$32 = y + z, y \cong z, \text{ so}$$

$$y = \frac{1}{2}(32) = 16$$

$$20 = (4)(5)$$

16 = (4)(4) (3, 4, 5) is a Pythagorean Triple, so

$$h = (4)(3) = 12$$

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{12}{2}(8 + 40)$$

$$A = 6(48) = 288$$

$$7 A_{\Delta} = \frac{1}{2}(bh)$$

$$A_{\Delta} = \frac{1}{2}(12)(3) = 18$$

8 Since 10 cm equals one decimeter, the lengths of the sticks

John has are 9 dm and 5 dm long.

$$A_{\text{kite}} = \frac{1}{2}d_1d_2 \quad \text{Total cost} = A_{\text{kite}} \cdot \text{amt per sq dm}$$

$$A_{\text{kite}} = \frac{1}{2}(9)(5) \quad \text{Total cost} = \frac{45}{2} \text{ dm} \cdot 3 \text{¢ per sq dm}$$

$$A_{\text{kite}} = \frac{45}{2} \text{ dm} \quad \text{Total cost} = \frac{135}{2} \text{¢}$$

$$\text{Total cost} = 67 \frac{1}{2} \text{¢}$$

$$9 A_{\text{reg poly}} = \frac{1}{2}ap$$

$$A_{\text{reg poly}} = \frac{1}{2}(7)(56) = 196$$

$$10 C = 2\pi r \quad A_{\odot} = \pi r^2$$

$$16\pi = 2\pi r \quad A_{\odot} = \pi(8)^2$$

$$8 = r \quad A_{\odot} = 64\pi$$

$$11 \text{ Side} = \frac{1}{2}(\text{semiperimeter}) = \frac{1}{2}(18) = 9$$

$$A_{\text{square}} = s^2$$

$$A_{\text{square}} = 9^2 = 81 \text{ sq m}$$

$$12 r = \frac{1}{2}(d) \quad A_{\odot} = \pi r^2$$

$$r = \frac{1}{2}(14) \quad A_{\text{semicircle}} = \frac{1}{2}\pi r^2$$

$$r = 7 \quad A_{\text{semicircle}} = \frac{49}{2}\pi \approx 77.0 \text{ sq m}$$

$$13 a \quad A_{\text{sector}} = \frac{m \text{ arc}}{360} \cdot \pi r^2$$

$$= \frac{90}{360} \cdot \pi \cdot 6^2$$

$$= \frac{1}{4}\pi \cdot 36 = 9\pi$$

$$b \quad A_{\text{outer } \odot} = \pi r^2 \quad A_{\text{inner } \odot} = \pi(3)^2$$

$$= \pi(5)^2 \quad = 9\pi$$

$$= 25\pi$$

$$\text{Shaded Area} = 25\pi - 9\pi = 16\pi$$

$$c \quad A_{\text{square}} = s^2 \quad A_{\odot} = \pi r^2$$

$$= 10^2 \quad A_{\odot} = \pi(5)^2 = 25\pi$$

$$= 100$$

$$A_{\text{shaded region}} = 100 - 25\pi$$

$$14 a \quad A_{\text{sector}} = \left(\frac{m \text{ arc}}{360}\right)\pi r^2$$

$$A_{\text{sector}} = \left(\frac{60}{360}\right)\pi(12)^2$$

$$A_{\text{sector}} = \left(\frac{1}{6}\right)144\pi = 24\pi$$

$$b \quad A_{\text{sector}} = \left(\frac{m \text{ arc}}{360}\right)\pi r^2$$

$$A_{\text{sector}} = \left(\frac{40}{360}\right)\pi(12)^2$$

$$A_{\text{sector}} = \left(\frac{1}{9}\right)144\pi = 16\pi$$

$$c \quad A_{\text{sector}} = \left(\frac{m \text{ arc}}{360}\right)\pi r^2$$

$$A_{\text{sector}} = \left(\frac{10}{360}\right)\pi(12)^2$$

$$A_{\text{sector}} = \left(\frac{1}{36}\right)144\pi = 4\pi$$

$$15 a \quad A_{\Delta} = \frac{1}{2}(bh)$$

$$A_{\Delta} = \frac{1}{2}(5)(3) = \frac{15}{2} \quad A_{\Delta} = \frac{1}{2}(12)(2) = \frac{24}{2}$$

$$\text{Ratio of areas} = 15:24 = 5:8$$

$$b \quad \text{Ratio of areas of similar } \Delta s = \frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{18}{24}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{3}{4}\right)^2$$

$$\frac{A_1}{A_2} = \frac{9}{16}$$

$$\text{Ratio of areas} = 9:16$$

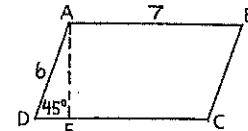
16 Both triangles have the same height, so their bases must

be equal for their areas to be equal. The base of ΔACD is 8. Subtracting 8 from 6 gives the coordinates for b, (-2, 0).

17 The \perp to the base forms two 9-40-41 Δ s, so $h = 40$.

$$A_{\Delta} = \frac{1}{2}(b \cdot h)$$

$$A_{\Delta} = \frac{1}{2}(18)(40) = 360$$



18 Draw alt \overline{AE} making

$$\Delta ADE \text{ a } 45^\circ 45^\circ 90^\circ \Delta.$$

$$AD = 6 \text{ so } AE = 3\sqrt{2}$$

$$A = b \cdot h$$

$$A = (7)(3\sqrt{2}) = 21\sqrt{2}$$

19 Perimeter = 52, side = 13

Find the length of the other diagonal. In a rhombus, the diagonals are \perp and bis each other.

A rt Δ is formed.

$$A = \frac{1}{2}d_1d_2$$

Use Pythagorean triple

$$= \frac{1}{2}(10)(24) = 120$$

(5, 12, 13). The other diagonal is $2(5) = 10$.

20 side = $\frac{1}{3}$ (perimeter) $A = \frac{s^2}{4}\sqrt{3}$

side = $\frac{1}{3}(21) = 7$ $A = \frac{(7)^2}{4}\sqrt{3}$
 $A = \frac{49}{4}\sqrt{3}$

21 By drawing in altitudes two

$30^\circ 60^\circ 90^\circ$ Δs are formed,

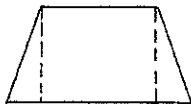
making the height = $7\sqrt{3}$

and the \cong sides each 14.

$$\begin{aligned} A_{\text{isos trap}} &= \frac{1}{2}(b_1 + b_2)h \\ &= \frac{1}{2}(18 + 4)h \\ &= \frac{1}{2}(22)7\sqrt{3} = 77\sqrt{3} \end{aligned}$$

Perimeter = base₁ + base₂ + side₁ + side₂

Perimeter = $4 + 16 + 14 + 14 = 50$



22 a The radius of the circle, $\sqrt{41}$, is the hyp of a right triangle with sides 4, 5, and $\sqrt{41}$. Then $C = 2\pi\sqrt{41} \approx 40.2$.

b $A = \pi(\sqrt{41})^2 \approx 128.8$

23 $A_{\text{square}} = \frac{1}{2}(d_1 d_2)$

a The diagonals of a square are \cong , so

$$A_{\text{square}} = \frac{(26)(26)}{2} = \frac{676}{2} = 338$$

b Area of square is 18, so

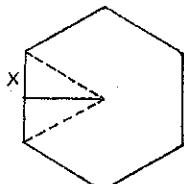
$$\begin{aligned} 18 &= \frac{1}{2}(d_1 d_2) && \text{The diagonals are } \cong, \text{ so} \\ 18 &= \frac{1}{2}(d)(d) = \frac{1}{2}(d^2) && \text{each diagonal} = 6 \\ 36 &= d^2, d = 6. \end{aligned}$$

24 The apothem of the figure is $\frac{1}{2}$ the span.

apothem = $\frac{1}{2}(36) = 18$

The apothem forms a $30^\circ 60^\circ 90^\circ$ Δ with x and a radius

$$\begin{aligned} x &= \frac{\text{apothem}}{\sqrt{3}} && \text{base} = 2x \\ &= \frac{18}{\sqrt{3}} && = 2(6\sqrt{3}) \\ &= 6\sqrt{3} && = 12\sqrt{3} \end{aligned}$$



$$\begin{aligned} A &= \frac{1}{2}a \cdot p \\ &= \frac{1}{2}(18)(72\sqrt{3}) \end{aligned}$$

$$\begin{aligned} \text{Perimeter} &= 6(\text{base}) \\ &= 6(12\sqrt{3}) \\ &= 72\sqrt{3} \end{aligned}$$

25 a $\frac{4}{x} = \frac{10}{8}$

$10x = 32$

$x = 3.2$

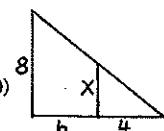
$A_{\Delta} = \frac{1}{2}bh$

$A_{\text{lg}\Delta} = \frac{1}{2}(8 \cdot 10)$

$A_{\text{lg}\Delta} = 40$

$A_{\text{sm}\Delta} = \frac{1}{2}(3 \cdot 2 \cdot 4)$

$A_{\text{sm}\Delta} = 6.4$



$A_{\text{shaded}} = A_{\text{lg}\Delta} + A_{\text{sm}\Delta}$

$A_{\text{shaded}} = 40 - 6.4 = 33.6$

b $30^\circ 60^\circ 90^\circ$ Δ formed with radius of \odot and Δ .

side opp $30^\circ \angle = 3$

side opp $60^\circ \angle = 3\sqrt{3}$

side of equil $\Delta = 6\sqrt{3}$

$$A = \frac{s^2}{4}\sqrt{3} \quad A_{\odot} = \pi r^2 \quad A_{\text{shaded}} = A_{\Delta} - A_{\odot}$$

$$A = \frac{(6\sqrt{3})^2}{4}\sqrt{3} \quad A_{\odot} = \pi(3)^2 \quad A_{\text{shaded}} = 27\sqrt{3} - 9\pi$$

$A = 27\sqrt{3}$

$A_{\odot} = 9\pi$

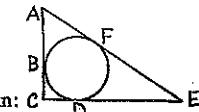
c By two tangent theorem:

$AB = AF = 2$ Using Pythagorean: C

$ED = EF = 3 \quad AC^2 + CE^2 = AE^2$

$BC = DC = x \quad (2+x)^2 + (x+3)^2 = 5^2$

$x = 1$



The circumscribed Δ is a 3-4-5 Δ .

$$A = \frac{1}{2}bh \quad A_{\odot} = \pi r^2 \quad A_{\text{shaded}} = A_{\Delta} - A_{\odot}$$

$$A = \frac{1}{2}(3 \cdot 4) \quad A_{\odot} = \pi \cdot 1 \quad A_{\text{shaded}} = 6 - \pi$$

$A = 6 \quad A_{\odot} = \pi$

26 a The height of both Δ s is the same. If the base of the Δ is bisected and the shaded Δ has a base x , then the base of the whole $\Delta = 2x$.

$$\frac{A_{\text{whole}\Delta}}{A_{\text{shaded}\Delta}} = \frac{\frac{1}{2}b \cdot h}{\frac{1}{2}b \cdot h} = \frac{\frac{1}{2}2xh}{\frac{1}{2}xh} = \frac{2}{1} = 2:1$$

b Using Angle Bisector Theorem

$$\frac{2}{5} = \frac{5}{3}$$

$5x = 6$

$x = \frac{6}{5}$

The base of the whole $\Delta = x + 3$.

Base = $\frac{6}{5} + 3 = \frac{21}{5}$

The height of both Δ s is the same, so the height of each can be set equal to h.

$$\frac{A_{\text{whole}\Delta}}{A_{\text{shaded}\Delta}} = \frac{\frac{1}{2}b \cdot h}{\frac{1}{2}b \cdot h} = \frac{\frac{1}{2}(\frac{21}{5})h}{\frac{1}{2}(\frac{15}{5})h} = \frac{\frac{21}{5}}{\frac{15}{5}} = \frac{7}{5} = 7:5$$

c. Using Midline Theorem, the base of the shaded Δ is \parallel to and is $\frac{1}{2}$ of the base of the whole Δ .

Using Reflexive \angle and 2 \angle s proven \cong by \parallel lines \Rightarrow corr. \angle s \cong , the shaded $\Delta \sim$ whole Δ by AA~.

By Similar Figures Theorem:

$$\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2 = \left(\frac{10}{5}\right)^2 = \frac{100}{25} = 4:1$$

27 Area of segment = $A_{\text{sector}} - A_{\Delta}$

$$\text{a } A_{\text{sector}} = \left(\frac{\text{m arc}}{360}\right)\pi r^2 \quad A_{\Delta} = \frac{1}{2}bh$$

$$A_{\text{sector}} = \frac{1}{4}\pi(36) = 9\pi \quad A_{\Delta} = \frac{1}{2}(6)(6)$$

$$A_{\text{segment}} = 9\pi - 18 \quad A_{\Delta} = 18$$

$$\text{b } A_{\text{sector}} = \frac{1}{6}(\pi)(36) \quad A_{\Delta} = \frac{s^2}{4}\sqrt{3}$$

$$A_{\text{sector}} = 6\pi \quad A_{\Delta} = \frac{36}{4}\sqrt{3} = 9\sqrt{3}$$

$$A_{\text{segment}} = 6\pi - 9\sqrt{3}$$

28 a Because the two bases of the trap are \parallel , use \parallel lines \Rightarrow alt. int. $\angle s$ \cong twice to get $\triangle I \sim \triangle II$ by AA~. Using Similar Figures Theorem,

$$\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2 = \left(\frac{8}{18}\right)^2 = \frac{16}{81} = 16:81$$

b $\triangle I \sim \triangle II$ by AA~ (inscribed $\angle s$ intercept same arc)

$$\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2 = \left(\frac{6}{9}\right)^2 = \frac{4}{9} = 4:9$$

c $\triangle I \sim \triangle II$ by AA~

$$\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2 = \left(\frac{10}{10}\right)^2 = 1:1$$

29 For the circle, $C = 2\pi r$.

$$100 = 2\pi r; r = \frac{100}{2\pi} \approx 15.92$$

$$A = \pi(15.92)^2 \approx 796.23$$

For the square, $P = 4s$.

$$100 = 4s; 25 = s$$

$$A = s^2 = 25^2 = 625$$

Thus, the circle has greater area.

30 $\angle ARM \cong \angle MBC$ because \parallel lines \Rightarrow alt. int. $\angle s \cong$.

$\angle BMC \cong \angle AMR$ because vertical angles are \cong . $\overline{RM} \cong \overline{MB}$ by definition of midpoint. Thus $\triangle ARM \cong \triangle MBC$ by ASA postulate.

$$A_{\text{BRAT}} = A_{\text{BMAT}} + A_{\Delta ARM}$$

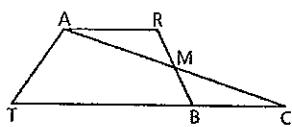
$$A_{\Delta ATC} = A_{\text{BMAT}} + A_{\Delta MBC}$$

But $A_{\Delta ARM} = A_{\Delta MBC}$

\therefore By substitution,

$$A_{\Delta ATC} = A_{\text{BMAT}} + A_{\Delta ARM}, \text{ and}$$

$$A_{\text{BRAT}} = A_{\Delta ACT}$$



31 a Since \overline{AR} is vertical, $x = 12$. Since the altitudes \overline{RC} and \overline{AB} are horizontal, $y_1 = 5$ and $y = y_2$. Since the trapezoid is isosceles, $\triangle APB \cong \triangle RTC$ by HL. Therefore, $PB = CT$ and

$$y_2 = 18 - CT$$

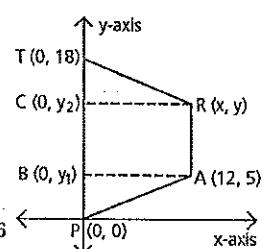
$$= 18 - PB$$

$$= 18 - y_1$$

$$= 18 - 5 = 13$$

So $R(x, y) = R(12, 13)$

$$\text{b } A_{\text{trap}} = \frac{1}{2}(12)(18 + 8) = 156$$



32 a $I \sim II \sim (I+II)$

$$\text{Ratio of areas of } \sim \Delta s = \frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$$

$$\frac{A_{II}}{A(I+II)} = \left(\frac{6}{9}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\frac{A_I}{A(I+II)} = \frac{9}{9} - \frac{4}{9} = \frac{5}{9}$$

$$\frac{A_I}{A_{II}} = \frac{5}{4} \text{ or } 5:4$$

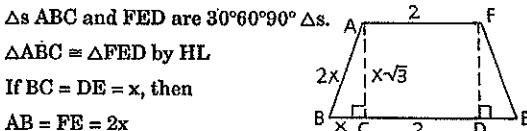
b $I \sim (I+II)$

$$\frac{A_I}{A(I+II)} = \left(\frac{10}{16}\right)^2 = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$$

$$\frac{A_{II}}{A(I+II)} = \frac{64}{64} - \frac{25}{64} = \frac{39}{64}$$

$$\frac{A_I}{A_{II}} = \frac{25}{39} \text{ or } 25:39$$

33 If $AF = 2$, then $CD = 2$.



$$AF + FE + DE + CD + BC + AB = 70$$

$$2 + 2x + x + 2 + x + 2x = 70$$

$$4 + 6x = 70$$

$$6x = 66$$

$$x = 11$$

$$AC = x\sqrt{3} \text{ (30°60°90° \Delta)}$$

$$AC = 11\sqrt{3} \text{ (height)}$$

$$\text{Base 1} = 2$$

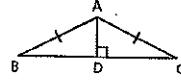
$$\text{Base 2} = BC + CD + DE$$

$$= x + 2 + x = 24$$

$$A_{\text{trap}} = \frac{h}{2}(b_1 + b_2)$$

$$A_{\text{trap}} = \frac{11\sqrt{3}}{2}(26) = 143\sqrt{3}$$

$$A_{\text{trap}} = \frac{11\sqrt{3}}{2}(26) = 143\sqrt{3}$$



34 Given: $\angle BAC \supseteq \angle QPR$

$\triangle ABC$ and $\triangle PQR$ are isos, with bases \overline{BC} and \overline{QR} .

Prove: $\text{Area } \triangle ABC = \text{area } \triangle PQR$

We can let $\angle BAC = 2x$, which will make $\angle QPR = 180 - 2x$.

Thus $\angle DAC = x$ and $\angle SPR = 90 - x$. They are thus complementary $\angle s$. But since $\triangle CDA$ is a rt \triangle , we know that $\angle C$ is comp to $\angle DAC$ and thus $\angle C \cong \angle SPR$.

$\therefore \triangle ADC \cong \triangle RSP$ by AAS and thus have equal areas.

Similarly, $\triangle BDA \cong \triangle PSQ$ and thus have equal areas.

$\therefore \text{area } \triangle ABC = \text{area } \triangle PQR$

- 35 To find the radius of the lg \odot , draw a square to connect the centers of the 4 small \odot s. Then connect the center of the lg \odot with the centers of 2 sm \odot s to form Δ . Legs are \cong because they are radii lg \odot - radii sm \odot . Rt \angle and $45^\circ 45^\circ 90^\circ \Delta$ formed, hyp = 6, each leg = $3\sqrt{2}$.

$$\text{radius sm } \odot = 3$$

$$\text{radius lg } \odot = 3\sqrt{2} + 3$$

$$\text{Shaded area} = A_{\text{lg } \odot} - 4(A_{\text{sm } \odot})$$

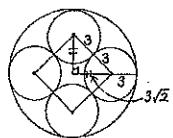
$$\text{Shaded area} = \pi R^2 - 4(\pi r^2)$$

$$\text{Shaded area} = \pi(3 + 3\sqrt{2})^2 - 4\pi(3^2)$$

$$\text{Shaded area} = \pi(9 + 18\sqrt{2} + 18) - 4(9\pi)$$

$$\text{Shaded area} = 27\pi + 18\pi\sqrt{2} - 36\pi$$

$$\text{Shaded area} = 18\pi\sqrt{2} - 9\pi$$



- 36 Draw an equilateral Δ , and from the area of the largest Δ , subtract the areas of the 3 sectors.

$$A_{\Delta} = \frac{s^2}{4}\sqrt{3} \quad A_{\text{sector}} = \frac{m \text{ arc}}{360} \cdot \pi r^2$$

$$A_{\text{lg } \Delta} = \frac{20^2}{4}\sqrt{3} \quad A_{\text{sector}} = \frac{1}{6} \cdot 100\pi$$

$$= 100\sqrt{3} \quad A_{\text{sector}} = \frac{50}{3}\pi$$

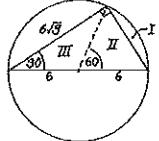
There are 3 sectors so $3(\frac{50}{3})\pi = 50\pi$

$$\text{Shaded Area} = 100\sqrt{3} - 50\pi$$

- b Divide the shaded region into 3 parts and find the area of each part. The lg Δ formed is a $30^\circ 60^\circ 90^\circ \Delta$.

$$\text{hyp} = 12; \text{side opp } 30^\circ \angle = 6; \text{side opp } 60^\circ \angle = 6\sqrt{3}$$

Region II is an equiangular Δ , so all sides = 6.



Region I

$$A_I = A_{\text{sector}} - A_{\text{eq } \Delta}$$

$$A_I = \frac{m \text{ arc}}{360} \pi r^2 - \frac{s^2}{4}\sqrt{3}$$

$$A_I = \frac{60}{360}\pi(6^2) - \frac{6^2}{4}\sqrt{3}$$

$$\text{Shaded Area} = A_I + II + III = 6\pi - 9\sqrt{3}$$

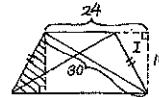
Region II + III

$$A_{II} + III = \frac{1}{2}bh$$

$$A_{II} + III = \frac{1}{2}(6)(6\sqrt{3})$$

$$A_{II} + III = 18\sqrt{3}$$

- 37 If the diag of a trap are \cong , it is isosceles. The area of the shaded region = A_I because the 2 Δ s are \cong (HHL).



To find the base,

$$18 = 6 \cdot 3$$

$$30 = 6 \cdot 5 \quad (3, 4, 5 \text{ is a Pythagorean Triple})$$

$$\text{so base} = 6 \cdot 4 = 24$$

Therefore

$$A_{\text{trap}} = A_{\text{rect}} = b \cdot h$$

$$A_{\text{trap}} = (24)(18) = 432$$

- 38 a $\Delta BGC \sim \Delta DGE$ (AA~)

Therefore the alts are prop to the bases (3:5).

In Region I, base = $5x$ and alt = $5h$.

In $ACDF$, base = $6x$ and alt = $8h$.

$$\frac{A_I}{A_{\square}} = \frac{\frac{1}{2}bh}{bh}$$

$$\frac{A_I}{A_{\square}} = \frac{\frac{1}{2}(5x)(5h)}{(6x)(8h)}$$

$$\frac{A_I}{A_{\square}} = \frac{25xh}{48xh}$$

$$\frac{A_I}{A_{\square}} = \frac{25}{96} \quad \text{Ratio of area I to } A_{ACDF} = 25:96.$$

$$\begin{aligned} b \quad A_{II} &= A_{\square} - A_{\triangle ECD} - A_{\triangle ABG} \\ &= (6x)(8h) - \frac{1}{2}(5x)(8h) - \frac{1}{2}(3x)(3h) \\ &= 48xh - 20xh - \frac{9}{2}xh \\ &= \frac{47}{2}xh \end{aligned}$$

$$\text{Prob} = \frac{A_{II}}{A_{\square}} = \frac{\frac{47}{2}xh}{48xh} = \frac{47}{96}$$

- 39 A tangent line is \perp to a radius drawn to the pt of contact so,

$\triangle PTA$ is a rt Δ .

If $AB = 12$, then $PB = 12$.

$PT = 12$ because it is also a radius.

$\triangle PTA$ is a $30^\circ 60^\circ 90^\circ \Delta$ because it has a rt \angle and $PT = \frac{1}{2}$ of the hypotenuse, \overline{PA} . $TA = PT(\sqrt{3}) = 12\sqrt{3}$

Shaded Area = $A_{\Delta} - A_{\text{sector}}$

$$\text{Shaded Area} = \frac{1}{2}bh - \left(\frac{m \text{ arc}}{360}\right)\pi r^2$$

$$\text{Shaded Area} = \frac{1}{2}(12\sqrt{3})(12) - \left(\frac{60}{360}\pi(12)^2\right)$$

$$\text{Shaded Area} = 72\sqrt{3} - \frac{1}{6}(144\pi)$$

$$\text{Shaded Area} = 72\sqrt{3} - 24\pi$$

- 40 Gremilda has $\frac{3}{4}$ of a 12 m radius \odot ,
plus $\frac{1}{4}$ of both a 2 m radius \odot and a

4 m radius \odot .

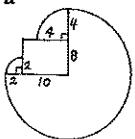
$$\frac{3}{4} \cdot \pi(12)^2 = \frac{3}{4} \cdot 144\pi = 108\pi$$

$$\frac{1}{4} \cdot \pi(2)^2 = \frac{1}{4} \cdot 4\pi = \pi$$

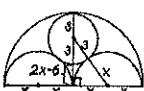
$$\frac{1}{4} \cdot \pi(4)^2 = \frac{1}{4} \cdot 16\pi = 4\pi$$

$$\text{Grazing Area} = 108\pi + \pi + 4\pi = 113\pi \text{ sq m}$$

Since the barn is a rectangle, it forms rt \angle s and takes up $\frac{1}{4}$ of each circle formed.



- 41 Solve for x by using a rt Δ .



$$x^2 + (2x - 6 + 3)^2 = (x + 3)^2$$

$$x^2 + (2x - 3)^2 = x^2 + 6x + 9$$

$$x^2 + 4x^2 - 12x + 9 = x^2 + 6x + 9$$

$$4x^2 - 18x = 0$$

$$2x - 9 = 0$$

$$x = \frac{9}{2}$$

$$A_{\odot} = \pi r^2$$

$$A_{\text{semicircle}} = \frac{1}{2}(81\pi)$$

$$A_{\text{med circle}} = \frac{81}{4}\pi$$

$$A_{\text{sm circle}} = 9\pi$$

$$\text{Shaded area} = \frac{162}{4}\pi - \frac{81}{4}\pi - \frac{36}{4}\pi = \frac{45}{4}\pi \text{ or } 11\frac{1}{4}\pi$$

- 42 $A_{\text{square}} = s^2$ $A_{\text{cross}} = 5 \cdot (A_{\text{square}})$

$$A_{\text{square}} = (4)^2$$

$$A_{\text{cross}} = 5 \cdot (16)$$

$$A_{\text{square}} = 16$$

$$A_{\text{cross}} = 80$$

To find radius of \odot , find \overline{AB} .

$$(AC)^2 + (BC)^2 = (AB)^2 \quad \text{radius} = \frac{1}{2}(AB)$$

$$(12)^2 + (4)^2 = (AB)^2 \quad \text{radius} = \frac{1}{2}(4\sqrt{10})$$

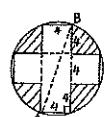
$$144 + 16 = (AB)^2 \quad \text{radius} = 2\sqrt{10}$$

$$160 = (AB)^2$$

$$4\sqrt{10} = AB$$

$$A_{\odot} = \pi r^2 = \pi(2\sqrt{10})^2 = 40\pi$$

$$\text{Shaded area} = A_{\odot} - A_{\text{cross}} = 40\pi - 80$$



- 43 $40 - 12 = 28$, so the bases

of the rt Δ s can be labeled
 x and $28 - x$.

