6.5. Advanced Algebra
Quadratics

DATE: 1/10

Target 5B. Solve quadratic equations graphically and algebraically
Target 5C. Use the discriminant to determine the number and type of roots and verify by graphing.
Target 5D. Solve quadratic equations with complex solutions

QUADRATIC FORMULA: The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant = $b^2 - 4ac$

The discriminant determines the number and type of solution(s) to expect.

Use the discriminant to verify the number and type of roots of the quadratic equation (use the chart on the last page). Then, solve the quadratic equation by using the Quadratic Formula. Finally, use the Nspire to check your solutions.

Two Real Rational Roots

1. $x^2 - 12x = 28$

$-28 \quad -28$

$\underset{a}{1}x^2 \underbrace{-12}_{b}x \underbrace{-28}_{c} = 0$

$a = 1$
$b = -12$
$c = -28$

$b^2 - 4ac = (-12)^2 - 4(1)(-28)$
$= 256$ ✓

256 is positive # and perfect square
∴ Two Real Rational Roots

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-28)}}{2(1)}$

$= \dfrac{12 \pm \sqrt{256}}{2}$

$= \dfrac{12 \pm 16}{2}$

$\dfrac{12 + 16}{2} = \dfrac{28}{2} = 14$ ✓

$\dfrac{12 - 16}{2} = \dfrac{-4}{2} = -2$ ✓

2. $x^2 - 8x = 33$

$-33 \quad -33$

$x^2 - 8x - 33 = 0$

subtract 33 to both sides 1st

$a = 1$
$b = -8$
$c = -33$

You try it!

:)

One Real Rational Root

3. $x^2 + 22x + 121 = 0$

$a=1$
$b=22$
$c=121$

$b^2-4ac = (22)^2 - 4(1)(121)$
$= 0$

0 means that you have 1 real rational root

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-22 \pm \sqrt{(22)^2-4(1)(121)}}{2(1)}$$

$$= \frac{-22 \pm \sqrt{0}}{2}$$

$$= \frac{-22 \pm 0}{2}$$

$$\frac{-22+0}{2} = -11$$

$$\frac{-22-0}{2} = -11$$

4. $x^2 - 34x + 289 = 0$

$a=1$
$b=-34$
$c=289$

You try it!

Two Real Irrational Roots

5. $2x^2 + 4x - 5 = 0$

$a=2$
$b=4$
$c=-5$

$b^2-4ac = (4)^2-4(2)(-5)$
$= 56$

56 is positive non-perfect square

$\therefore$ Two Real Irrational Roots

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{(4)^2-4(2)(-5)}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{56}}{4}$$

$$\frac{-4+\sqrt{56}}{4} \approx 0.871$$

$$\frac{-4-\sqrt{56}}{4} \approx -2.871$$

6. $2x^2 - 6x + 2 = 0$

$a=2$
$b=-6$
$c=2$

You try it!

Two Complex Roots

7. $x^2 - 4x = -13$

$ +13 \quad +13$

$x^2 - 4x + 13 = 0$

$a = 1$
$b = -4$
$c = 13$

$b^2 - 4ac = (-4)^2 - 4(1)(13)$
$= -36$

-36 is a neg. number
$\therefore$ Two Complex Roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$\sqrt{-36}$
$= \sqrt{-1 \cdot 36}$
$= 6i$

$i = \sqrt{-1}$
$\sqrt{36} = 6$

$\frac{4-6i}{2} = \frac{4}{2} - \frac{6i}{2} = 2 - 3i$

$\frac{4+6i}{2}$

$\frac{4}{2} + \frac{6i}{2} = 2 + 3i$

8. $x^2 + 13 = 6x$

$ -6x \quad -6x$

$x^2 - 6x + 13 = 0$

$a = 1$
$b = -6$
$c = 13$

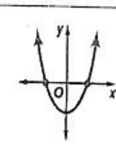
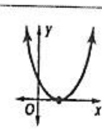
You try it!

Concept Summary — *Solving Quadratic Equations*

Method	Can be Used	When to Use
Graphing	sometimes	Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.
Factoring	sometimes	Use if the constant term is 0 or if the factors are easily determined. **Example** $x^2 - 3x = 0$
Square Root Property	sometimes	Use for equations in which a perfect square is equal to a constant. **Example** $(x + 13)^2 = 9$
Completing the Square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is even. **Example** $x^2 + 14x - 9 = 0$
Quadratic Formula	always	Useful when other methods fail or are too tedious. **Example** $3.4x^2 - 2.5x + 7.9 = 0$

Key Concept — *Discriminant*

Consider $ax^2 + bx + c = 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real, rational root	
$b^2 - 4ac < 0$	2 complex roots