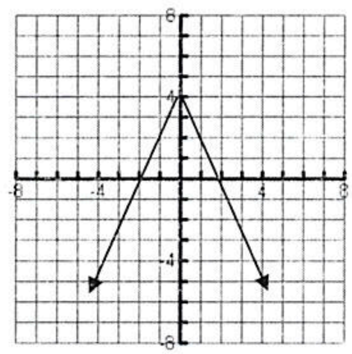
Name: Solutions Period:_______

Honors Advanced Algebra: 1st Semester Review

Target 1.A – Graph, transform, and identify key features of absolute value functions.

1. What is the domain of the graphed absolute value function?



Domain: $(-\infty, \infty)$

Range: $(-\infty, 4)$

2. Which is the vertex for the equation $y = -2|x-7| + 2$?

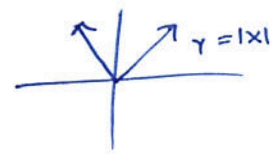
$f(x) = a|x-h| + k$

$(7, 2)$

3. Write the equation whose graph is the function $f(x) = |x|$ reflected about the x-axis, translated 2 units left, vertically stretched by a factor of 3, and translated down 6 units?

→ negative

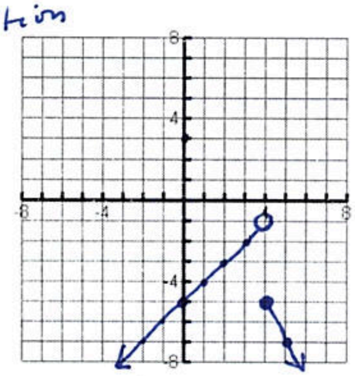
$f(x) = -3|x+2| - 6$



Target 1.B – Graph piecewise and step functions.

4. Graph this function: $f(x) = \begin{cases} x-5 & if\ x < 4 \\ -2x+3 & if\ x \geq 4 \end{cases}$

open ○ — condition — closed ●



$y = x - 5$ slope $\frac{1\uparrow}{1\rightarrow}$ y-int: -5

$y = -2x + 3$ slope $\frac{-2\downarrow}{1\rightarrow}$ y-int: 3

5. Evaluate $f(0)$ for the function: $f(x) = \begin{cases} \frac{1}{3}x - 5 \text{ if } x < 4 \\ -2x + 3 \text{ if } x \geq 4 \end{cases}$

$0 < 4$, so evaluate $f(0)$ @ $\frac{1}{3}x - 5$

$f(0) = \frac{1}{3}(0) - 5 = 0 - 5 = -5$

Solve for any variable 1st $(2, 3, 5)$

Target 1.C – Solve linear systems in 3 variables

Equation Number:

6. Solve this system of linear equations $\begin{cases} ① \; 2x + 3y - z = -10 \\ ② \; -x - 2y + 2z = 14 \\ ③ \; 3x + y + z = 8 \end{cases}$

① $2x + 3y - z = -10$

② $2(-x - 2y + 2z = 14)$ $\rightarrow$

$$\begin{array}{r} 2x + 3y - z = -10 \\ -2x - 4y + 4z = 28 \\ \hline ④ \; -y + 3z = 18 \end{array}$$

② $3(-x - 2y + 2z = 14)$

③ $3x + y + z = 8$ $\rightarrow$

$$\begin{array}{r} -3x - 6y + 6z = 42 \\ 3x + y + z = 8 \\ \hline ⑤ \; -5y + 7z = 50 \end{array}$$

Combine eq ④ and ⑤

④ $-5(-y + 3z = 18)$

⑤ $-5y + 7z = 50$ $\rightarrow$

$$\begin{array}{r} 5y - 15z = -90 \\ -5y + 7z = 50 \\ \hline -8z = -40 \end{array}$$

$\boxed{z = 5}$

Since $z = 5$, substitute into eq ④ or ⑤ to find y.

$-y + 3(5) = 18 \Rightarrow -y + 15 = 18 \Rightarrow -y = 3 \Rightarrow \boxed{y = -3}$

Find x:

$3x + y + z = 8$

$3x + (-3) + (5) = 8$

$3x + 2 = 8$

$\Rightarrow \boxed{x = 2}$

7. Find three numbers that satisfy these requirements: The sum of the first number and two times the second number minus the third number is 13. The sum of three times the first number and the second number and three times the third number is -7. The sum of four times the first number, three times the second number, and the third number is 17.

Let x be the 1st #

Let y be the 2nd #

Let z be the 3rd #

① $x + 2y - z = 13$

② $3x + y + 3z = -7$

③ $4x + 3y + z = 17$

Solve by hand.

Target 2.A - Perform arithmetic operations, including adding, subtracting, and multiplying with complex numbers.

8. Perform the indicated operations: $(5-2i)-(3+6i)$

Distribute 1st

$5 - 2i - 3 - 6i$

$5 - 3 = -2$ $\qquad -2i - 6i = -8i$

$\boxed{-2 - 8i}$

9. Perform the indicated operations: $(5 + 2i)(2 - 3i)$.

multiply

Remember: $i^2 = -1$

	5	$+2i$
2	10	$4i$
$-3i$	$-15i$	$-6i^2$ $-6(-1)$ 6

$10 + 6 + 4i - 15i$

$\boxed{16 - 11i}$

Target 2.B - Graph, transform and identify the key features of a polynomial function and make connections between algebraic and graphical forms.

10. Identify the zeros and local min and max of the graph of this polynomial: $h(x) = x^3 - 7x^2 - 6x + 72$

Use Nspire to Graph and Analyze key features

Zeros: $x = -3$, $x = 4$, $x = 6$

Max: $(-0.40, 73.22)$

Min: $(5.06, -8.03)$

→ Assume no multiplicities

Target 2.C - Create polynomials functions given factors and zeros.

11. Create an equation, in standard form, that has the zeros of 2, -4, and 5.

Factored form: $y = (x-2)(x+4)(x-5)$

	x	$+4$
x	x^2	$4x$
-2	$-2x$	-8

x^2+2x-8

	x^2	$2x$	-8
x	x^3	$2x^2$	$-8x$
-5	$-5x^2$	$-10x$	$+40$

$\boxed{\text{Standard form: } y = x^3-3x^2-18x+40}$

12. Create the cubic polynomial, in standard form, that has the zeros of 4 and $-i$.

Zeros: $4, -i, i$ → recall complex conjugate

Factors: $(x-4)$, $\underbrace{(x+i), (x-i)}_{\text{multiply 1st}}$

	x	$+i$
x	x^2	ix
$-i$	$-ix$	$-i^2$ $-(-1)$ $+1$

x^2+1

	x^2	$+1$
x	x^3	x
-4	$-4x^2$	-4

$\boxed{\text{Standard form: } f(x) = x^3-4x^2+x-4}$

Target 2.D - Apply the Remainder Theorem to determine the factors and zeros of a polynomial function.

13. Write the polynomial $f(x) = x^3-2x^2-9x+18$ in factored form if $x=2$ is a zero.
$(x-2)$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -9 & 18 \\ & \downarrow & 2 & 0 & -18 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$x^2-9 = (x-3)(x+3)$

Polynomial in factored form:

$\boxed{f(x) = (x-2)(x+3)(x-3)}$

14. Divide using synthetic division: $(x^3-4x^2+8x-8) \div (x-2)$.

$$\frac{x^3-4x^2+8x-8}{x-2} = \boxed{x^2-2x+4}$$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 8 & -8 \\ & \downarrow & 2 & -4 & 8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

x^2-2x+4

Target 2.E - Solve polynomials algebraically and graphically by using technology.

15. Solve the polynomial equation.

$$x^3+3x^2 = 40x$$
$$-40x \quad -40x$$

$x^3+3x^2-40x=0$

$x(x^2+3x-40)=0$

$\boxed{x=0}$ or $x^2+3x-40=0$

$(x+8)(x-5)=0$

$x+8=0$ or $x-5=0$

$\boxed{x=-8}$ or $\boxed{x=5}$

16. Solve the polynomial equation.

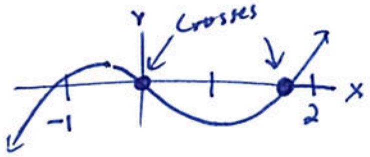
$$4x^3-12x^2-37x-15=0$$

① Graph using Nspire

② Analyze zeros: $x=-\frac{1}{2}$, $x=-\frac{3}{2}$, $x=5$

Target 2.F - Analyze a polynomial function in multiple representations (equation, table or graph) within a context and make conclusions on the features.

17. A twist in a river can be modeled by the function $f(x)=\frac{1}{2}x^3-\frac{1}{4}x^2-x$. A city wants to build a road that goes directly along the x-axis and that reflects a domain of $-1\le x\le 3$. How many bridges would it have to build for each time the road crosses the river?



On domain interval $[-1, 3]$ the road crosses the river @ 2 real solutions, so build 2 bridges

Domain : $[-1, 3]$

18. A roller coaster is being created and reflects the polynomial $f(x)=(2x-3)(2x+5)(x+4)(x-9)$. The boarding area will be placed at one of the zeros. If the boarding area can only be placed in the interval of $-10\le x\le 10$, what are the possible locations of the loading zones?

$2x-3=0$ $2x+5=0$ $x+4=0$ $x-9=0$

$2x=3$ $2x=-5$ $\boxed{x=-4}$ $\boxed{x=9}$

Zeros: $\boxed{x=3/2}$ $\boxed{x=-5/2}$

Interval $[-10, 10]$ has 4 zeros, so 4 possible locations

Target 3.A - Perform operations with rational expressions to demonstrate the analogy with integers.

19. Perform the given operations.

$$\frac{3}{8x}+\frac{5}{4x^2} = \frac{3}{2\cdot2\cdot2\cdot x}\cdot\frac{x}{x}+\frac{5}{2\cdot2\cdot x\cdot x}\cdot\frac{2}{2} = \frac{3x}{8x^2}+\frac{10}{8x^2} = \boxed{\frac{3x+10}{8x^2}}$$

20. Perform the given operations.

$$\frac{3x+2}{5x-20}\cdot(x^2-11x+28) = \frac{3x+2}{5\cancel{(x-4)}}\cdot\frac{\cancel{(x-4)}(x-7)}{1}$$

$$= \boxed{\frac{(3x+2)(x-7)}{5}}$$

Target 3.B - Graph, transform and identify the key features of the graph of a rational function.

21. Determine (if any) vertical asymptote, horizontal asymptote, and hole of:

$$f(x)=\frac{1x-3}{1x+2}$$

V.A. @ $x+2=0 \Rightarrow \boxed{x=-2}$ (Non-removable discontinuity, Infinite discontinuity)

H.A. @ $\boxed{y=\frac{1}{1}=1}$

$\boxed{\text{No Holes}}$ (No removable discontinuities)

22. Determine (if any) vertical asymptote, horizontal asymptote, and hole of:

$$g(x)=\frac{(2x+7)\cancel{(x-5)}}{\cancel{(x-5)}(x-6)} = \frac{2x+7}{x-6}$$

$x-5=0 \Rightarrow \boxed{x=5 \quad \text{Hole}}$

V.A. @ $x-6=0 \Rightarrow \boxed{x=6}$

H.A. @ $\boxed{y=\frac{2}{1}=2}$

Target 3.C - Solve rational equations and identify, if any, extraneous solutions.

23. Solve the equation.

$$\frac{7}{x+4}=\frac{x-2}{x+4}$$

multiply by LCD: $(x+4)$ to both sides

$$\frac{7}{\cancel{x+4}}\cdot\cancel{(x+4)} = \frac{x-2}{\cancel{x+4}}\cdot\cancel{(x+4)} \Longrightarrow 7=x-2 \Rightarrow \boxed{x=9}$$

24. Solve the equation.

$$\frac{-4x-42}{x^2+3x-18}=\frac{2}{x+6}+\frac{x}{x-3}$$

LCD: $(x+6)(x-3)$

$$\frac{-4x-42}{(x+6)(x-3)} = \frac{2}{x+6} + \frac{x}{x-3}$$

$$\frac{-4x-42}{\cancel{(x+6)(x-3)}}\cdot\cancel{(x+6)(x-3)} = \frac{2}{\cancel{x+6}}\cancel{(x+6)}(x-3) + \frac{x}{\cancel{x-3}}(x+6)\cancel{(x-3)}$$

$\Rightarrow -4x-42=2(x-3)+x(x+6)$

$-4x-42=2x-6+x^2+6x$

$-4x-42=x^2+8x-6$

$+4x+42 \quad\quad +4x+42$

$0=x^2+12x+36$

$0=(x+6)(x+6)$

$0=x+6$ or $0=x+6$

$\therefore x=-6$ (but extran)

CHECK: $\ldots=\frac{2}{-6+6}+\ldots$

0

☹

$\boxed{\text{NO SOLUTION}}$

Target 3.D - Solve systems involving rational equations and identify, if any, extraneous solutions.

25. Solve the following systems of equations.

$$y=\frac{27}{x}$$
$$y=3x$$

LCD: x

$\frac{27}{x}=3x$

$\frac{27}{\cancel{x}}\cdot\cancel{x}=3x\cdot x$

$\frac{27}{3}=\frac{3x^2}{3}$

$9=x^2$

$\boxed{\pm 3=x}$

$x=3 \Rightarrow y=3(3)$, $y=9$

$\boxed{\therefore (3,9)}$

$x=-3 \Rightarrow y=3(-3)$, $y=-9$

$\boxed{\therefore (-3,-9)}$

Target 4.A - Use properties of integer exponents and apply those to rational exponents.

26. Rewrite the following expressions using properties of exponents.

$x^{\frac{3}{8}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{2}}$ Add exponents : need common denominator

$$x^{\frac{3}{8} + \frac{1}{4}\cdot\frac{2}{2} + \frac{1}{2}\frac{4}{4}} = x^{\frac{3}{8}+\frac{2}{8}+\frac{4}{8}} = \boxed{x^{\frac{9}{8}}}$$

27. Simplify.

$$\left(49x^{12}\right)^{\frac{1}{2}} = \left(7^2 x^{12}\right)^{\frac{1}{2}} = 7^{2\cdot\frac{1}{2}} \cdot x^{12\cdot\frac{1}{2}} = \boxed{7x^6}$$

Target 4.B - Convert between exponential and radical forms.

28. Rewrite the following expression in radical form.

$$3^{\frac{5}{7}} = \boxed{\sqrt[7]{3^5}} \text{ or } \boxed{\sqrt[7]{243}}$$

29. Rewrite the following expression so that it no longer has a radical.

$$\left(\sqrt[4]{2x}\right)^7 = \left((2x)^{\frac{1}{4}}\right)^7 = \boxed{(2x)^{\frac{7}{4}}}$$

Target 4.C - Make decisions about the results of adding and multiplying combinations of rational and irrational numbers.

30. Evaluate the expression and identify the answer as a rational, irrational, or complex term.

$\sqrt{8}\cdot\sqrt{2}$

$= \sqrt{16}$

$= \boxed{4}$ $\boxed{\text{Rational}}$

Target 4.C - Make decisions about the results of adding and multiplying combinations of rational and irrational numbers.

31. Evaluate the expression and describe the solution as rational, irrational, or complex.

$2\sqrt{5} - 7\sqrt{5}$

Solution: $-5\sqrt{5}$

Type of solution: Irrational

Target 4.D - Solve an equation involving radicals or rational exponents and identify, if any extraneous solutions.

32. Solve the following equation.

$-7+\sqrt{2x-1}=-4$

$+7 \qquad\qquad +7$

$\left(\sqrt{2x-1}\right)^2 = 3^2$ Square both sides

$2x-1=9$

$2x=10$

$\boxed{x=5}$

CHECK: $-7+\sqrt{2(5)-1}=-4$

$-7+\sqrt{9}=-4$

$-7+3=-4$

$-4=-4$ ✓

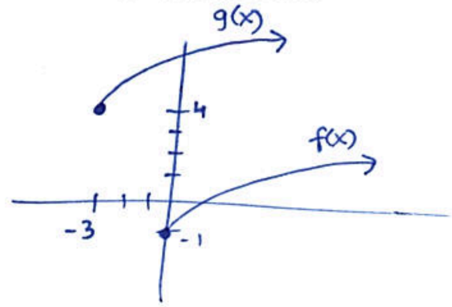
Target 4.E - Graph, transform and identify the key features of the square root and cube root functions.

33. Describe the transformation from $f(x)$ to $g(x)$.

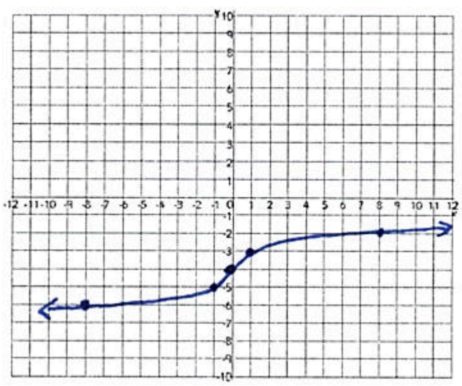
$f(x)=\sqrt{x}-1$

$g(x)=\sqrt{x+3}+4$

Translate 3 units left and 5 units up.



34. Graph the function $h(x)=\sqrt[3]{x}-4$.



Domain:
$(-\infty,\infty)$

Range:
$(-\infty,\infty)$

x	y
-8	-6
-1	-5
0	-4
1	3
8	-2

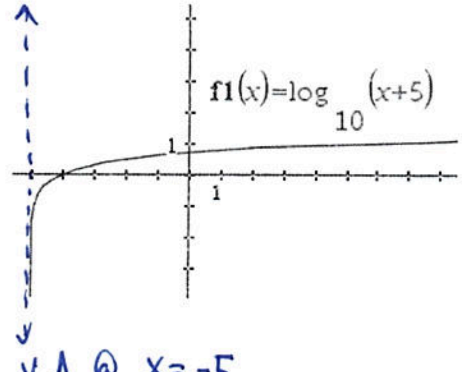
Target 5.A - Graph, transform and identify the key features of the graph of an exponential and logarithmic function.

35. Identify the y-intercept of $f(x)=5^x-3$

Y - Intercept @ $(0,-2)$

$f(0)=5^0-3=1-3=-2$

36. Identify the domain and range of the function $h(x)=\log(x+5)$



V.A. @ $x=-5$

∴ Domain: $(-5,\infty)$

∴ Range: $(-\infty,\infty)$

Target 5.B - Explain why exponential and logarithmic equations can be written in equivalent forms.

37. Write the equivalent form of $a = b^4$ using the properties of logarithms.

$\boxed{\log_b a = 4}$ "Canoe"

38. Write the equivalent form of $m = \ln(5)$ using the properties of logarithms.

$m = \log_e 5$ $\boxed{e^m = 5}$

Target 5.C - Evaluate logarithms using change of base formula with technology.