Using the Pythagorean Theorem

$$x^2 + h^2 = (25)^2$$

$$(28 - x)^2 + h^2 = (17)^2$$

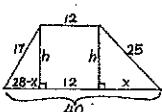
$$x^2 + h^2 = 625 \quad (\text{Solving}$$

$$784 - 56x + x^2 + h^2 = 289 \quad \text{simultaneous}$$

$$-784 + 56x = 336 \quad \text{equations)}$$

$$56x = 1120$$

$$x = 20$$



$$x^2 + h^2 = 625 \quad A_{\text{trap}} = \frac{h}{2}(b_1 + b_2)$$

$$(20)^2 + h^2 = 625 \quad A_{\text{trap}} = \frac{15}{2}(12 + 40)$$

$$400 + h^2 = 625 \quad A_{\text{trap}} = \frac{15}{2}(52) = 390$$

$$h = \sqrt{225} = 15$$

$$44 \quad A_{\text{reg poly}} = \frac{1}{2}ap$$

$$A_{\text{jig hexagon}} = \frac{1}{2}(\frac{5}{2}\sqrt{3})(30) = \frac{75}{2}\sqrt{3}$$

$$A_{\text{sm hexagon}} = \frac{1}{2}(2\sqrt{3})(24) = 24\sqrt{3}$$

$$A_{\text{wax}} = \frac{75}{2}\sqrt{3} - 24\sqrt{3} = \frac{27}{2}\sqrt{3}$$

$$\% \text{ wax} = \frac{\frac{27}{2}\sqrt{3}}{\frac{75}{2}\sqrt{3}} = \frac{27}{75} = 36\%$$

- 45 The length of chord \overline{AB} is $\sqrt{68}$ by the distance formula.

Since $m\widehat{AB} = 90^\circ$, the chord is also one side of an inscribed square, the diagonals of which intersect at the circle's center. The radius of the circle is $\frac{1}{2}$ the length of a

diagonal, or $\frac{\sqrt{136}}{2}$. Thus, $A_{\odot} = \pi \left(\frac{\sqrt{136}}{2}\right)^2 = \pi \frac{136}{4} =$

$$\pi 34 \approx 106.8$$

Chapter 12 SURFACE AREA AND VOLUME

Pages 563–564 (Section 12.1)

1 a $A_{\text{rect}} = \ell \cdot h$

$$15 \cdot 10 = 150 \quad 150 + 150 + 75 + 75 + 50 + 50 = 550 \text{ sq cm}$$

$$15 \cdot 5 = 75$$

$$5 \cdot 10 = 50$$

b $12 \cdot 7 = 84 \quad 84 + 84 + 36 + 36 + 21 + 21 = 282 \text{ sq mm}$

$$12 \cdot 3 = 36$$

$$7 \cdot 3 = 21$$

c $18 \cdot 9 = 162 \quad 162 + 162 + 162 + 81 + 81 = 810 \text{ sq in.}$

$$18 \cdot 9 = 162$$

$$9 \cdot 9 = 81$$

2 a $\ell = 10, a = 3, b = 5, c = 7$

L.A. = $A_{\text{rect}} + A_{\text{rect}} + A_{\text{rect}}$

$$A_{\text{rect}} = b \cdot h \quad A_{\text{rect}} = b \cdot h \quad A_{\text{rect}} = b \cdot h$$

$$A_{\text{rect}} = \ell \cdot a \quad A_{\text{rect}} = \ell \cdot b \quad A_{\text{rect}} = \ell \cdot c$$

$$A_{\text{rect}} = 10 \cdot 3 \quad A_{\text{rect}} = 10 \cdot 5 \quad A_{\text{rect}} = 10 \cdot 7$$

$$A_{\text{rect}} = 30 \quad A_{\text{rect}} = 50 \quad A_{\text{rect}} = 70$$

$$\text{L.A.} = 30 + 50 + 70 = 150$$

b $\ell = 14, a = 2, b = 3, c = 4$

L.A. = $A_{\text{rect}} + A_{\text{rect}} + A_{\text{rect}}$

$$A_{\text{rect}} = b \cdot h \quad A_{\text{rect}} = b \cdot h \quad A_{\text{rect}} = b \cdot h$$

$$A_{\text{rect}} = \ell \cdot a \quad A_{\text{rect}} = \ell \cdot b \quad A_{\text{rect}} = \ell \cdot c$$

$$A_{\text{rect}} = 14 \cdot 2 \quad A_{\text{rect}} = 14 \cdot 3 \quad A_{\text{rect}} = 14 \cdot 4$$

$$A_{\text{rect}} = 28 \quad A_{\text{rect}} = 42 \quad A_{\text{rect}} = 56$$

$$\text{L.A.} = 28 + 42 + 56 = 126$$

3 a $A_{\text{rect}} = b \cdot h \quad A_{\text{rect}} = b \cdot h \quad A_{\text{rect}} = b \cdot h$

$$A_{\text{rect}} = 11 \cdot 17 \quad A_{\text{rect}} = 11 \cdot 17 \quad A_{\text{rect}} = 11 \cdot 16$$

$$A_{\text{rect}} = 187 \quad A_{\text{rect}} = 187 \quad A_{\text{rect}} = 176$$

$$\text{Lateral Area} = A_{\text{rect}} + A_{\text{rect}} + A_{\text{rect}}$$

$$\text{LA} = 187 + 187 + 176 = 550$$

b Draw an altitude. The rt Δ formed is a 8-15-17 Δ , so altitude is 15.

$$A_{\Delta} = \frac{1}{2}bh$$

$$A_{\Delta} = \frac{1}{2}(16)(15) = 120$$

c Total Area = LA + 2(A_{base})

$$\text{TA} = 550 + 2(120)$$

$$\text{TA} = 550 + 240 = 790$$

4 a $s = 6, \ell = 5$

All rectangles are = because the Δ is equilateral.

$$\text{Lateral Area} = 3(A_{\text{rect}}) \quad A_{\text{bases}} = 2(A_{\Delta})$$

$$\text{LA} = 3(b \cdot h) \quad A_{\text{bases}} = 2\left(\frac{s^2}{4}\sqrt{3}\right)$$

$$\text{LA} = 3(5 \cdot 6) \quad A_{\text{bases}} = 2\left(\frac{6^2}{4}\sqrt{3}\right)$$

$$\text{LA} = 90 \quad A_{\text{bases}} = 2(9\sqrt{3})$$

$$\text{LA} = 90 \quad A_{\text{bases}} = 18\sqrt{3}$$

$$\text{Total Area} = \text{LA} + A_{\text{bases}} = 90 + 18\sqrt{3}$$

b $s = 12, \ell = 10$

$$\text{Lateral Area} = 3(A_{\text{rect}}) \quad A_{\text{bases}} = 2(A_{\text{eq}\Delta})$$

$$\text{LA} = 3(b \cdot h) \quad A_{\text{bases}} = 2\left(\frac{s^2}{4}\sqrt{3}\right)$$

$$\text{LA} = 3(10 \cdot 12) \quad A_{\text{bases}} = 2\left(\frac{12^2}{4}\sqrt{3}\right)$$

$$\text{LA} = 360 \quad A_{\text{bases}} = 72\sqrt{3}$$

$$\text{Total Area} = \text{LA} + A_{\text{bases}} = 360 + 72\sqrt{3}$$

5 a $A = 6(s^2)$ b $A = 6(s^2)$

$$A = 6(5^2) \quad A = 6(7^2)$$

$$A = 6(25) = 150 \quad A = 6(49) = 294$$

6 a Area of sides is $(6)(8) = 48$

$$(6)(7) = 42$$

$$(8)(7) = 56$$

$$2(48) + 2(42) + 56 = A$$

$$236 = A$$

b Area of one side is $(6)(8) = 48$

To find the sides of the Δ bases, use the $30^\circ 60^\circ 90^\circ \Delta$. Side opp $30^\circ = 6$, side opp $60^\circ = 6\sqrt{3}$ and hyp is 12.

$$A_{\Delta} = \frac{1}{2}(6)(6\sqrt{3}) = 18\sqrt{3}$$

$$A_{\text{other rect}} = (12)(8) = 96$$

$$\text{and } (6\sqrt{3})(8) = 48\sqrt{3}$$

$$\text{Total Area} = 96 + 48 + 48\sqrt{3} + 18\sqrt{3}$$

$$= 144 + 66\sqrt{3}$$

7 a Rt square prism

$$A_{\text{rect}} = b \cdot h \quad A_{\text{base}} = A_{\text{square}} = s^2$$

$$A_{\text{rect}} = 20 \cdot 6 = 120 \quad A_{\text{base}} = 6^2 = 36$$

$$\text{Lateral Area} = 4(A_{\text{rect}})$$

$$\text{LA} = 4(120) = 480$$

$$\text{Total Area} = \text{LA} + 2(A_{\text{base}})$$

$$\text{TA} = 480 + 2(36)$$

$$\text{TA} = 480 + 72 = 552$$

b Lateral Area = $A_{rect} + A_{rect} + A_{rect}$

$$A_{rect} = b \cdot h \quad A_{rect} = b \cdot h \quad A_{rect} = b \cdot h$$

$$A_{rect} = 10 \cdot 4 \quad A_{rect} = 10 \cdot 5 \quad A_{rect} = 10 \cdot 3$$

$$A_{rect} = 40 \quad A_{rect} = 50 \quad A_{rect} = 30$$

$$LA = 40 + 50 + 30 = 120$$

$$\text{Total Area} = LA + 2(A_{base}) \quad \text{Use Pythagorean}$$

$$TA = 120 + 2(6) = 132 \quad \text{Triple (3-4-5)}$$

to find $h = 4$.

$$A_{\Delta} = \frac{1}{2}bh$$

$$A_{\Delta} = \frac{1}{2}(3 \cdot 4) = 6$$

c Lateral Area = $A_{rect} + A_{rect} + A_{rect}$

$$A_{rect} = b \cdot h \quad A_{rect} = b \cdot h \quad A_{rect} = b \cdot h$$

$$A_{rect} = 50 \cdot 13 \quad A_{rect} = 50 \cdot 13 \quad A_{rect} = 50 \cdot 24$$

$$A_{rect} = 650 \quad A_{rect} = 650 \quad A_{rect} = 1200$$

$$LA = 650 + 650 + 1200 = 2500$$

Use Pythagorean Triple (5-12-13) to find $h = 5$.

$$A_{\Delta} = \frac{1}{2}bh$$

$$A_{\Delta} = \frac{1}{2}(24 \cdot 5) = 60$$

$$\text{Total Area} = LA + 2(A_{base})$$

$$TA = 2500 + 2(60)$$

$$TA = 2620$$

d Lateral Area = $6(A_{rect})$

$$LA = 6(b \cdot h)$$

$$LA = 6(10 \cdot 6)$$

$$LA = 360$$

$$A_{base} = A_{reg \ hex} = 6 \left(\frac{s^2}{4} \sqrt{3} \right)$$

$$A_{reg \ hex} = 6 \left(\frac{(6)^2}{4} \sqrt{3} \right)$$

$$A_{reg \ hex} = 6(9\sqrt{3})$$

$$A_{reg \ hex} = 54\sqrt{3}$$

$$\text{Total Area} = LA + 2(A_{base})$$

$$TA = LA + 2(54\sqrt{3})$$

$$TA = 360 + 108\sqrt{3}$$

8 Draw dotted line on top of prism to divide into rectangles.

$$\text{Area of top and bottom} = 2(4)(2) + 2(7)(2)$$

$$= 16 + 28 = 44$$

$$\text{Area of front} = (4)(7) + (5)(7) + 2(2)(7)$$

$$= 28 + 35 + 28 = 91$$

$$\text{Area of back rects} = (7)(6) + (7)(7)$$

$$= 42 + 49 = 91$$

$$\text{Total Area} = 44 + 91 + 91 = 226$$

9 Lateral Area = $2(11 \cdot 13) + 2(11 \cdot 15)$

$$LA = 616$$

Find area of bases.

$$x^2 + y^2 = 13^2 \quad x^2 + y^2 = 169$$

$$x^2 + (y+4)^2 = 15^2 \quad x^2 + y^2 + 8y + 16 = 225$$

To solve equations, mult first by -1 and add.

$$-x^2 - y^2 = -169$$

$$x^2 + y^2 + 8y + 16 = 225$$

$$8y + 16 = 56$$

$$8y = 40$$

$$y = 5, x = 12$$

$$\text{Area of base} = \frac{1}{2}(24)(9) - \frac{1}{2}(24)(5)$$

$$= 108 - 60 = 48$$

$$\text{Total Area} = 616 + 2(48) = 712$$

10 To find the LA take the length of one edge of the base and multiply it by 10 to get the area of one rect. Do this four other times, each time multiplying a different edge of the base by 10. When this is done, add the areas of all five rectangles to get the LA.

It would also be correct to add all the edges of the base together before multiplying by 10. This would give us the perimeter which could then be multiplied by 10 for the LA. Both methods work because either way, each edge is being multiplied by 10.

$$LA = \text{perimeter of base} \cdot \text{lateral edge}$$

$$LA = 17 \cdot 10$$

$$LA = 170$$

11 a 6 faces painted = 0, 5 painted = 0, 4 painted = 0,

$$3 \text{ painted} = 8, 2 \text{ painted} = 12, 1 \text{ painted} = 6, \text{none}$$

$$\text{painted} = 1.$$

8 cubes like I.

12 like II.

6 like III.

1 in center.

$$b \text{ Probability} = \frac{8+12}{27} = \frac{20}{27}$$

c Each cube is 2 by 2, so the area of each = 4.

Type of cube	No. of unpainted faces	No. of cubes	Total no. of unpainted faces
I	3	8	24
II	4	12	48
III	5	6	30
in center	6	1	6
			108



$$\text{Total area} = 108 \times 4 = 432$$

Pages 567–568 (Section 12.2)

- 1 a Slant height is 12 because the alt \perp divides the Δ face into 2 rt Δ s of the 5-12-13 family.
 $\text{Lateral face} = \frac{1}{2}(10)(12) = 60.$
 b $(60)(4) = 240$ c $(10)(10) + 240 = 340$

- 2 a Since it is regular, all the lateral faces are equal. (8-15-17) is a Pythagorean triple, so the height is 15.
 $A_{\Delta} = \frac{1}{2}(b \cdot h)$
 $A_{\Delta} = \frac{1}{2}(16 \cdot 15)$
 $A_{\Delta} = \frac{1}{2}(240) = 120$

b Because the figure is a reg polygon, the triangular base is also regular, which means it is equilateral.

$$A_{eq\Delta} = \frac{s^2}{4}\sqrt{3}$$

$$A_{eq\Delta} = \frac{16^2}{4}\sqrt{3}$$

$$A_{eq\Delta} = 64\sqrt{3}$$

$$c \quad TA = 3(\text{Area of one lateral face}) + A_{base}$$

$$TA = 3(120) + 64\sqrt{3}$$

$$TA = 360 + 64\sqrt{3}$$

- 3 a The base is not regular.

b Use Pythagorean Triples to find each slant height.

The faces have $b = 14$, $h = 24$ and $b = 30$, $h = 20$.

$$A = \frac{1}{2}(14)(24) = 168$$

$$A = \frac{1}{2}(30)(20) = 300$$

$$\begin{aligned} \text{Lateral Area} &= 2(168) + 2(300) \\ &= 336 + 600 = 936 \end{aligned}$$

$$c \quad \text{Total Area} = \text{LA} + A_{base}$$

$$TA = 936 + (14)(30)$$

$$TA = 936 + 420 = 1356$$

- 4 a No b 9

$$\begin{aligned} c \quad A_{base} &= A_{square} = s^2 \quad \text{The LA of the bottom} \\ A_{base} &= (10)^2 = 100 \qquad \qquad \qquad = 4(A_{square}) \\ &\qquad \qquad \qquad = 4(s^2) \\ &\qquad \qquad \qquad = 4(10^2) = 400 \end{aligned}$$

$$\text{LA of the reg pyramid} = 4(A_{\Delta})$$

(5-12-13) is a Pythagorean triple, so the slant height of the Δ is 12.

$$\text{LA reg pyramid} = 4(\frac{1}{2}bh)$$

$$\text{LA reg pyramid} = 4(\frac{1}{2})(10)(12)$$

$$\text{LA reg pyramid} = 2(120) = 240$$

$$TA = A_{base} + \text{LA}_{\text{bottom prism}} + \text{LA}_{\text{reg pyramid}}$$

$$TA = 100 + 400 + 240 = 740$$

- 5 a Use the Pythagorean triple (6-8-10) to find slant

height = 8.

$$A_{\Delta} = \frac{1}{2}bh$$

$$A = \frac{1}{2}(12 \cdot 8) = 48$$

$$\text{LA} = (48)(4) = 192$$

- b Use the Pythagorean triple (3-4-5) to find slant

height = 4.

$$A = \frac{1}{2}(6)(4) = 12$$

$$\text{LA} = (12)(4) = 48$$

$$c \quad A = 6^2 = 36 \quad d \quad A = 12^2 = 144 \quad e \quad 36:144 = 1:4$$

$$f \quad A = \frac{1}{2}(12 + 6)(4) = 36$$

- 6 If the square $4(A_{\Delta}) = \text{LA}$

base is 25, $4(\frac{1}{2}bh) = \text{LA}$

$$s = \sqrt{25} = 5. \quad 4(\frac{1}{2})(5)(8) = \text{LA}$$

$$80 = \text{LA}$$

$$TA = \text{LA} + A_{base}$$

$$TA = 80 + 25 = 105$$

- 7 a Use the Pythagorean triple (3-4-5 family) to find side

of base = $9 \times 2 = 18 \quad P = 4(18) = 72$

$$b \quad \text{LA} = 4(\frac{1}{2}bh)$$

$$\text{LA} = 4(\frac{1}{2})(18)(12)$$

$$\text{LA} = 432$$

$$c \quad \text{Area of base} = 18^2 = 324 \quad d \quad \text{TA} = 432 + 324 = 756$$

- 8 a $RQ = \frac{1}{2}(CB)$

$$RQ = \frac{1}{2}(14)$$

$RQ = 7 \quad (7-24-25)$ is a Pythagorean Triple, so $\overline{PR} = 25$

$$\text{LA} = 4(A_{\Delta})$$

$$\text{LA} = 4(\frac{1}{2}bh) \quad A_{base} = A_{square} = s^2$$

$$\text{LA} = 2(14)(25) \quad A_{base} = 14^2 = 196$$

$$\text{LA} = 700 \quad A_{base} = 196$$

$$TA = \text{LA} + A_{base}$$

$$TA = 700 + 196 = 896$$

b (8-15-17) is a Pythagorean Triple, so $\overline{RQ} = 8$,

$$\overline{CB} = 2(RQ) = 16, \overline{CB} \cong \overline{DC} = 16$$

$$\text{LA} = 4(A_{\Delta})$$

$$\text{LA} = 4(\frac{1}{2}bh) \quad A_{base} = A_{square} = s^2$$

$$\text{LA} = 2(16)(17) \quad A_{base} = 16^2$$

$$\text{LA} = 2(272) = 544 \quad A_{base} = 256$$

$$TA = \text{LA} + A_{base}$$

$$TA = 544 + 256 = 800$$

9 a Rt $\triangle PQR$ has sides 8, 15. \overline{PR} is hypotenuse, so it = 17.

Rt $\triangle PQS$ has sides 6, 8. \overline{PS} is hypotenuse which = 10.

$$LA = 2\left(\frac{1}{2}\right)(12)(17) + 2\left(\frac{1}{2}\right)(30)(10) = 504$$

$$TA = 504 + (30)(12) = 864$$

b Using Pythagorean theorem for rt $\triangle TSC$,

$$TC^2 = 24^2 + 30^2$$

$$TC^2 = 576 + 900$$

$$TC^2 = 1476$$

$$TC = 6\sqrt{41}, CQ = 3\sqrt{41}.$$

Use Pythagorean theorem on rt $\triangle PQG$.

$$(PQ)^2 + (3\sqrt{41})^2 = 25^2$$

$$(PQ)^2 + 369 = 625$$

$$(PQ)^2 = 256$$

$$PQ = 16$$

10 a Use Pythagorean Theorem

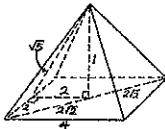
$$(AP)^2 + (DP)^2 = (AD)^2$$

$$(1)^2 + (DP)^2 = 9$$

$$1 + (DP)^2 = 9 \quad 2(DP) = DF$$

$$(DP)^2 = 8 \quad 2(2\sqrt{2}) = DF$$

$$DP = 2\sqrt{2} \quad 4\sqrt{2} = DF$$



b Because $\triangle CDP$ is a $45^\circ 45^\circ 90^\circ$ Δ and $DP = 2\sqrt{2}$, $CP = 2$.

$$(AP)^2 + (CP)^2 = (AC)^2$$

$$1^2 + 2^2 = (AC)^2$$

$$1 + 4 = (AC)^2$$

$$5 = (AC)^2$$

$$\sqrt{5} = AC$$

$$c \quad CP = \frac{1}{2}(DE) \quad A_{base} = s^2$$

$$2 = \frac{1}{2}(DE) \quad A_{base} = (DE)^2$$

$$4 = DE \quad A_{base} = (4)^2 = 16$$

d Because the pyramid is made up of 4 $\cong \Delta$ s,

$$4(A_\Delta) = LA$$

$$4\left(\frac{1}{2}\right)(bh) = LA$$

$$4\left(\frac{1}{2}\right)(4\sqrt{5}) = LA$$

$$2(4\sqrt{5}) = LA$$

$$8\sqrt{5} = LA$$

$$11 a \quad TA = 4\left(\frac{6^2}{4}\right)\sqrt{3} = 4(9\sqrt{3}) = 36\sqrt{3}$$

$$b \quad AB = 6$$

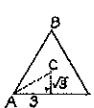
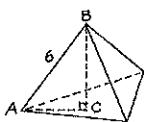
$$AC = 2\sqrt{3}$$

$$(2\sqrt{3})^2 + BC^2 = 6^2$$

$$12 + BC^2 = 36$$

$$BC^2 = 24$$

$$BC = 2\sqrt{6}$$



12 a Because the figure consists of 8 equilateral Δ s,

$$8(A_{eq\Delta}) = TA$$

$$8\left(\frac{s^2}{4}\sqrt{3}\right) = TA$$

$$2(6^2\sqrt{3}) = TA$$

b The plane CDEF is a square and so $\triangle CDE$ is a

$45^\circ 45^\circ 90^\circ$ Δ . If $CD = DE = 6$, then $CE = 6\sqrt{2}$ (diagonal of a square).

c ADBF is a square and so $\triangle ADB$ is a $45^\circ 45^\circ 90^\circ$ Δ . If $AD = DB = 6$, then $AB = 6\sqrt{2}$.

d Quad ACBE forms a vertical plane through the figure that is a square.

13 A regular hexahedron is a cube.

Pages 572-574 (Section 12.3)

$$1 a \quad TA = 4\pi r^2 \quad c \quad TA = 4\pi 3^2$$

$$TA = 4\pi 7^2 \quad TA = 36\pi$$

$$TA = 196\pi$$

$$b \quad TA = 4\pi 3^2 \quad d \quad TA = 4\pi \left(\frac{5}{2}\right)^2$$

$$TA = 36\pi \quad TA = 4\pi \left(\frac{25}{4}\right) = 25\pi$$

$$2 a \quad LA_{cone} = \frac{1}{2}C \cdot \ell \quad A_{base} = \pi r^2$$

$$LA = \pi r \ell \quad A_{base} = \pi(3)^2$$

$$LA = \pi(3)(8) = 24\pi \quad A_{base} = 9\pi$$

$$TA = LA + A_{base}$$

$$TA = 24\pi + 9\pi = 33\pi$$

$$b \quad LA_{cylinder} = C \cdot h \quad A_{base} = \pi r^2$$

$$LA = 2\pi rh \quad A_{base} = \pi(7)^2$$

$$LA = 2\pi(7)(10) = 140\pi \quad A_{base} = 49\pi$$

$$TA = LA + 2(A_{base})$$

$$TA = 140\pi + 2(49\pi)$$

$$TA = 140\pi + 98\pi = 238\pi$$

$$c \quad LA_{cylinder} = C \cdot h \quad A_{base} = \pi r^2$$

$$LA = 2\pi rh \quad A_{base} = \pi(4)^2$$

$$LA = 2\pi(4)(5) \quad A_{base} = 16\pi$$

$$LA = 40\pi$$

$$TA = LA + 2(A_{base})$$

$$TA = 40\pi + 2(16\pi)$$

$$TA = 40\pi + 32\pi = 72\pi$$

$$d \quad LA_{cone} = \frac{1}{2}C \cdot \ell \quad A_{base} = \pi r^2$$

$$LA = \pi r \ell \quad A_{base} = \pi(1)^2$$

$$LA = \pi(1)(3) \quad A_{base} = \pi$$

$$TA = LA + (A_{base})$$

$$TA = 3\pi + \pi = 4\pi$$

3 $TA = 4\pi r^2$

$$144\pi = 4\pi r^2$$

$$36 = r^2$$

$$6 = r$$

4 a $TA = LA \text{ of cone} + LA \text{ of cylinder} + A \text{ of cylinder's base}$

$$LA_{\text{cone}} = \frac{1}{2}C \cdot \ell$$

$$LA_{\text{cylinder}} = C \cdot h$$

$$LA_{\text{cone}} = \pi r^2$$

$$LA_{\text{cylinder}} = 2\pi rh$$

$$LA_{\text{cone}} = \pi(2)(5)$$

$$LA_{\text{cylinder}} = 2\pi(2)(5)$$

$$LA_{\text{cone}} = 10\pi$$

$$LA_{\text{cylinder}} = 20\pi$$

$$A_{\text{base}} = \pi r^2$$

$$A_{\text{base}} = \pi 2^2 = 4\pi$$

$$TA = 10\pi + 20\pi + 4\pi = 34\pi$$

b $TA = A_{\text{hemisphere}} + A_{\text{base}}$

$$A_{\text{hemisphere}} = \frac{1}{2}4\pi r^2$$

$$A_{\text{hemisphere}} = 2\pi(6)^2$$

$$A_{\text{hemisphere}} = 2\pi(36)$$

$$A_{\text{hemisphere}} = 72\pi$$

$$TA = 72\pi + 36\pi = 108\pi$$

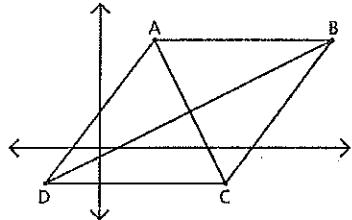
5 a $m \text{ of } \overline{AC} = \frac{6 - (-2)}{3 - 7} = \frac{8}{-4} = -2$

$$m \text{ of } \overline{BD} = \frac{6 - (-2)}{13 - (-3)} = \frac{8}{16} = \frac{1}{2}$$

b mdpt of $\overline{AC} = \left(\frac{7+3}{2}, \frac{-2+6}{2}\right) = (5, 2)$

$$\text{mdpt of } \overline{BD} = \left(\frac{13-3}{2}, \frac{6-2}{2}\right) = (5, 2)$$

$\therefore \overline{AC}$ and \overline{BD} are perp bis and ABCD is a rhombus.



6 A of $\frac{1}{2} \text{ cyl} = LA + A_{\text{base} 1} + A_{\text{base} 2} + A_{\text{cut surface}}$

$$LA = \frac{1}{2}(C \cdot h)$$

$$A_{\text{base}} = \frac{1}{2}\pi r^2$$

$$LA = \frac{1}{2}(2\pi rh)$$

$$A_{\text{base}} = \frac{1}{2}\pi(5)^2$$

$$LA = \frac{1}{2}(2)\pi(5)(2)$$

$$A_{\text{base}} = \frac{25}{2}\pi$$

$$LA = 10\pi$$

$$A_{\text{cut surface}} = \ell \cdot w$$

$$A_{\text{cut surface}} = 2 \cdot 10$$

$$A_{\text{cut surface}} = 20$$

$$A_{\frac{1}{2} \text{ cyl}} = 10\pi + 2\left(\frac{25}{2}\right)\pi = 20$$

$$A_{\frac{1}{2} \text{ cyl}} = 35\pi + 20$$

7 a The radius = 5 because (5-12-13) is a Pythagorean Triple.

$$LA_{\text{cone}} = \frac{1}{2}C \cdot \ell$$

$$LA_{\text{cone}} = \pi r \ell$$

$$LA_{\text{cone}} = \pi(5)(13)$$

$$LA_{\text{cone}} = 65\pi$$

$$TA = LA + A_{\text{base}}$$

$$TA = 65\pi + 25\pi = 90\pi$$

b 8 = 2 · 4 (3-4-5) is a Pythagorean Triple

$$10 = 2 \cdot 5$$

$$6 = 2 \cdot 3 \text{ (diameter)}$$

$$\text{diameter} = 2 \cdot 3 = 6 \quad r = \frac{1}{2}d = 3$$

$$LA_{\text{cylinder}} = C \cdot h$$

$$LA_{\text{cylinder}} = 2\pi rh$$

$$LA_{\text{cylinder}} = 2\pi(3)(8)$$

$$LA_{\text{cylinder}} = 48\pi$$

$$TA = LA + 2(A_{\text{base}})$$

$$TA = 48\pi + 2(9\pi)$$

$$TA = 48\pi + 18\pi = 66\pi$$

$$c \text{ Radius} = \frac{1}{2}(30) = 15$$

(8-15-17) is a Pythagorean Triple, so $\ell = 17$

$$LA_{\text{cone}} = \frac{1}{2}C \cdot \ell$$

$$LA = \pi r \cdot \ell$$

$$LA = \pi(15)(17)$$

$$LA = 255\pi$$

$$TA = LA + A_{\text{base}}$$

$$TA = 255\pi + 225\pi = 480\pi$$

8 Since the tower is 10 m in total height, the sides of rt Δ formed are 4 and 3. Use Pythagorean triple (3-4-5) and the slant height is 5.

$$LA_{\text{cone}} = \frac{1}{2}C \cdot \ell$$

$$LA_{\text{cone}} = \pi r \ell$$

$$LA_{\text{cone}} = \pi(4)(5)$$

$$LA_{\text{cone}} = 20\pi$$

$$A_{\text{total painted}} = A_{\text{cone}} + A_{\text{cyl}}$$

$$A_{\text{total painted}} = 20\pi + 56\pi = 76\pi$$

$$(76)(3.14) = 238.64 \text{ sq m}$$

$\sim 238.64 \div 10 \approx 23.864$, about 24 cans of paint

9 The label is the LA and LA of cylinder is $C \cdot h$.

$$LA = C \cdot h$$

$$r = \frac{1}{2} \text{diameter} = 4$$

$$LA = 2\pi rh$$

$$LA = 2\pi(4)(14) = 8\pi \text{ cm by 14 cm}$$

- 10 Diameter = 6, radius = 3. Use Pythagorean triple 3-4-5 to find height of cone = 4, slant height of cone = 5, height of cylinder = $17 - 4 - 3 = 10$.

$$A_{cyl} = 2\pi rh \quad A_{cone} = \pi r^2$$

$$A_{cyl} = 2\pi \cdot 3 \cdot 10 \quad A_{cone} = \pi \cdot 3 \cdot 5$$

$$A_{cyl} = 60\pi \quad A_{cone} = 15\pi$$

$$A_{hemisphere} = \frac{1}{2}(4\pi r^2)$$

$$A_{hemisphere} = 2\pi r^2$$

$$A_{hemisphere} = 18\pi$$

$$TA = A_{cyl} + A_{cone} + A_{hemisphere}$$

$$TA = 60\pi + 15\pi + 18\pi = 93\pi$$

11 a $A_{kite} = \frac{1}{2}d_1 \cdot d_2 = \frac{1}{2}(16)(8) = 64$

b From mdpt form, mid $\overline{KE} = (2, 4)$, mid $\overline{KI} = (-2, 4)$, and mid $\overline{ET} = (2, -4)$. Area $\square = 4 \cdot 8 = 32$

12 $TA = LA_{prism} + LA_{cyl} + 2(A_{prism \ base} - A_{cyl \ base})$

$$TA = 4(b \cdot h) + 2\pi rh + 2(b \cdot h) - \pi r^2$$

$$TA = 4(10)(8) + 2\pi(3)(10) + 2[(8)(8) - \pi 3^2]$$

$$TA = 4(80) + 60\pi + 2(64 - 9\pi)$$

$$TA = 320 + 60\pi + 128 - 18\pi$$

$$TA = 448 + 42\pi$$

- 13 Draw in the rest of the cone and find the area.

$$LA = \pi(8)(10) = 80\pi$$

Subtract area of top cone.

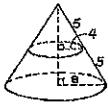
$$LA = \pi(4)(5) = 20\pi$$

$$80\pi - 20\pi = 60\pi$$

radius of large $\odot = 8$, small $\odot = 4$

slant ht large = 10, small = 5, (converse of Midline Th)

$$TA = 16\pi + 64\pi + (\pi \cdot 8 \cdot 10 - \pi \cdot 4 \cdot 5) = 140\pi$$



- 14 a Forms the frustum of a cone.

Extend the frustum to form

a cone. There are two

$30^\circ 60^\circ 90^\circ$ Δ s formed with

|| bases.

In the large Δ , hypotenuse = ℓ of cone = 12

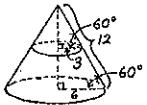
In small Δ , hypotenuse = $2(3) = 6$

$$TA = LA_{large \ cone} - LA_{small \ cone} + A_{base \ 1} + A_{base \ 2}$$

$$TA = \pi r \cdot \ell - \pi r \cdot \ell + \pi r^2 + \pi r^2$$

$$TA = \pi(6)(12) - \pi(3)(6) + \pi(6)^2 + \pi(3)^2$$

$$TA = 72\pi - 18\pi + 36\pi + 9\pi = 99\pi$$



- b Forms cylindrical shell.

$$\text{radius } sm \odot = 2$$

$$\text{radius lg } \odot = 2 + 1 = 3$$

$$\text{height} = 5$$



$$TA = LA_{large \ cyl} + LA_{small \ cyl} + 2(A_{large \ base} - A_{small \ base})$$

$$TA = 2\pi rh + 2\pi rh + 2(\pi r^2 - \pi r^2)$$

$$TA = 2\pi(3)(5) + 2\pi(2)(5) + 2[\pi(3)^2 - \pi(2)^2]$$

$$TA = 2\pi(15) + 20\pi + 2(9\pi - 4\pi)$$

$$TA = 30\pi + 20\pi + 10\pi = 60\pi$$

- c Forms same solid as problem 9.

$$\text{radius} = 8$$

$$\text{height cyl} = 10$$

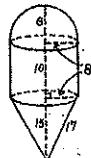
$$\text{slant height cone} = 17$$

$$TA = \frac{1}{2}(A_{sphere}) + LA_{cylinder} + LA_{cone}$$

$$TA = \frac{1}{2}(4\pi r^2) + 2\pi rh + \pi r\ell$$

$$TA = \frac{1}{2}[4\pi(8)^2] + 2\pi(8)(10) + \pi(8)(17)$$

$$TA = 128\pi + 160\pi + 136\pi = 424\pi$$



Pages 579-582 (Section 12.4)

1 a $V_{cylinder} = Bh$

$$V = 25\pi \cdot 12 = 300\pi$$

b $V_{prism} = B \cdot h$

$$V = (9 \cdot 10)8 = 720$$

2 $V_{cylinder} = Bh \quad V_{rect \ box} = Bh$

$$= \pi r^2 h \quad = \ell wh$$

$$= \pi(1)^2(5) \quad = (4)(3)(\frac{1}{2})$$

$$= 5\pi \quad = 6$$

$$V_{pedestal} = V_c + V_r$$

$$= 5\pi + 6$$

$$= (5\pi + 6) m^3$$

3 $V = Bh$

$$V = 51 \cdot 7 = 357$$

4 Volume = ℓwh

$$V = (5)(6)(10)$$

$$V = 300$$

$$TA = LA + 2(A_{base})$$

$$= 2(bh) + 2(bh) + 2(bh)$$

$$= 2(6)(10) + 2(5)(10) + 2(6)(5)$$

$$= 2(60) + 2(50) + 2(30)$$

$$= 120 + 100 + 60 = 280$$

5 a $V = e^3 \quad b V = e^3 \quad c V = e^3$

$$V = 7^3 = 343$$

$$125 = e^3$$

$$5 = e$$

$$6 \quad V = Bh$$

$$286 = 13h$$

$$22 = h$$

$$7 \quad a \quad V = 7 \times 5 \times \frac{8}{12} \approx 23 \text{ cu ft}$$

$$b \quad 23.33 \times 62.4 \approx 1456 \text{ lb}$$

$$8 \quad (2, 0, 8)$$

$$9 \quad V = Bh$$

$$= \frac{1}{2}(24)(5)(10)$$

$$= 600$$

$$TA = LA + B$$

$$= 2(13)(10) + (10)(24) + 2(\frac{1}{2})(24)(5)$$

$$= 260 + 240 + 120 = 620$$

$$10 \quad a \quad V = Bh \quad TA = LA + 2B$$

$$= \pi r^2 h \quad TA = 2\pi rh + 2\pi r^2$$

$$= \pi(\frac{9}{2})^2(12) \quad TA = 2\pi(\frac{9}{2})(12) + 2\pi(\frac{9}{2})^2$$

$$= \frac{81}{4}\pi(12) \quad TA = 108\pi + 40.5\pi$$

$$V = 243\pi \quad TA = 148.5\pi$$

$$b \quad V = \frac{1}{2}Bh$$

$$= \frac{1}{2}\pi r^2 h$$

$$= \frac{1}{2}\pi(6)^2(20)$$

$$= (18)\pi(20)$$

$$V = 360\pi$$

$$TA = \frac{1}{2}(LA + B_1 + B_2) + A_{\text{rect}}$$

$$= \frac{1}{2}(2\pi rh + \pi r^2 + \pi r^2) + bh$$

$$= \frac{1}{2}(2\pi rh + 2\pi r^2) + bh$$

$$= \pi rh + \pi r^2 + bh$$

$$= \pi(6)(20) + \pi(6)^2 + (6 + 6)(20)$$

$$= 120\pi + 36\pi + 12(20)$$

$$= 156\pi + 240$$

$$11 \quad \text{Let } x \text{ be the length of a side. } V_{\text{cube}} = x^3. \text{ A cube has 6 square faces. Each face has an area of } x^2. TA = 6x^2.$$

$$V_{\text{cube}} = TA$$

$$x^3 = 6x^2$$

$$x^3 - 6x^2 = 0$$

$$x^2(x - 6) = 0$$

$$x = 0 \text{ or } x = 6, x \neq 0, \text{ so } x = 6$$

$$12 \quad V_{\text{prism}} = Bh$$

$$V = (\frac{1}{2}ap)h$$

$$V = \frac{1}{2}(3\sqrt{3})(36)(10)$$

$$V = 54\sqrt{3}(10) = 540\sqrt{3}$$

$$TA = LA + Abases$$

$$TA = 6(6)(10) + 2(\frac{1}{2})(3\sqrt{3})(36)$$

$$TA = 360 + 108\sqrt{3}$$

13 The face diagonal forms a $45^\circ 45^\circ 90^\circ \Delta$, so an edge is $5\sqrt{2}$.

$$V_{\text{cube}} = s^3$$

$$= (5\sqrt{2})^3$$

$$V = 250\sqrt{2}$$

$$14 \quad V_{\text{batter}} = \ell wh$$

$$810 = \ell wh$$

$$810 = (10)(12)(h)$$

$$810 = 120h$$

$$h = \frac{810}{120}$$

$$h = \frac{27}{4} = 6.75 \text{ cm}$$

$$15 \quad V_{\text{prism}} = Bh$$

$$= (9)(7)(3) = 189$$

$$16 \quad V_{\text{pan}} = \ell wh \quad V_{\text{cyl}} = Bh$$

$$= (15)(10)(5) \quad = \pi r^2 h$$

$$= 750 \text{ cu cm} \quad = \pi(4)^2(15)$$

$$= 240\pi$$

$$V_{\text{cyl}} \approx 240(3.14)$$

$$V_{\text{cyl}} \approx 753.6 \text{ cu cm}$$

Yes. It will overflow by $\approx 3.6 \text{ cu cm}$.

$$17 \quad a \quad V_{\text{lunch box}} = V_{\text{prism}} + V_{\text{cyl}}^1$$

$$V_{\text{prism}} = Bh = (6)(8)(10) = 480$$

$$V_{\text{cyl}}^1 = \frac{1}{2}Bh = \frac{1}{2}(9\pi)(10) = 45\pi$$

$$V_{\text{lunch box}} = 480 + 45\pi \approx 621 \text{ cu in.}$$

$$b \quad TA = A_5 \text{ rectangles} + A_{\text{cyl}}^1$$

$$= 2(8)(10) + 6(10) + 2(6)(8) + \frac{1}{2}(6\pi)(10) + 9\pi$$

$$= 160 + 60 + 96 + 30\pi + 9\pi$$

$$TA \text{ of metal} = 316 + 39\pi \approx 439 \text{ sq in.}$$

18 Diameter of outer cylinder = $10 + 1 + 1 = 12$

Radius of outer cylinder = 6

$$\begin{aligned} V_{\text{outer cyl}} &= Bh \\ &= \pi r^2 h \\ &= \pi(6)^2(20) \\ &= 720\pi \text{ cu ft} \end{aligned}$$

Because the bottom will be 1 ft thick, height inner cyl = height outer cyl - 1 ft = 19 ft and the radius inner cyl = $\frac{1}{2}(10) = 5$

$$\begin{aligned} V_{\text{inner cyl}} &= Bh \\ &= \pi r^2 h \\ &= \pi(5)^2(19) \\ &= 475\pi \text{ cu ft} \end{aligned}$$

$$\begin{aligned} \text{Amt cement} &= V_{\text{outer cyl}} - V_{\text{inner cyl}} \\ &= 720\pi \text{ cu ft} - 475\pi \text{ cu ft} \end{aligned}$$

$$\text{Amt cement} = 245\pi \approx 770 \text{ cu ft}$$

19 $V_{\text{prism}} = Bh$

$$\begin{aligned} &= A \text{ sector } h \\ &= \frac{40}{360}\pi(20)^2(15) \\ V_{\text{prism}} &= \frac{1}{9}\pi(400)(15) = \frac{2000}{3}\pi \\ TA &= \frac{40}{360}(2\pi \cdot 20 \cdot 15) + 2[\frac{40}{360}\pi(20)^2 + 15 \cdot 20] \\ &= \frac{1}{9}(40\pi)(15) + 2(\frac{1}{9})(400\pi) + 2(15)(20) \\ &= \frac{600}{9}\pi + \frac{800}{9}\pi + 600 = \frac{1400}{9}\pi + 600 \\ TA &= \frac{1400}{9}\pi + 600 \end{aligned}$$

