

8.1. Honors Geometry

DATE: 1 | 21 | 14

Target 7A. Apply the product and ratio theorems and calculate geometric means

Ratio is a comparison between two quantities, a quotient of two numbers.

Ex: $\frac{\text{Boys}}{\text{Girls}} = \frac{10}{12} = \frac{5}{6}$, 5:6, 5 to 6

$\frac{\text{Rise}}{\text{Run}}$ Famous in Algebra

Proportion is an equation stating two or more ratios are equal.

Ex: $\frac{12}{16} = \frac{x}{y} = \frac{3}{4}$, $\frac{x+4}{5} = \frac{20}{10}$

Means-Extremes Product Theorem states the product of the means is equal to the product of the extremes.

Given: $\frac{a}{b} = \frac{c}{d}$, $b \neq 0$, $d \neq 0$

Prove: $ad = bc$

1st, 4th terms - extremes
2nd, 3rd terms - means

Statement	Reason
① $\frac{a}{b} = \frac{c}{d}$	① Given
② $bd \cdot \frac{a}{b} = bd \cdot \frac{c}{d}$	② Multiplication property
③ $ad = bc$	③ Algebra ✓

Means-Extremes Ratio Theorem

Given: $ad = bc, (a, b, c, d) \neq 0$

Prove: $\frac{a}{b} = \frac{c}{d}$

$\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a}{c} = \frac{b}{d}, \frac{b}{a} = \frac{d}{c}$

Statement	Reason
① $ad = bc$ $(a, b, c, d) \neq 0$	① Given
② $\frac{ad}{b\cancel{d}} = \frac{bc}{\cancel{d}d}$	② Division Property
③ $\frac{a}{b} = \frac{c}{d}$	③ Algebra

Geometric Mean (Mean Proportional)

↳ when **means are =** in a proportion.

Ex: $\frac{1}{4} = \frac{4}{16}$ → mean

← mean

Arithmetic Mean

↳ Avg. of #s {5, 3}

Ex: $\frac{5+3}{2} = \frac{8}{2} = 4$

∴ AM = 4

Examples

1. Find the fourth term (fourth proportional) of a proportion if the first three terms are 2, 3, and 4.

4th ?

$$\frac{2}{3} = \frac{4}{x} \Rightarrow 2x = 12$$

$$\Rightarrow x = 6 \checkmark$$

↓ 1st ↓ 2nd ↓ 3rd

2. Find the mean proportional(s) between 4 and 16.

$$\frac{4}{x} = \frac{x}{16} \Rightarrow x^2 = 64$$

$$x = \pm 8 \checkmark$$

There are
2 solutions

3. If $3x = 4y$, find the ratio of x to y and y to x .

$$\frac{3x}{3y} = \frac{4y}{3y} \Rightarrow \frac{x}{y} = \frac{4}{3} \checkmark$$

$$x \text{ to } y \Rightarrow \frac{x}{y} = \frac{4}{3}$$

$$y \text{ to } x \Rightarrow \frac{y}{x} = \frac{3}{4}$$

4. