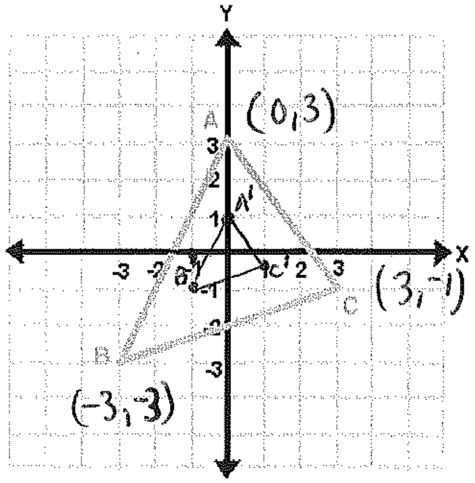
Honors Geometry Name: Key

Semester 2 Final Exam Review

Target 6A. Perform dilation with a given center and scale factor on a figure in the coordinate plane.

1) What are the coordinates of the image under dilation with center at the origin of scale factor 1/3?

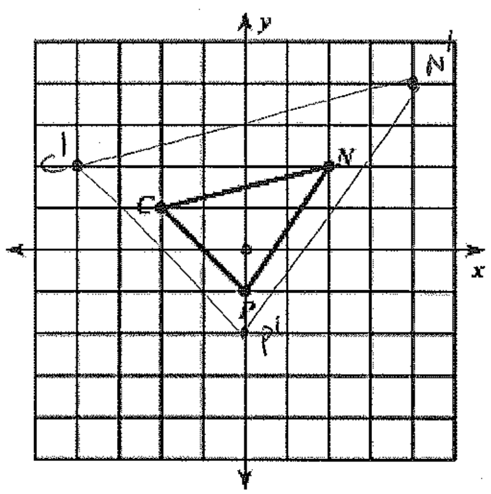
Reduction



$\underbrace{\begin{matrix} A(0,3) \\ B(-3,-3) \\ C(3,-1) \end{matrix}}_{\text{Pre-image coordinates}} \begin{matrix} \xrightarrow{\times \frac{1}{3}} \\ \xrightarrow{\times \frac{1}{3}} \\ \xrightarrow{\times \frac{1}{3}} \end{matrix} \underbrace{\begin{matrix} A'(0,1) \\ B'(-1,-1) \\ C'(1,-\frac{1}{3}) \end{matrix}}_{\text{Image coordinates}}$

2) What are the coordinates of the image under dilation with center at the origin of scale factor 2?

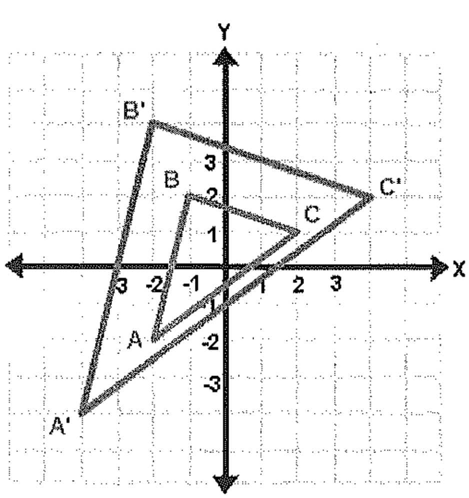
Enlargement



$\underbrace{\begin{matrix} C(-2,1) \\ N(2,2) \\ P(0,-1) \end{matrix}}_{\text{Pre-image coordinates}} \begin{matrix} \xrightarrow{\times 2} \\ \xrightarrow{\times 2} \\ \xrightarrow{\times 2} \end{matrix} \underbrace{\begin{matrix} C'(-4,2) \\ N'(4,4) \\ P'(0,-2) \end{matrix}}_{\text{Image coordinates}}$

Target 6B. Verify the characteristics of dilations.

3) What is the scale factor of the dilation (with center at the origin) shown below?



$A(-2,-2) \xrightarrow{\times 2} A'(-4,-4)$

$B(-1,2) \xrightarrow{\times 2} B'(-2,4)$

$C(2,1) \xrightarrow{\times 2} C'(4,2)$

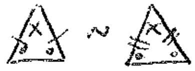
Enlargement

$\therefore$ Scale factor 2

Target 6C. Define similarity using rigid motions and dilations

For questions 4 – 7 answer Always, Sometimes, or Never.

4) If the vertex angles of two isosceles triangles are congruent, then the triangles are similar.



Always

5) If two triangles are congruent, then they are similar.

Always

6) If two triangles are similar, then they are congruent.

Sometimes

7) If three angles of one triangle are congruent to three angles of another triangle, then the triangles are similar.

Always

Target 6D. Identify corresponding sides and corresponding angles of similar figures by demonstrating that in a pair of similar triangles, corresponding angles are congruent.

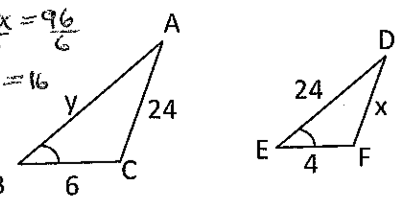
Use the diagram below to answer question 8 and 9.

8) Given ΔABC ~ ΔDEF, find the value of x and y.

$\frac{6}{4} \times \frac{24}{x} \Rightarrow \frac{6x}{6} = \frac{96}{6} \Rightarrow x = 16$

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \Rightarrow \frac{y}{24} = \frac{6}{4} = \frac{24}{x}$

$\frac{y}{24} \times \frac{6}{4} \Rightarrow \frac{4y}{4} = \frac{144}{4} \Rightarrow y = 36$

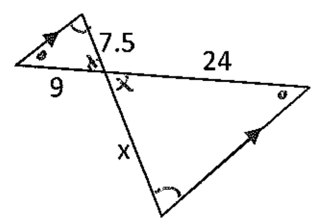


9) What is ratio, in simplest terms, of corresponding sides from the smaller triangle to the bigger triangle?

$\frac{EF}{BC} = \frac{\text{Smaller}}{\text{Bigger}} = \frac{4}{6} = \frac{2}{3}$

10) Using the diagram on the right, find the value of x.

$\frac{7.5}{x} \times \frac{9}{24} \Rightarrow \frac{9x}{9} = \frac{180}{9} \Rightarrow x = 20$



11) ΔABC ~ ΔDEF. Find the values of m and n.

∠B ≅ ∠E

$5n + 50 = 115$

$-50 \quad -50$

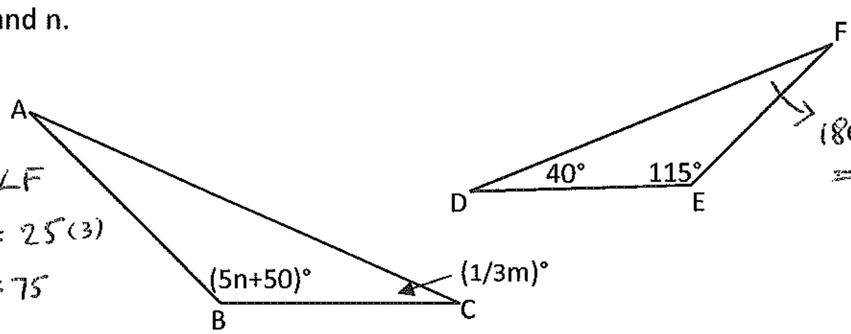
$\frac{5n}{5} = \frac{65}{5}$

$n = 13$

∠C ≅ ∠F

$(3)\frac{1}{3}m = 25(3)$

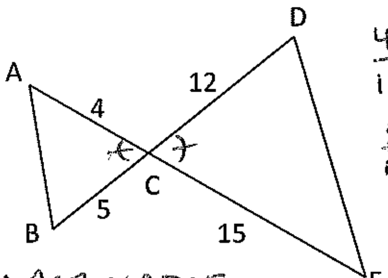
$m = 75$



$180 - 40 - 115 = 25$

Target 6E. Determine that two figures are similar by verifying that angle measures are preserved and corresponding sides are proportional.

For problems 12 - 15, determine if the triangles are similar state the reason why. Then, write the similarity statement.

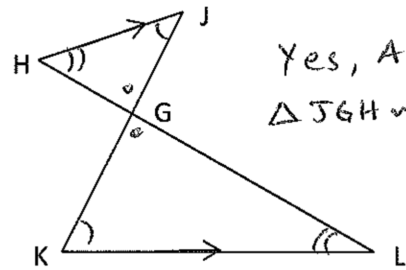
12) 

$\frac{4}{12} = \frac{1}{3}$

$\frac{5}{15} = \frac{1}{3}$

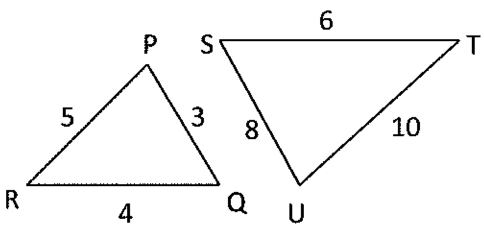
Yes, SAS ~

$\triangle ACB \sim \triangle DCE$

13) 

Yes, AA ~

$\triangle JGH \sim \triangle KGL$

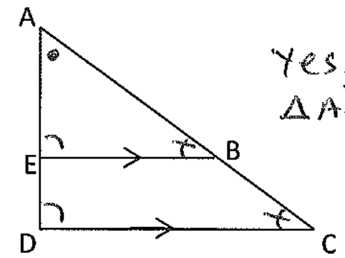
14) 

$\frac{3}{6} = \frac{1}{2}$

$\frac{4}{8} = \frac{1}{2}$ Yes, SSS ~

$\frac{5}{10} = \frac{1}{2}$

$\triangle PQR \sim \triangle TSU$

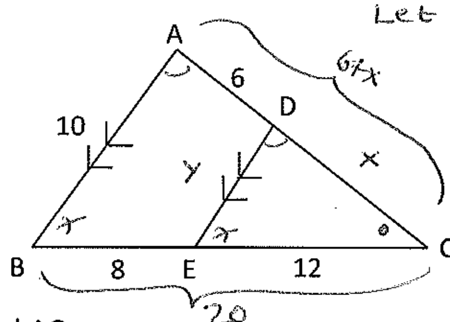
15) 

Yes, AA ~

$\triangle AEB \sim \triangle ADC$

Target 6G. Use theorems, postulates, or definitions to prove theorems about triangles.

Use the diagram below to answer question 15.



Let DC = x and DE = y

16) Find DC and DE.

$\triangle CDE \sim \triangle CAB \Rightarrow \frac{CD}{CA} = \frac{DE}{AB} = \frac{CE}{CB}$

$\Rightarrow \frac{x}{6+x} = \frac{y}{10} = \frac{12}{20}$

$\frac{y}{10} \times \frac{12}{20}$

$20y = 120$

$\frac{20y}{20} = \frac{120}{20}$

$y = 6$

METHOD 1:

$\frac{x}{(6+x)} \times \frac{12}{20}$

$20x = 12(6+x)$

$20x = 72 + 12x$

$-12x \quad -12x$

$8x = 72$

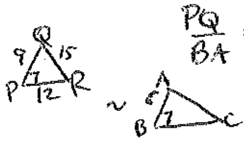
$\frac{8x}{8} = \frac{72}{8}$

$x = 9$

METHOD 2 to find x – SIDE SPLITTER

$\frac{x}{6} \times \frac{12}{8} \Rightarrow 8x = 72 \Rightarrow x = 9$

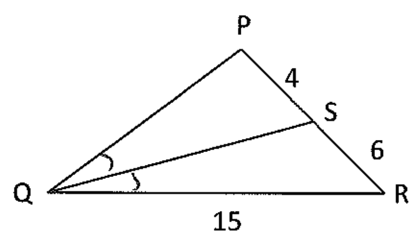
17) Given ΔPQR ~ ΔBAC, AB = 6, PR = 12, PQ = 9, and QR = 15, find AC.



$\frac{PQ}{BA} = \frac{QR}{AC} = \frac{PR}{BC} \Rightarrow \frac{9}{6} = \frac{15}{AC} = \frac{12}{BC}$

$\frac{9}{6} \times \frac{15}{AC} \Rightarrow \frac{9AC}{9} = \frac{90}{9} \Rightarrow AC = 10$

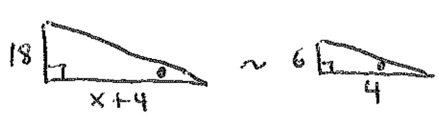
18) Given $\overrightarrow{QS}$ bisects $\angle PQR$, find QP.

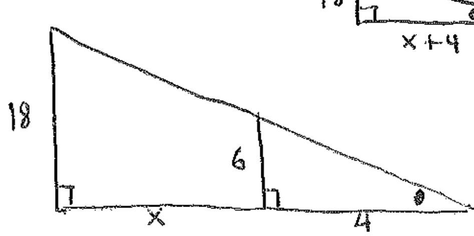


By side-splitter thm, we have

$\frac{15}{QP} \times \frac{6}{4} \Rightarrow 6 \cdot QP = 60 \Rightarrow QP = 10$

19) The light from an 18 foot street light creates a 4 foot shadow in front of a 6 foot sign. How far is the sign from the lamppost? Draw a picture.





$\frac{18}{6} = \frac{(x+4)}{4}$

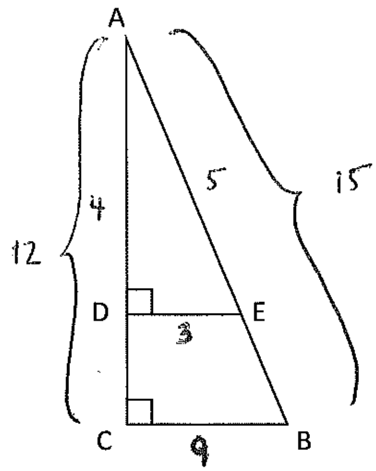
$\frac{3}{1} \times \frac{(x+4)}{4}$

$12 = x + 4$

$8 = x$

(3, 4, 5) (5, 12, 13) (8, 15, 17) (7, 24, 25) (9, 40, 41) → Know these!

Use the triangle on the right to answer questions 20 and 21. Decide whether >, <, =, or inconclusive gives a correct statement comparing the quantity on the left to the quantity on the right. Figures are drawn to scale.



20) Sin B $\boxed{>}$ Sin A

$\frac{4}{5}$ $\frac{3}{5}$

$\sin B = \frac{opp}{hyp} = \frac{12}{15} = \frac{4}{5}$

$\sin A = \frac{opp}{hyp} = \frac{9}{15} = \frac{3}{5}$

21) Tan B $\boxed{>}$ Cos A

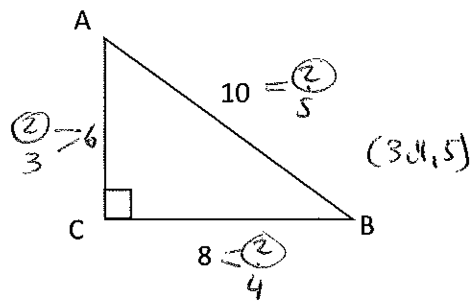
$\frac{4}{3}$ $\frac{4}{5}$

$\tan B = \frac{opp}{adj} = \frac{12}{9} = \frac{4}{3}$

$\cos A = \frac{adj}{hyp} = \frac{12}{15} = \frac{4}{5}$

22) Write the trigonometric ratio of cos A.

$\cos A = \frac{6}{10} = \frac{3}{5}$

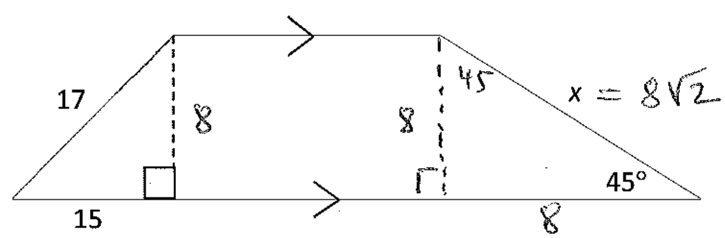


23) Write the trigonometric ratio of sin B.

$\sin B = \frac{6}{10} = \frac{3}{5}$

24) Find the exact value of *x*.

(8, 15, 17)

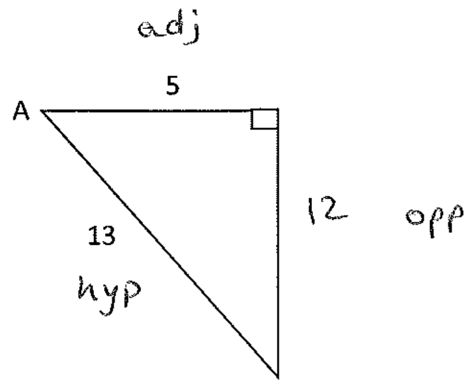


25) From the given triangle, find the m∠A rounded to the nearest tenth.

$\cos \angle A = \frac{5}{13} \Rightarrow \angle A = \cos^{-1}\left(\frac{5}{13}\right)$

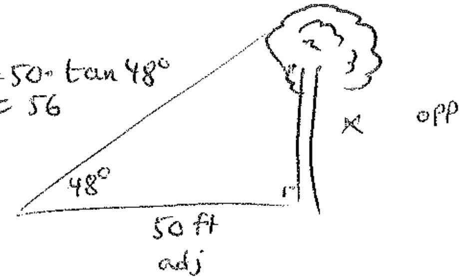
$\angle A = 67.4°$

Can we use sin or tan here? Why?



26) At a point on the ground 50 feet from the foot of a tree, the angle of elevation to the top of a tree is 48°. Find the height of the tree.

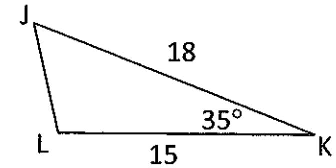
$\tan 48° = \frac{x}{50} \Rightarrow x = 50 \cdot \tan 48°$
≈ 56



The height of the tree is ≈ 56 ft.

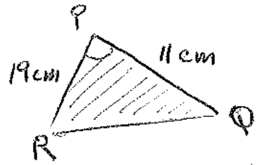
Target 7G: Calculate the area of a triangle using the formula $A = \frac{1}{2}ab \cdot sinC$ using any angle of the triangle.

27) Find the area of triangle JKL to the nearest tenth.



$A = \frac{1}{2}(18)(15)\sin 35° \approx 77.4$ units^2

28) The area of ΔPQR is 102 cm^2. If PQ = 11cm and PR = 19 cm, find the measure of ∠P to the nearest tenth.



$A = \frac{1}{2} PQ \cdot PR \cdot \sin \angle P$

$102 = \frac{1}{2}(11)\cdot(19)\cdot \sin \angle P$

$102 = \frac{1}{2}(209)\sin \angle P$

$204 = 209 \sin \angle P$

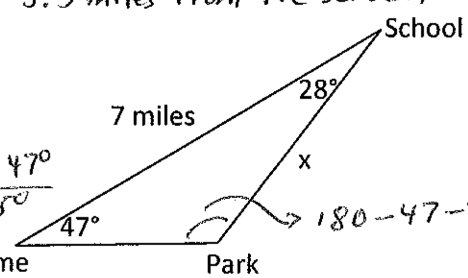
$\frac{204}{209} = \sin \angle P$

$\Rightarrow \angle P = \sin^{-1}\left(\frac{204}{209}\right)$

$\angle P = 77.4°$

Target 7H: Derive the Law of Sines and Law of Cosines and use them to solve real world problems.

29) Karina wants to calculate the distance a park is located from her school. She makes two angle measurements from 7 miles apart as represented by the triangle below. To the nearest tenth of a mile, how far is the park from school? The park is 5.3 miles from the school.



LAW of SINES $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

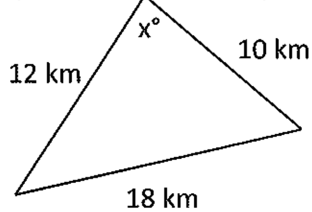
$\frac{\sin 105°}{7} = \frac{\sin 47°}{x} \Rightarrow \frac{x \cdot \cancel{\sin 105°}}{\cancel{\sin 105°}} = \frac{7 \cdot \sin 47°}{\sin 105°}$

$180 - 47 - 28 = 105$

$\Rightarrow x = 5.3$

Target 7I: Distinguish between the Law of Sines and Law of Cosines, and use them to find unknown angles measures in triangles.

30) To the nearest tenth, find the value of x.



LAW of Cosines $a^2 = b^2 + c^2 - 2bc\cos A$

$18^2 = 12^2 + 10^2 - 2(12)(10)\cos A$

$-12^2 - 10^2 \quad -12^2 \quad -10^2$

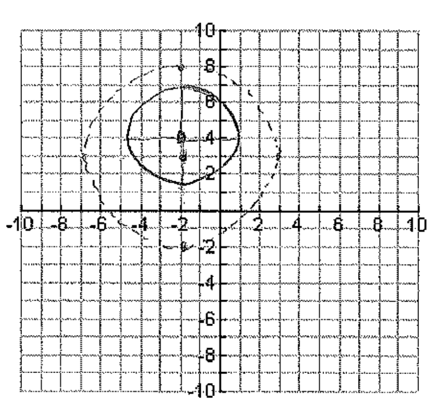
$\frac{18^2 - 12^2 - 10^2}{-2(12)(10)} = \frac{-2(12)(10)\cos A}{-2(12)(10)}$

$\frac{18^2 - 12^2 - 10^2}{-2(12)(10)} = \cos A \Rightarrow \angle A = \cos^{-1}\left(\frac{18^2 - 12^2 - 10^2}{-2(12)(10)}\right)$

$\angle A = 109.5°$

Target 8A: Prove that all circles are similar using transformations.

31) Given Circle C and Circle D, what type of transformation would show that the larger circle is similar to the smaller circle?



Circle C with center (–2, 4) and radius 3

Circle D with center (-2, 3) and radius 5.

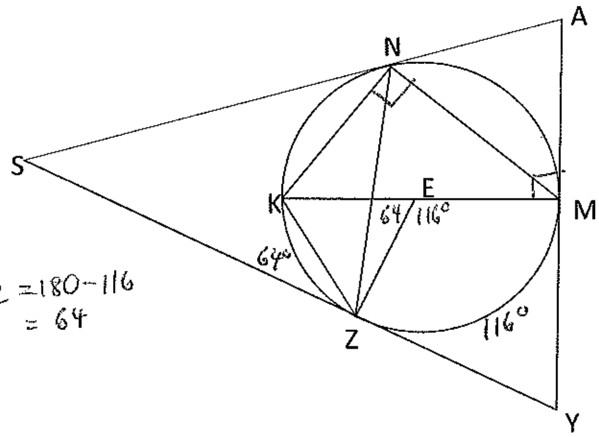
- Translation of 1 unit up &
- Dialation (reduction) with scale factor $\frac{3}{5}$

Target 8B: Describe the relationship between a central angle , inscribed angle, and circumscribed angle and the arc they intercept.

Use the diagram and the given information for problems 32 – 36.

Given: Circle E is inscribed in ΔSYA and $m\widehat{MZ} = 116°$

$\angle Y + m\widehat{MZ} = 180$
$\angle Y + 116 = 180$
$-116 \quad -116$
$\angle Y = 64°$



$\widehat{KZ} = 180 - 116$
$= 64$

Find each of the following:

32) $m\angle Y$ ___64°___

33) $m\angle KNM$ ___90°___

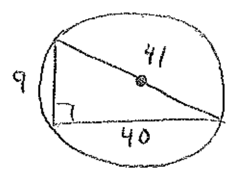
34) $m\angle ZNM$ ___$\frac{1}{2}(116) = 58°$___

35) $m\angle MEZ$ ___116°___

36) $m\angle EMA$ ___90°___

Target 8C: Given a triangle, construct an inscribed circle and a circumscribed circle.

37) A circle is circumscribed about a right triangle with side lengths of 9, 40, and 41. What is the radius of the circle?



$\frac{41}{2} = 20.5$

38) List the steps to construct an inscribed circle.

You list them!

39) List the steps to construct a circumscribed circle.

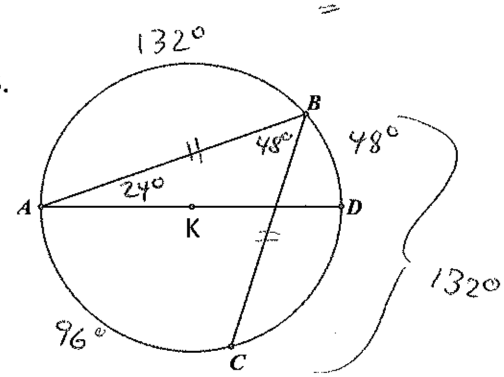
You list them!

Target 8D: Explain and utilize the properties of arcs.

$\widehat{AB} = 180 - 48$
$=$

40) In circle K, $m\angle BAD = 24°$ and $\overline{AB} \cong \overline{BC}$. Find the measure of $\angle B$.

Since $\overline{AB} \cong \overline{BC}$, this implies $\widehat{AB} \cong \widehat{BC}$
Since $\widehat{AB} = 132°$, $\widehat{BC} = 132°$
$360 - 2(132) = 96° = \widehat{AC}$
$\angle B = \frac{1}{2}(96) = 48°$



$(h, k) =$ center , $r =$ radius

Target 8E: Identify the center and radius of a circle given its equation in standard form.

41) What is the standard equation of a circle with center (-66, 13) and a radius of 12?

$(x-h)^2 + (y-k)^2 = r^2 \Rightarrow (x-(-66))^2 + (y-(13))^2 = 12^2$

$\Rightarrow (x+66)^2 + (y-13)^2 = 144$

Target 8F: Define and identify a tangent line and construct a tangent line from a point outside the circle.

42) BA and BC are tangent segments. BC = 2x + 5, AB = 7x – 30, and OA = 4x – 3. What is the circumference of circle O?

$\overline{BA}, \overline{BC}$ tangents $\Rightarrow \overline{BA} \cong \overline{BC}$ Tang.-Tang. Theorem

$2x+5 = 7x-30$

$5 = 5x - 30$

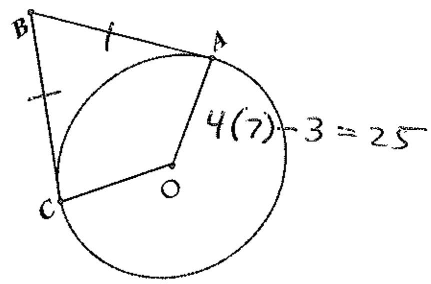
$35 = 5x$

$7 = x$

$C = 2\pi r$

$= 2\pi(25)$

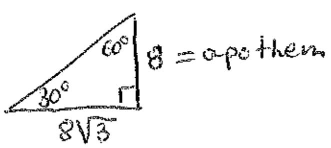
$= 50\pi$ units



Target 9A: Inscribe a regular polygon in a circle and break it into many congruent triangles to find its area.

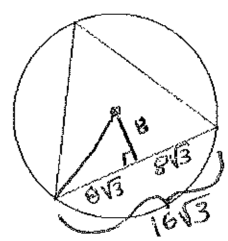
Target 9B: Explain how to use the dissection method on regular polygons to generate an area formula for regular polygons. $A = \frac{1}{2}ap$

43) Find the area of the inscribed equilateral triangle with apothem of 8.



$A = \frac{1}{2}(8)(48\sqrt{3})$

$= (4)(48\sqrt{3}) = 192\sqrt{3}$ units^2



$P = 3(16\sqrt{3})$

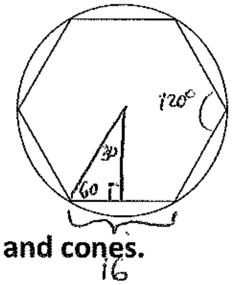
$= 48\sqrt{3}$

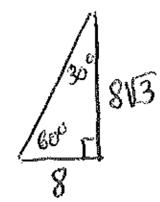
Can we solve this another way?

44) Find the area of the inscribed regular hexagon with side length 16.

Side = 16 $\Rightarrow P = 6(16) = 96$

$A = \frac{1}{2}(8\sqrt{3})(96) = (4\sqrt{3})(96) = 384\sqrt{3}$ units^2





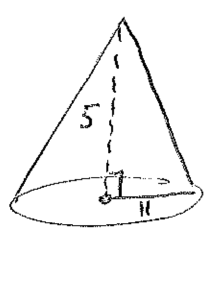
Target 9C: Calculate the base area and volume of prisms, cylinders, pyramids, and cones.

- **Understand that the formula of a pyramid is 1/3 the volume of a prism with the same base area and height.**
- **Understand that the volume of a cone is 1/3 the volume of a cylinder with the same base area and height.**

45) The diameter of the base of a cone is 22 while the height of the cone is 5. Find the volume of the cone.

$d = 22$

$\Rightarrow r = 11$



$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(11)^2(5) = \frac{605}{3}\pi$ units^3

46) The square pyramid below has a volume 1875 m^3 and height 9. What is the value of s?

$V = \frac{1}{3} B \cdot h$

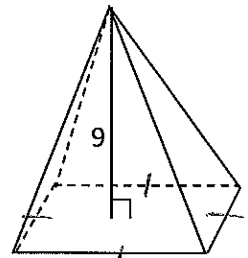
$1875 = \frac{1}{3} s^2 \cdot 9$

$\frac{1875}{3} = \frac{3s^2}{3}$

$\sqrt{625} = \sqrt{s^2}$

$25 = s$

square pyramid $\Rightarrow$
B = area of base = s^2

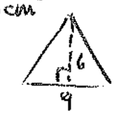


$s = 25$

47) The following triangular prism had a smaller triangular prism piece removed. Calculate the volume of this solid. Round to the nearest tenth if needed.

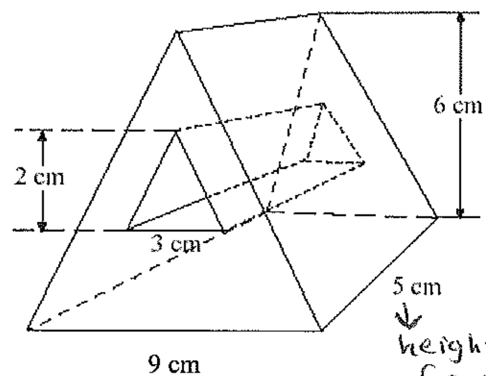
B = area of base = $\frac{1}{2}bh$

$V_{Big} = B \cdot h = (\frac{1}{2}(9)(6))(5) = 135 \text{ cm}^3$



$V_{Small} = B \cdot h = (\frac{1}{2}(3)(2))(5) = 15 \text{ cm}^3$

$V_{Solid} = V_{Big} - V_{Small} = 135 - 15 = 120 \text{ cm}^3$



height of prism

Target 9D: Explain that the volume of a pyramid is 1/3 the volume of a prism with the same base and same height and that the volume of a cone is 1/3 the volume of a cylinder with the same base are and height

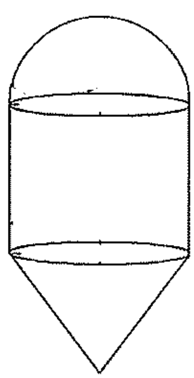
48) A prism has a base area of 99 mm^2 and a height of 17 mm. What is the volume of the pyramid that has the same base and height?

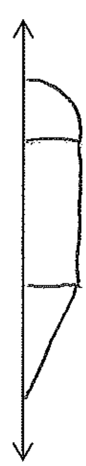
$V_{prism} = B \cdot h = (99)(17) = 1683 \text{ mm}^3$

$V_{pyramid} = \frac{1}{3} V_{prism} = \frac{1}{3}(1683) = 561 \text{ mm}^3$

Target 9E: Rotate a 2D figure and identify the 3D object that is formed.

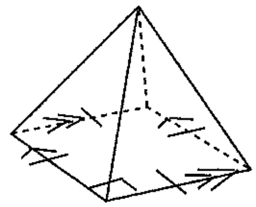
49) What 2D plane figure can be rotated around the line to create the following 3D solid? Draw the figure.



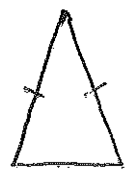


Target 9F: Identify the shapes of 2D cross-sections of 3D objects.

Use the following pyramid for questions 50 and 51.

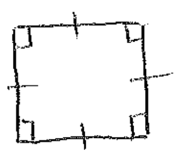


50) Draw the 2D figure that is the vertical cross section (through the vertex) of the pyramid.



Isosceles triangle

51) Draw the 2D figure that is the horizontal cross section of the pyramid.



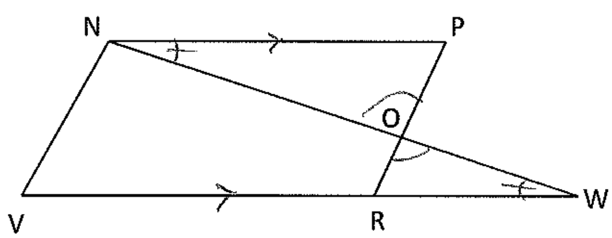
square

For problems 52-55, complete a two-column proof.

Mark up your diagram!

52) Given: NPRV is a parallelogram

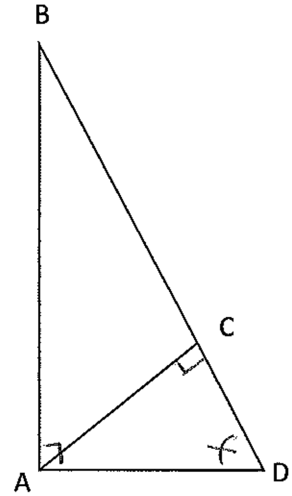
Prove: $\Delta ORW \sim \Delta OPN$



Statement	Reason
① NPRV ∥-gram	① Given
② $\overleftrightarrow{NP} \parallel \overleftrightarrow{VR}$	② ∥-gram ⇒ opposite sides ∥ (Def. of ∥-gram).
③ $\angle PNO \cong \angle RWO$	③ ∥ lines ⇒ Alternate interior $\angle$s $\cong$.
④ $\angle PON \cong \angle ROW$	④ Vertical $\angle$'s are $\cong$.
⑤ $\Delta ORW \sim \Delta OPN$	⑤ AA~ (step 3 & 4)

53) Given: $\overline{AB} \perp \overline{AD}$, $\angle ACD$ is a right angle

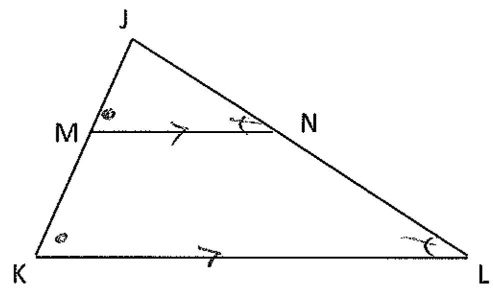
Prove: $\triangle ABD \sim \triangle CAD$