20 a $V_{\text{cube}} = (s)^3$ $V_{\text{hole}} = Bh$
 $V_{\text{cube}} = (4)^3$ $V_{\text{hole}} = \pi r^2 h$
 $V_{\text{cube}} = 64 \text{ cu cm}$ $V_{\text{hole}} = \pi(1)^2(4)$
 $V_{\text{cube}} \approx 4(3.14) \approx 12.56 \text{ cu cm}$
 $V_{\text{ice}} = V_{\text{cube}} - V_{\text{hole}}$
 $V \approx 64 - 12.56 \text{ cu cm}$
 $V \approx 51.4 \text{ cu cm}$

b $V 10 \text{ cubes} \approx 10(51.44 \text{ cu cm}) \approx 514.4 \text{ cu cm}$
 $V 10 \text{ melted cubes} \approx 514.4 \text{ cu cm} - 11\%(514.4 \text{ cu cm})$
 $V 10 \text{ melted cubes} \approx (.89)(514.4 \text{ cu cm}) \approx 457.8 \text{ cu cm}$
c $TA = 6(\text{A face of the cube}) - 2(\text{A base of cyl}) + LA_{\text{cyl}}$
 $TA = 6(s^2) - 2(\pi r^2) + Ch$
 $= 6(4)^2 - 2(\pi(1)^2) + 2\pi(1)(4)$
 $= 6(16) - 2\pi + 8\pi$
 $\approx 96 - 2(3.14) + 8(3.14)$
 $TA \approx 114.8 \text{ sq cm}$

d $TA \text{ of a cube without hole} = 6(s^2)$
 $TA = 6(4^2)$
 $TA = 96 \text{ sq cm}$

$$\frac{114.8}{96} \approx 1.20$$

His claim is not true, 114.8 sq cm vs. 96 sq cm.

21

$$\begin{aligned} V_{\text{prism}} &= Bh \\ V &= [\frac{1}{2}\pi(6)^2 - 2(\frac{1}{2}\pi(3)^2)](10) \\ V &= (18\pi - 9\pi)10 = 90\pi \end{aligned}$$

22 Separate the cylinder into a cylinder and a half cylinder.

The height of the top cylinder is $12 - 8 = 4$.

$$\begin{aligned} \text{Total Volume} &= V_{\text{cyl 1}} + V_{\text{cyl 2}} \\ &= Bh_1 + \frac{1}{2}Bh_2 \\ &= \pi r^2 h_1 + \frac{1}{2}(\pi r^2 h_2) \\ &= \pi(3)^2(8) + \frac{1}{2}\pi(3)^2(4) \\ &= 72\pi + 18\pi \end{aligned}$$

$$\text{Total Volume} = 90\pi$$

Pages 585-588 (Section 12.5)

1. $V_{\text{pyramid}} = \frac{1}{3}Bh$
 $= \frac{1}{3}(\frac{14^2}{4})\sqrt{3}(30)$
 $= \frac{1}{3}(49\sqrt{3})(30) = 490\sqrt{3}$

2. $V_{\text{cone}} = \frac{1}{3}Bh$ (5-12-13) is a Pythagorean Triple, so $h = 12$
 $V_{\text{cone}} = \frac{1}{3}(\pi r^2)h$
 $= \frac{1}{3}\pi(5)^2(12)$
 $= \frac{1}{3}(25\pi)12 = 100\pi \approx 314.16$

3 a $V_{\text{pyramid}} = \frac{1}{3}B \cdot h$
 $V_{\text{pyramid}} = \frac{1}{3}(100)(12) = 400$
b $TA = LA + A_{\text{base}}$
 $= LA + 100$ Height of Δ is 13, using rt Δ
 $= 4(\frac{1}{2})(10)(13) + 100$
 $TA = 260 + 100 = 360$

4 $V_{\text{pyramid}} = \frac{1}{3}Bh$
 $42 = \frac{1}{3}(14)h$
 $42 = \frac{14}{3}h, h = 9$

5 a $V_{\text{cone}} = \frac{1}{3}Bh$ $r = 9$ from Pythagorean triple 9-40-41
 $V_{\text{cone}} = \frac{1}{3}(81\pi)(40)$
 $= 1080\pi$
b $LA = \pi r \ell$ $c TA = LA + A_{\text{base}}$
 $= \pi(9)(41)$
 $LA = 369\pi$ $TA = 450\pi$
 $= 369\pi + 81\pi$

$$\begin{aligned}
 6 \quad \text{Total Volume} &= V_{\text{rect box}} + V_{\text{pyramid}} \\
 &= Bh_1 + \frac{1}{3}Bh_2 \\
 &= 25(20) + \frac{1}{3}(25)(24 - 20) \\
 &= 500 + \frac{1}{3}(25)(4) \\
 \text{Total Volume} &= 500 + \frac{100}{3} = \frac{1600}{3} \approx 533 \text{ cu m}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad V_{\text{cyl}} &= Bh \quad \text{The height of the cone is 4} \\
 &= 9\pi(50) \quad (\text{Pythagorean Triple 3-4-5}) \\
 &= 450\pi \\
 V_{\text{cone}} &= \frac{1}{3}Bh \\
 &= \frac{1}{3}(9\pi)(4) \\
 V_{\text{cone}} &= 12\pi
 \end{aligned}$$

$$\text{Total Volume} = 450\pi + 12\pi = 462\pi \approx 1451 \text{ cu ft}$$

$$\begin{aligned}
 8 \quad \text{The } \Delta \text{ formed by the alt of the cone, a radius, and a slant} \\
 \text{height is } 30^\circ 60^\circ 90^\circ \Delta. \quad \text{radius} = \frac{1}{2}(12) = 6, \text{ alt} = h = 6\sqrt{3} \\
 V_{\text{cone}} &= \frac{1}{3}Bh \\
 &= \frac{1}{3}(\pi r^2)h \\
 &= \frac{1}{3}\pi(6)^2(6\sqrt{3}) \\
 &= \frac{1}{3}(36\pi)(6\sqrt{3}) = 72\pi\sqrt{3} \approx 391.8
 \end{aligned}$$

$$\begin{aligned}
 9 \quad V_{\text{pyramid}} &= \frac{1}{3}Bh \quad \text{Side of square found} \\
 &= \frac{1}{3}(5\sqrt{2})^2(12) \quad \text{using } 45^\circ 45^\circ 90^\circ \Delta. \\
 &= \frac{1}{3}(50)(12) = 200 \quad \text{Height found using} \\
 &\quad 5-12-13 \text{ rt } \Delta.
 \end{aligned}$$

$$\begin{aligned}
 10 \quad a \quad \text{Since PABCD is a reg pyramid and P is (5, 5, 12),} \\
 C \text{ must be } 5+5 \text{ along the } y\text{-axis. } C = (0, 10, 0). \\
 b \quad V = \frac{1}{3}(100)(12) = 400
 \end{aligned}$$

$$\begin{aligned}
 11 \quad V_{\text{sm cone}} &= \frac{1}{3}(36\pi)(4) = 48\pi \\
 V_{\text{lg cone}} &= \frac{1}{3}(36\pi)(10) \approx 120\pi \\
 \text{Remaining Volume} &= V_{\text{lg cone}} - V_{\text{sm cone}} \\
 &= 120\pi - 48\pi = 72\pi
 \end{aligned}$$

$$\begin{aligned}
 12 \quad V_{\text{rocket}} &= V_{\text{cyl}} + V_{\text{cone}} \\
 V_{\text{rocket}} &= Bh + \frac{1}{3}Bh \\
 &= (\pi r^2)h + \frac{1}{3}(\pi r^2)h \\
 &= \pi(8)^2(50) + \frac{1}{3}\pi(8)^2(15) \\
 &= 64\pi(50) + \frac{1}{3}(64\pi)15 \\
 &= 3200\pi + 320\pi = 3520\pi
 \end{aligned}$$

(8-15-17 is a Pythagorean Triple, so the height of the cone is 15.) If 60% contains fuel, 40% remains.

$$40\% \text{ of } 3520\pi = (.4)(3520\pi) = 1480\pi = 4423$$

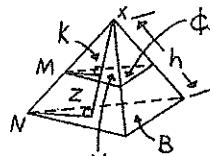
$$\begin{aligned}
 13 \quad V_{\text{prism}} &= Bh \quad V_{\text{pyramid}} = \frac{1}{3}Bh \\
 V_{\text{prism}} &= (60)(10) = 600 \quad V_{\text{pyramid}} = \frac{1}{3}(60)(6) = 120 \\
 600 + 120 &= 720 \\
 \text{Total Volume} &= 720 \text{ cu m}
 \end{aligned}$$

$$\begin{aligned}
 14 \quad a \quad \text{The small } \Delta \text{ is } \sim \text{ to the whole } \Delta \text{ by AA-}, \text{ so} \\
 \frac{x}{x+10} &= \frac{12}{12+8} \quad 20x = 12x + 120 \\
 \frac{x}{x+10} &= \frac{12}{20} \quad 8x = 120 \\
 & \quad x = 15 \\
 b \quad 12 &= 4 \cdot 3 \\
 15 &= 5 \cdot 3 \quad (3-4-5) \text{ is a Pythagorean triple.} \\
 \text{radius} &= 3 \cdot 3 = 9 \\
 x+10 &= 15+10=25 \quad 25 = 5 \cdot 5 \\
 12+8 &= 20 \quad 20 = 4 \cdot 5 \text{ (3-4-5) is a} \\
 & \quad \text{Pythagorean Triple.} \\
 \text{radius} &= 3 \cdot 5 = 15
 \end{aligned}$$

The radii are 9 and 15.

$$\begin{aligned}
 c \quad V_{\text{cone}} &= \frac{1}{3}Bh \quad d \quad V_{\text{cone}} = \frac{1}{3}Bh \\
 &= \frac{1}{3}(\pi r^2)h \quad = \frac{1}{3}(\pi r^2)h \\
 &= \frac{1}{3}(\pi(9)^2)12 \quad = \frac{1}{3}(\pi(15)^2)20 \\
 V &= 324\pi \approx 1018 \quad V = 1500\pi \approx 4712
 \end{aligned}$$

$$\begin{aligned}
 e \quad V_{\text{frustum}} &= V_{\text{large cone}} - V_{\text{small cone}} \\
 V &= 1500\pi - 324\pi = 1176\pi \approx 3695
 \end{aligned}$$



- 15 Given: The pyramid as shown
Prove: $\frac{\epsilon}{3} = \left(\frac{k}{h}\right)^2$
- | | |
|---------------------------------------|--|
| 1 Pyramid as shown | 1 Given |
| 2 $\angle XYM$ is a rt \angle . | 2 Altitudes form rt \angle s. |
| 3 $\angle XZN$ is a rt \angle . | 3 Same as 2 |
| 4 $\angle XYM \cong \angle XZN$ | 4 All rt \angle s are \cong . |
| 5 $\angle MXY \cong \angle MXN$ | 5 Reflexive prop |
| 6 $\triangle WYM \cong \triangle XZN$ | 6 AA- |
| 7 $\frac{k}{h} = \frac{XM}{ZN}$ | 7 If 2 Δ s \sim , the ratios of
corres sides are $=$. |

$$\begin{aligned}
 8 \quad \left(\frac{XM}{ZN}\right)^2 &= \frac{\epsilon}{B} \\
 9 \quad \left(\frac{k}{h}\right)^2 &= \frac{\epsilon}{B} \quad 9 \text{ Substitution prop}
 \end{aligned}$$

$$\begin{aligned}
 16 \quad 6s^2 &= 150 \\
 s^2 &= 25 \\
 s &= 5 \\
 V &= 5^3 = 125 \text{ cu in.}
 \end{aligned}$$

- 17 a The slant height forms

two $30^\circ 60^\circ 90^\circ$ Δ s,

$$\text{slant height} = 3\sqrt{3}$$

In the base, the Δ

formed is a $30^\circ 60^\circ 90^\circ$ Δ .

$$x = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$x^2 + h^2 = (\text{slant height})^2$$

$$(\sqrt{3})^2 + h^2 = (3\sqrt{3})^2$$

$$3 + h^2 = 27$$

$$h = \sqrt{24} = 2\sqrt{6}$$

$$V_{\text{pyramid}} = \frac{1}{3}Bh$$

$$= \frac{1}{3}(\frac{s^2}{4}\sqrt{3})(2\sqrt{6})$$

$$= \frac{1}{3}(9)(2)(\sqrt{18})$$

$$= 6\sqrt{18} = 18\sqrt{2}$$

b If side is s , slant height = $\frac{s}{2}\sqrt{3}$, $h = \frac{s}{3}\sqrt{6}$

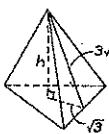
$$V_{\text{pyramid}} = \frac{1}{3}Bh$$

$$= \frac{1}{3}(\frac{s^2}{4}\sqrt{3})(\frac{s}{3}\sqrt{6})$$

$$= (\frac{1}{3}\frac{s^3}{12})(\sqrt{18})$$

$$= \frac{1}{3}\frac{s^3}{12}(3\sqrt{2})$$

$$V_{\text{pyramid}} = \frac{s^3}{12}\sqrt{2}$$



- 18 The octahedron can form 2 square pyramids; then,

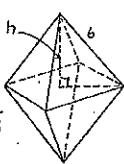
$$V_{\text{pyramid}} = \frac{1}{3}Bh$$

$$= \frac{1}{3}(6)^2(3\sqrt{2})$$

$$= (12)(3\sqrt{2}) = 36\sqrt{2}$$

$$V_{2 \text{ pyramids}} = 2(36\sqrt{2}) = 72\sqrt{2}$$

$$V_{\text{octahedron}} = 72\sqrt{2}$$



- 19 The height of the pyramid is 12 because (5-12-13) is a Pythagorean Triple. The base of the pyramid is an isosceles Δ . The alt to the base of an isosceles Δ bisects the base. 3-4-5 is a Pythagorean