39. Evaluate the logarithm. $\log_3(243) = r$

$3^r = 243 \Rightarrow 3^r = 3^5$ $\boxed{r = 5}$

40. Evaluate the logarithm. $\log_2(100)$

Use calculator: $\boxed{\log_2 100 \approx 6.64}$

Target 5.D - Solve exponential equations in base 2, 10 and e by using the properties of logarithms.

41. Solve the equation.

$\log(14x + 20) = 3$

method I

$\cancel{10}^{\cancel{\log_{10}} 14x+20} = 10^3$

$14x+20 = 1000$
$-20 \quad -20$
$14x = 980$
$\frac{14x}{14} = \frac{980}{14}$
$\boxed{x = 70}$

method II "Canoe"

$\log_{10} 14x+20 = 3$

$10^3 = 14x+20$
$1000 = 14x+20$
$980 = 14x$
$\boxed{70 = x}$

42. Solve the equation.

$2^{2x-5} = 30$

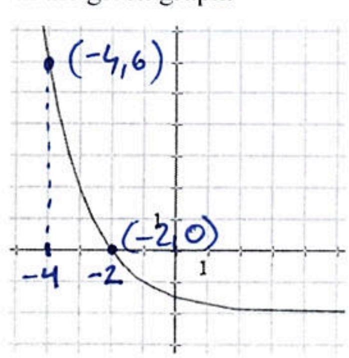
$\cancel{\log_2 2}^{2x-5} = \log_2 30$

$2x-5 = \log_2 30 \Rightarrow 2x = \log_2 30 + 5 \Rightarrow \boxed{x = \frac{\log_2 30 + 5}{2} \approx 4.95}$

Target 5.E - Calculate and interpret the average rate of change of a function as the function relates to a real world situation.

43. Calculate the average rate of change from $x = -4$ to $x = -2$ in the given graph.

$\frac{\Delta y}{\Delta x}$ (slope)



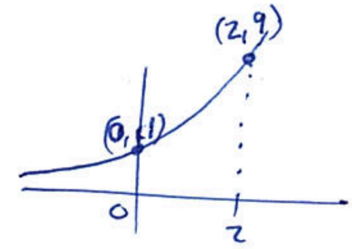
Avg rate of change $= \frac{\Delta y}{\Delta x}$

$x_1 \; y_1$
$(-4, 6)$
$x_2 \; y_2$
$(-2, 0)$

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0-6}{-2-(-4)} = \frac{-6}{-2+4} = \frac{-6}{2} = \boxed{-3}$

44. Calculate the average rate of change of the interval $\underset{}{\overset{x\text{-interval}}{[0, 2]}}$ of the function $y = 3^x$.



$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-1}{2-0} = \frac{8}{2} = \boxed{4}$$

when $x=0$: $y = 3^0 = 1 \Rightarrow (0, 1)$

when $x=2$: $y = 3^2 = 9 \Rightarrow (2, 9)$

Target 5.F - Solve problems using the formula for the sum of a finite geometric series.

Use the geometric series formula to answer the questions.

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

45. A soldier who rescues her King in battle requests a single gold coin as a reward for the first day, three gold coins on the second day then nine on the third day, then twenty-seven and so on for ten days total. How many gold coins in all did the soldier earn throughout the ten days?

1st Day — 2nd Day — 3rd Day — 4th Day

$a_1 = 1$ 3   9   27   ...

$\frac{3}{1} = 3$ $\frac{9}{3} = 3$ $\frac{27}{9} = 3$

Common ratio $= r = 3$; $a_1 = 1$; $n = 10$ days

$$S_{10} = \frac{1(1-3^{10})}{1-3} = \frac{-59048}{-2} = \boxed{29,524}$$

46. Find the sum of the first nine terms in the geometric series 2, 4, 8,....

$r = 2 = \frac{4}{2} = \frac{8}{4}$

$a_1 = 2$

$n = 9$

$$S_9 = \frac{2(1-2^9)}{1-2} = \frac{2(1-512)}{1-2} = \frac{2(-511)}{-1} = \boxed{1022}$$