Statement	Reason
① $\overline{AB} \perp \overline{AD}$	① Given
② $\angle BAD$ rt. $\angle$	② Definition of $\perp$.
③ $\angle ACD$ rt. $\angle$	③ Given
④ $\angle BAD \cong \angle ACD$	④ Right $\angle$s are $\cong$.
⑤ $\angle BDA \cong \angle ADC$	⑤ Reflexive property of $\angle$s.
⑥ $\triangle ABD \sim \triangle CAD$	⑥ AA~ (Step 4 & 5)

54) Given: $\overleftrightarrow{MN} \parallel \overleftrightarrow{KL}$

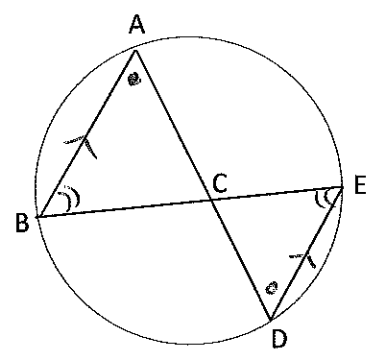
Prove: $\frac{JM}{JK} = \frac{MN}{KL}$



statement	Reason
① $\overleftrightarrow{MN} \parallel \overleftrightarrow{KL}$	① Given
② $\angle JMN \cong \angle JKL$	② ∥ lines ⇒ Corresponding $\angle$s are $\cong$
③ $\angle JNM \cong \angle JLK$	③ ∥ lines ⇒ Corresponding $\angle$s are $\cong$ (or same as above ☺)
④ $\triangle JMN \sim \triangle JKL$	④ AA ~ (step 2 & 3)
⑤ $\frac{JM}{JK} = \frac{MN}{KL}$	⑤ Corresponding sides of similar $\triangle$s are proportional

55) Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{ED}$

Prove: (BA)(EC) = (ED)(BC)

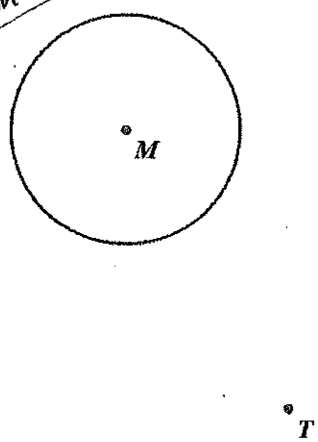


Statement	Reason
① $\overleftrightarrow{AB} \parallel \overleftrightarrow{ED}$	① Given
② $\angle BAC \cong \angle EDC$	② ∥ lines ⇒ Alternate interior $\angle$s $\cong$
③ $\angle ABC \cong \angle DEC$	③ Same as above
④ $\triangle ABC \sim \triangle DEC$	④ AA ~ (step 2 & 3)
⑤ $\frac{AB}{DE} = \frac{BC}{EC} = \frac{AC}{DC}$	⑤ Corresponding sides of similar $\triangle$s are proportional
⑥ $AB \cdot EC = DE \cdot BC$	⑥ Means–Extremes Product Theorem

☺

56) Construct a tangent line from T to circle M.

Know these constructions



Write out the steps.

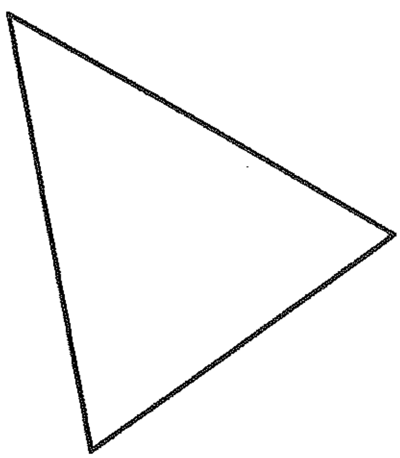
① Draw a line connecting pt. M to pt. T

② Construct ⊥ bisector of $\overline{MT}$

③ Place the compass on the midpoint of $\overline{MT}$, adjust it width to reach either endpoint, and draw an arc across the ⊙

④ Where the arc crosses the ⊙ will be the tangent points.

57) Construct an inscribed circle.



Write out the steps.

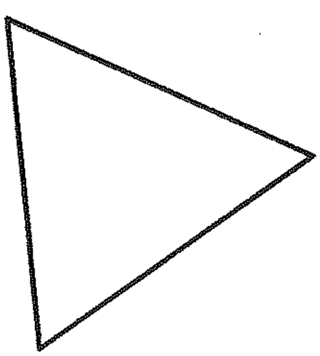
① Bisect any two ∠s. (Construct ∠ bisectors of any two ∠s)

② where they intersect is the center of the inscribed ⊙.

③ Construct a ⊥ from the center pt. to one side of the Δ

④ Place the compass on the center point, adjust it length to where the ⊥ crosses the Δ, and draw the inscribed ⊙!

58) Construct a circumscribed circle.



Write out the steps.

① Construct the ⊥ bisector of any two sides of the Δ.

② where the ⊥ bisectors intersect is the center of the circumscribed ⊙

③ Place compass on center pt., adjust its width to reach any vertex of Δ, and draw the circumscribed ⊙!