Triple, so the height is 4.

$$B = A_\Delta = \frac{1}{2}bh$$

$$V_{\text{pyramid}} = \frac{1}{3}Bh$$

$$B = \frac{1}{2}(6)(4) = 12$$

$$V_{\text{pyramid}} = \frac{1}{3}(12)(12) = 48$$



- 20 Using $\sim \Delta$ s,

$$\frac{x}{x+5} = \frac{6}{9}$$

$$9x = 6x + 30$$

$$3x = 30, x = 10$$

By Pythagorean Theorem,

$$h_{\text{small cone}} = 8$$

$$h_{\text{large cone}} = 12$$

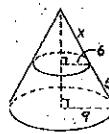
$$V_{\text{frustum}} = V_{\text{large cone}} - V_{\text{small cone}}$$

$$V_{\text{lg cone}} = \frac{1}{3}\pi r^2 h \quad V_{\text{sm cone}} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}(81\pi)(12) \quad = \frac{1}{3}(36\pi)(8)$$

$$= 324\pi \quad = 96\pi$$

$$V_{\text{frustum}} = 324\pi - 96\pi = 228\pi$$



Pages 589-592 (Section 12.6)

1 a $V_{\text{sphere}} = \frac{4}{3}\pi r^3$ b $V_{\text{sphere}} = \frac{4}{3}\pi r^3$
 $= \frac{4}{3}\pi(3^3)$ $= \frac{4}{3}\pi(9^3)$
 $= \frac{4}{3}\pi(27)$ $= \frac{4}{3}\pi(729)$

$$V_{\text{sphere}} = 36\pi \quad V_{\text{sphere}} = 927\pi$$

c $V_{\text{sphere}} = \frac{4}{3}\pi r^3$
 $= \frac{4}{3}\pi(5^3)$
 $= \frac{4}{3}\pi(125)$

$$V_{\text{sphere}} = \frac{500}{3}\pi$$

2 $V_{\text{sphere}} = \frac{4}{3}\pi r^3$ TA_{sphere} = $4\pi r^2$
 $= \frac{4}{3}\pi(6^3)$ $= 4\pi(6)^2$
 $= \frac{4}{3}\pi(216)$ $= 4\pi(36)$

$$V_{\text{sphere}} = 288\pi \quad TA_{\text{sphere}} = 144\pi$$

3 $V_{\text{cyl}} = B \cdot h$ $V_{\text{sphere}}^1 = \frac{1}{2}(\frac{4}{3}\pi r^3)$
 $V_{\text{cyl}} = 9\pi(15) = 135\pi$ $= \frac{1}{2}(\frac{4}{3}\pi)(27)$
 $V = 18\pi$

$$V_{\text{silo}} = 135\pi + 18\pi = 153\pi = 481 \text{ cu m}$$

4 a $V_{\text{cyl}} = Bh$ $r = \frac{1}{2}(14) = 7$
 $= \pi r^2 h$
 $V_{\text{cyl}} = \pi(7)^2(8) = 392\pi$

$$b V_{\text{hemisphere}} = \frac{1}{2}V_{\text{sphere}}$$
 $= \frac{1}{2}(\frac{4}{3}\pi 6^3)$

$$V_{\text{hemisphere}} = 144\pi$$

c $V_{\text{plastic}} = V_{\text{cyl}} - V_{\text{hemisphere}}$
 $V_{\text{plastic}} = 392\pi - 144\pi = 248\pi$

5 $V_{\text{sphere}} = \frac{4}{3}\pi r^3$
 $= \frac{4}{3}\pi(5^3)$
 $= \frac{4}{3}\pi(125)$

$$V_{\text{sphere}} = \frac{500}{3}\pi \approx (166)(3.14) \approx 523 \text{ cu ft}$$

6 $r = \frac{1}{2}(48) = 24$ $r = \frac{1}{2}(42) = 21$

$$\begin{aligned} V_{\text{sphere}} &= \frac{4}{3}\pi r^3 & V_{\text{sphere}} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(24)^3 & &= \frac{4}{3}\pi(21)^3 \\ &= \frac{4}{3}\pi(13824) & &= \frac{4}{3}\pi(9261) \end{aligned}$$

$$V_{\text{sphere}} = 18,432\pi$$

$$V_{\text{rubber used}} = 18,432\pi - 12,348\pi = 6084\pi \approx 19 \text{ cu cm}$$

7 a $V_{\text{sphere}}^1 = \frac{1}{2}(\frac{4}{3}\pi r^3)$ $V_{\text{cone}} = \frac{1}{3}Bh$
 $V_{\text{sphere}}^1 = \frac{1}{2}(\frac{4}{3}\pi)(216) = 144\pi$ $= \frac{1}{3}(36\pi)(8)$
 $V_{\text{cone}} = 96\pi$

$$\text{Total volume} = 144\pi + 96\pi = 240\pi$$

b $A_{\text{sphere}}^1 = \frac{1}{2} \cdot 4\pi r^2$

$$A_{\text{sphere}}^1 = \frac{1}{2}(4\pi)(36) = 72\pi$$

$$A_{\text{cone}} = \pi r \ell$$

$$\ell = \text{slant height} = 10$$

$$A_{\text{cone}} = \pi \cdot 6 \cdot 10$$

(Pythagorean triple 6-8-10)

$$A_{\text{cone}} = 60\pi$$

$$\text{Total surface area} = 72\pi + 60\pi = 132\pi$$

8 a $V_{\text{hemisphere}} = \frac{1}{2}V_{\text{sphere}}$ b $A \odot = \pi r^2$
 $= \frac{1}{2}(\frac{4}{3}\pi r^3)$ $A \odot = \pi(30)^2$
 $= \frac{2}{3}\pi(30)^3$ $A \odot = 900\pi \approx 2827 \text{ sq m}$
 $= \frac{2}{3}\pi(27,000)$

$$V_{\text{hemisphere}} = 18,000\pi \approx 56,549 \text{ cu m}$$

c $A_{\text{hemisphere}} = \frac{1}{2}A_{\text{sphere}}$
 $= \frac{1}{2}(4\pi r^2)$
 $= 2\pi(30)^2$

$$A_{\text{hemisphere}} = 1800\pi \text{ sq m}, A \odot = 900\pi \text{ sq m}$$

Twice as much paint is needed to cover the area of the hemisphere.

d $1800\pi = \pi r^2$

$$1800 = r^2$$

$$r = 30\sqrt{2} \approx 42 \text{ m}$$

9 The cold capsule is a cylinder and a sphere.

$$V_{\text{cyl}} = B \cdot h$$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$V_{\text{cyl}} = \frac{9}{4}\pi(8) = 18\pi$$

$$V_{\text{sphere}} = \frac{4}{3}\pi(\frac{9}{2})^3$$

$$V_{\text{sphere}} = \frac{4}{3}\pi(\frac{27}{8}) = \frac{9}{2}\pi$$

$$\text{Total Volume} = 18\pi + \frac{9}{2}\pi = 22\frac{1}{2}\pi \approx 71 \text{ cu mm}$$

10 a $V_{\text{sphere}} = \frac{4}{3}\pi r^3$ and the ratio of the radii is 2:5, so the ratio of the Volumes is $2^3:5^3$ or 8:125.

b $A_{\text{sphere}} = 4\pi r^2$ and the ratio of the radii is 2:5, so the ratio of the Areas is $2^2:5^2$ or 4:25.

11 a The hemisphere, cylinder, and cone all have a radius of 3.

$$\begin{aligned} V_{\text{hemi}} &= \frac{1}{2}V_{\text{sphere}} & V_{\text{cyl}} &= B \cdot h \\ &= \frac{1}{2}(\frac{4}{3}\pi r^3) & &= (\pi r^2)h \\ &= \frac{2}{3}\pi(3)^3 & &= (\pi \cdot 3^2)12 \end{aligned}$$

$$V_{\text{hemi}} = 18\pi$$

$$\begin{aligned} V_{\text{cone}} &= \frac{1}{3}B \cdot h \\ &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi(3^2)4 \end{aligned}$$

$$V_{\text{cone}} = 12\pi$$

$$\begin{aligned} TA &= V_{\text{hemi}} + V_{\text{cyl}} + V_{\text{cone}} \\ &= 18\pi + 108\pi + 12\pi = 138\pi \end{aligned}$$

b $LA_{\text{cyl}} = 2\pi rh$ $\ell = 5 \text{ in a } 3-4-5 \text{ rt } \Delta.$

$$LA_{\text{cyl}} = 2\pi 3(12)$$

$$LA_{\text{cone}} = \pi r \ell$$

$$LA_{\text{cyl}} = 72\pi$$

$$LA_{\text{cone}} = \pi(3)(5)$$

$$LA_{\text{cone}} = 15\pi$$

$$A_{\text{hemisphere}} = \frac{1}{2}(4\pi r^2)$$

$$A_{\text{hemisphere}} = 2\pi(3)^2$$

$$A_{\text{hemisphere}} = 18\pi$$

$$\text{Total surface area} = 72\pi + 15\pi + 18\pi = 105\pi$$

12 The radius of ice cream and cone is 2.

$$\begin{aligned} V_{\text{ice cream}} &= \frac{4}{3}\pi r^3 & V_{\text{cone}} &= \frac{1}{3}Bh \\ &= \frac{4}{3}\pi(2)^3 & &= \frac{1}{3}(\pi r^2)h \\ &= \frac{32}{3}\pi & &= \frac{1}{3}(\pi(2^2))9 \\ V_{\text{ice cream}} &= 10\frac{2}{3}\pi & &= \frac{1}{3}(36\pi) \\ & & & V_{\text{cone}} = 12\pi \end{aligned}$$

No. The cone will hold $12\pi \text{ cu cm}$ and the ice cream is only $10\frac{2}{3} \text{ cu cm}$.

13 a The radius of the largest inscribed sphere is $\frac{1}{2}$ the side of the cube. Since $V_{\text{cube}} = s^3$ and $s^3 = 1000$, $s = 10$ and the radius is 5.

$$\begin{aligned} V_{\text{sphere}} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi 5^3 \\ &= \frac{4}{3}(125)\pi = \frac{500}{3}\pi \approx 524 \text{ cu m} \end{aligned}$$

b. The radius of the smallest circumscribed sphere is $\frac{1}{2}$ the diagonal of the cube. From a $45^\circ 45^\circ 90^\circ \Delta$, the diagonal is $10\sqrt{3}$ and the radius = $5\sqrt{3}$.

$$\begin{aligned} V_{\text{sphere}} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(5\sqrt{3})^3 \\ &= \frac{4}{3}\pi(375\sqrt{3}) = 500\pi\sqrt{3} \approx 2721 \text{ cu m} \end{aligned}$$

$$14 \quad V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$V_{\text{cyl}} = Bh$$

$$V_{\text{cyl}} = \pi r^2(2r)$$

$$V_{\text{cyl}} = 2\pi r^3$$

$$\frac{V_{\text{sphere}}}{V_{\text{cyl}}} = \frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

$$15 \quad A \odot = \pi r^2$$

$$= \pi \cdot 6^2$$

$$= 36\pi$$

$$A_{\text{rect}} ABGH = 1 \cdot 12 = 12$$

$$\therefore \left(\frac{12}{36\pi} \cdot 100\right)\% = \frac{100}{3\pi}\% \approx 11\%$$

$$16 \quad V_{\text{hemi}} = \frac{1}{2}(\frac{4}{3}\pi r^3) = \frac{2}{3}\pi r^3$$

$$V_{\text{cone}} = \frac{1}{3}\pi r^2(r) = \frac{1}{3}\pi r^3$$

$$\frac{V_{\text{hemi}}}{V_{\text{cone}}} = \frac{\frac{2}{3}\pi r^3}{\frac{1}{3}\pi r^3} = \frac{2}{1}$$

17 The height of both prisms is the same, so by Cavalieri's principle, if $A_{\text{shell}} = A_{\text{cyl}}$, then $V_{\text{shell}} = V_{\text{cyl}}$.

$$A_{\text{shell}} = A_{\text{cyl}} - A_{\text{inner cyl}} \quad A_{\text{cyl}} = \pi(r\sqrt{3})^2$$

$$A_{\text{shell}} = \pi(2r)^2 - \pi r^2 \quad A_{\text{cyl}} = 3\pi r^2$$

$$A_{\text{shell}} = 3\pi r^2$$

$$18 \quad a \quad A_{\text{annulus}} = \pi R^2 - \pi d^2$$

The radius of the \odot is $\sqrt{R^2 - d^2}$ by Pythagorean Theorem.

$$A_{\text{circle}} = \pi(\sqrt{R^2 - d^2})^2$$

$$A_{\text{circle}} = \pi R^2 - \pi d^2$$

b By Cavalieri's principle the volume of the hemisphere is equal to the volume of the cylinder minus the volume of the cone.

$$V_{\text{hemi}} = V_{\text{cyl}} - V_{\text{cone}}$$

$$V_{\text{hemi}} = \pi R^2(R) - \frac{1}{3}\pi R^2(R)$$

$$V_{\text{hemi}} = \frac{2}{3}\pi R^3$$

Therefore, the volume of a sphere = $\frac{4}{3}\pi R^3$

Pages 594–597 Chapter 12 Review Problems

1. a Slant height is 4. b $LA = 2\pi(4)(7) = 56\pi$

$$LA = \frac{1}{2}(6)(4)(4) = 48$$

$$TA = LA + A_{\text{base}}$$

$$= 48 + 36$$

$$= 84$$

2. a	$V_{\text{cube}} = s^3$	d	$V_{\text{pyr}} = \frac{1}{3}Bh$
	$V_{\text{cube}} = 8^3$		$V_{\text{pyr}} = \frac{1}{3}(12)(5)$
	$V_{\text{cube}} = 512$		$V_{\text{pyr}} = 20$
b	$V_{\text{rect}} = Bh$	e	$V_{\text{prism}} = Bh$
	$V_{\text{rect}} = 24(4\frac{1}{2})$		$V_{\text{prism}} = 12(5)$
	$V_{\text{rect}} = 108$		$V_{\text{prism}} = 60$
c	$V_{\text{cyl}} = Bh$	f	$V_{\text{sphere}} = \frac{4}{3}\pi r^3$
	$V_{\text{cyl}} = 49\pi(2)$		$V_{\text{sphere}} = \frac{4}{3}\pi(8)$
	$V_{\text{cyl}} = 98\pi$		$V_{\text{sphere}} = \frac{32}{3}\pi$

3. a	$V_{\text{cyl}} = Bh$	TA _{cyl}	$LA + 2(A_{\text{base}})$
	$= \pi r^2 h$	TA	$Ch + 2(\pi r^2)$
	$= \pi(6)^2 10$		$= 2\pi rh + 2(\pi r^2)$
	$V_{\text{cyl}} = 360\pi$		$= 2\pi(6)(10) + 2\pi(6^2)$
			$= 120\pi + 2(36\pi)$
			$TA = 120\pi + 72\pi = 192\pi$

b The base of the prism is a rt Δ and $x = 12$ because 3-4-5 is a Pythagorean Triple.

$$V_{\text{prism}} = Bh$$

$$V_{\text{prism}} = \frac{1}{2}bh(h)$$

$$V_{\text{prism}} = \frac{1}{2}(9)(12)(10)$$

$$V_{\text{prism}} = 540$$

$$TA_{\text{prism}} = LA + 2(A_{\text{base}})$$

$$TA = bh + bh + bh + 2(\frac{1}{2}bh)$$

$$= 12(10) + 9(10) + 15(10) + 9(12)$$

$$TA = 120 + 90 + 150 + 108 = 468$$

c Radius = 5 because 5-12-13 is a Pythagorean Triple.

$V_{\text{cone}} = \frac{1}{3}Bh$	TA _{cone}	$LA + A_{\text{base}}$
$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$	TA _{cone}	$\frac{1}{2}Cl + \pi r^2$
$V_{\text{cone}} = \frac{1}{3}\pi(5)^2(12)$		$= \pi r\ell + \pi r^2$
$V_{\text{cone}} = \frac{1}{3}(25\pi)(12)$		$= \pi(5)13 + \pi(5)^2$
$V_{\text{cone}} = 100\pi$		$TA = 65\pi + 25\pi = 90\pi$

4 $V_{\text{rect}} = B \cdot h$

$$V_{\text{rect}} = (9)(5)2 = 90$$

5. a $V_{\text{rect box}} = \ell wh$

$$100 = 15(1\frac{1}{3})h$$

$$100 = 20h$$

$$5 = h$$

b $V_{\text{cube}} = x^3$

$$216 = x^3$$

$$6 = x$$

$$h = 6$$

6 By using the Pythagorean triple 8-15-17, the base diameter is 8.

$$V_{\text{cyl}} = B \cdot h$$

$$V_{\text{cyl}} = 16\pi(15) = 240\pi$$

7 Use the "Divide and Conquer" method.

$$V_{\text{rect box}} = \ell wh$$

$$V = (100)(25)(15)$$

$$V = 37,500 \text{ cu cm}$$

$$V_{\text{rect box}} = \ell wh$$

$$V = 100(25)(15 + 15)$$

$$V = 100(25)(30)$$

$$V = 75,000 \text{ cu cm}$$

$$V_{\text{rect box}} = \ell wh$$

$$V = (100)(100)(15 + 15 + 15)$$

$$V = 100(100)(45)$$

$$V = 450,000 \text{ cu cm}$$

$$V_{\text{concrete}} = 37,500 + 75,000 + 450,000 = 562,500 \text{ cu cm}$$

$$8 \text{ a } LA = \frac{1}{2}(10)(16)(4) = 320$$

$$\text{b } TA = LA + A_{\text{base}}$$

$$= 320 + 256 = 576$$

$$\text{c } V = \frac{1}{3}(256)(6) = 512$$

$$9 \quad TA_{\text{sphere}} = 4\pi r^2 \quad V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$36\pi = 4\pi r^2 \quad V_{\text{sphere}} = \frac{4}{3}\pi(3)^3$$

$$9 = r^2, r = 3 \quad V_{\text{sphere}} = \frac{4}{3}\pi(27) = 36\pi$$

10 a Slant height = 12

$$LA = \frac{1}{2}(10)(12)(3) = 180$$

$$A_{\text{base}} = \frac{1}{2}(10)(\sqrt{75})$$

$$= 25\sqrt{3}$$

$$TA = 180 + 25\sqrt{3}$$

$$\text{b } LA = \pi r \ell$$

$$= \pi(6)(10)$$

$$= 60\pi$$

$$TA = LA + A_{\text{base}}$$

$$= 60\pi + 36\pi$$

$$= 96\pi$$

$$\text{c } LA = 2\pi r h$$

$$= 2\pi(6.5)(15)$$

$$= 195\pi$$

$$TA = LA + A_{\text{base}}$$

$$= 195\pi + (2)(\pi)(6.5)^2$$

$$= 195\pi + 84\frac{1}{2}\pi$$

$$= 279\frac{1}{2}\pi$$

$$11 \text{ a } A = \frac{1}{2} \cdot 4\pi(5)^2 + \pi(5)^2$$

$$= 50\pi + 25\pi$$

$$= 75\pi$$

$$\text{b } A_{\text{half cone}} = \frac{1}{2}(\pi \cdot 5 \cdot 13 + \pi 5^2)$$

$$= \frac{1}{2}(65\pi + 25\pi)$$

$$= 45\pi$$

$$A\Delta = \frac{1}{2}(10)(12) = 60$$

$$TA = 45\pi + 60$$

$$12 V_{\text{cyl}} = Bh = 16\pi h$$

Since $A_{\text{base}} = \pi r^2$, $r = 4$ and diameter = 8, $C = \pi d = 8\pi$.

Therefore the length of the rectangle = 8π . The height must = 3, because $(3)(8\pi) = 24\pi$, the area of the rectangle.

$$V_{\text{cyl}} = 16\pi(3) = 48\pi$$

$$13 \quad A_{\text{rhombus}} = \frac{1}{2}(d_1 d_2) \quad V_{\text{pyramid}} = \frac{1}{3}Bh$$

$$A_{\text{rhombus}} = \frac{1}{2}(7)(6) \quad V_{\text{pyramid}} = \frac{1}{3}(21)(5)$$

$$A_{\text{rhombus}} = \frac{1}{2}(42) = 21 \quad V_{\text{pyramid}} = 35$$

$$14 \quad A_{\text{bases}} = \left(\frac{1}{2}ap\right)(2)$$

$$= \frac{1}{2}(6\sqrt{3})(72)(2)$$

$$= 432\sqrt{3}$$

$$LA = (12)(20)(6)$$

$$= 1440$$

$$TA = 1440 + 432\sqrt{3}$$

$$V = 216\sqrt{3}(20)$$

$$= 4320\sqrt{3} \text{ cu cm}$$

15 Draw the alt to the base of the isosceles Δ , bisecting it.

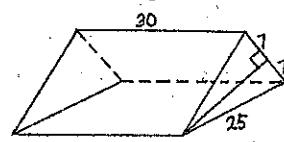
7-24-25 is a Pythagorean Triple, so the height = 24.

$$V_{\text{prism}} = Bh$$

$$V_{\text{prism}} = \frac{1}{2}(bh)h$$

$$V_{\text{prism}} = \left(\frac{1}{2}(14)(24)\right)(30)$$

$$V_{\text{prism}} = (168)(30) = 5040$$



$$16 V_{\text{castle rect}} = Bh = (50)(100)30 = 150,000$$

$$V_{\text{rectangular tower}} = Bh = (9)(50) = 450$$

$$V_{\text{pyramid on tower}} = \frac{1}{3}Bh = \frac{1}{3}(9)(3) = 9$$

$$V_{\text{cylinder tower}} = Bh = 9\pi(50) = 450\pi$$

$$V_{\text{hemisphere}} = \frac{1}{2}(\frac{4}{3}\pi r^3) = \frac{2}{3}\pi(27) = 18\pi$$

$$V_{\text{triangular tower}} = Bh = \frac{9}{4}\sqrt{3}(50) = \frac{450}{4}\sqrt{3}$$

$$V_{\text{pyramid on tower}} = \frac{1}{3}Bh = \frac{1}{3}(\frac{9}{4}\sqrt{3})(3) = \frac{9}{4}\sqrt{3}$$

$$V_{\text{cylinder tower}} = Bh = 9\pi(50) = 450\pi$$

$$V_{\text{cone on tower}} = \frac{1}{3}Bh = \frac{1}{3}(9\pi)3 = 9\pi$$

$$\text{Total Volume} = 150,459 + 927\pi + \frac{459}{4}\sqrt{3}$$

$$\begin{aligned}
 17 \quad V_{\text{rect}} &= Bh & V_{\text{cyl}} &= Bh \\
 V_{\text{rect}} &= 30(8) & V_{\text{cyl}} &= (1^2)\pi(8) \\
 V_{\text{rect}} &= 240 & V_{\text{cyl}} &= 8\pi \\
 & & V_{\text{cyl}} &= 8(3.14) \approx 25.12
 \end{aligned}$$

Remaining Volume = $240 - 25.12 \approx 215$

$$18 \quad V_{\text{prism}} = Bh = A_{\Delta}h \quad s = \frac{5+9+6}{2} = 10$$

$$V_{\text{prism}} = (\sqrt{s(s-a)(s-b)(s-c)})h$$

$$V_{\text{prism}} = (\sqrt{10(10-5)(10-9)(10-6)})h$$

$$V_{\text{prism}} = (\sqrt{200})h$$

$$V_{\text{prism}} = (10\sqrt{2})7 = 70\sqrt{2}$$

$$\begin{aligned}
 19 \quad A &= \frac{1}{4}(12\pi \cdot 10) + 2(6 \cdot 10) + 2(\frac{\pi}{4} \cdot 36) \\
 &= 120 + 48\pi
 \end{aligned}$$

20 To find B, find the area

of the shaded segment.

The inscribed Δ will be equilateral, so the radii will be 10 because all sides of an equilateral Δ are \equiv .



$$B = A_{\text{sector}} - A_{\text{eq}\Delta}$$

$$B = \left(\frac{m \text{ arc}}{360}\right)\pi r^2 - \frac{s^2}{4}\sqrt{3}$$

$$B = \left(\frac{60}{360}\right)\pi(10)^2 - \frac{10^2}{4}\sqrt{3}$$

$$B = \frac{1}{6}(100\pi) - 25\sqrt{3}$$

$$B = \frac{50}{3}\pi - 25\sqrt{3}$$

$$TA = 2(B) + A_{\text{rect}} + \frac{m \text{ arc}}{360}(\text{LA}_{\text{cyl}})$$

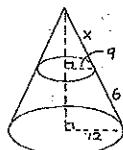
$$TA = 2\left(\frac{50}{3}\pi - 25\sqrt{3}\right) + bh + \frac{60}{360}Ch$$

$$TA = \frac{100}{3}\pi - 50\sqrt{3} + 10(30) + \left(\frac{1}{6}\right)2\pi(10)(30)$$

$$TA = \frac{100}{3}\pi - 50\sqrt{3} + 300 + 100\pi$$

$$TA = 300 + \frac{400}{3}\pi - 50\sqrt{3}$$

$$\begin{aligned}
 21 \quad \frac{x}{x+6} &= \frac{9}{12} \\
 12x &= 9x + 54 \\
 3x &= 54 \\
 x &= 18
 \end{aligned}$$



Using the Pythagorean Theorem, the height of the small cone = $9\sqrt{3}$ the height of the large cone = $12\sqrt{3}$

$$V_{\text{frustum}} = V_{\text{lg cone}} - V_{\text{sm cone}}$$

$$V_{\text{lg cone}} = \frac{1}{3}Bh = \frac{1}{3}(144\pi)(12\sqrt{3}) = 576\pi\sqrt{3}$$

$$V_{\text{sm cone}} = \frac{1}{3}Bh = \frac{1}{3}(81\pi)(9\sqrt{3}) = 243\pi\sqrt{3}$$

$$V_{\text{frustum}} = 576\pi\sqrt{3} - 243\pi\sqrt{3} = 333\pi\sqrt{3}$$

$$22 \quad \text{a The } \Delta \text{ is a } 30^\circ 60^\circ 90^\circ \Delta, \text{ so the radius of cone} = \frac{1}{2}(8) = 4$$

and the height = $4\sqrt{3}$.

$$V_{\text{cone}} = \frac{1}{3}Bh$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(4)^2(4\sqrt{3})$$

$$V = \frac{1}{3}(16\pi)4\sqrt{3} = \frac{64\pi\sqrt{3}}{3}$$

$$\text{b Radius small cyl} = 3$$

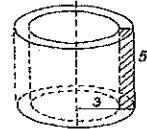
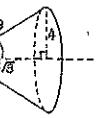
$$\text{radius large cyl} = 3 + 1 = 4$$

$$V = Bh$$

$$V = (\pi r^2 - \pi r^2)5$$

$$V = (\pi(4)^2 - \pi(3)^2)5$$

$$V = (16\pi - 9\pi)5 = 35\pi$$



Pages 598–603 Chapters 1–12 Cumulative Review

$$1 \quad 9x + x = 90$$

$$10x = 90, x = 9$$

The measure of the larger acute \angle is $9(9) = 81^\circ$.

$$2 \quad 2x + 3 + 4x - 5 + 8x - 19 = 28$$

$$14x - 21 = 28$$

$$14x = 49, x = \frac{7}{2}$$

$$AB = 2x + 3$$

$$BC = 4x - 5 \quad CA = 8x - 19$$

$$AB = 2\left(\frac{7}{2}\right) + 3 \quad BC = 4\left(\frac{7}{2}\right) - 5 \quad CA = 8\left(\frac{7}{2}\right) - 19$$

$$AB = \frac{14}{2} + 3 \quad BC = \frac{28}{2} - 5 \quad CA = \frac{56}{2} - 19$$

$$AB = 10 \quad BC = 9 \quad CA = 9$$

$\triangle ABC$ is isosceles.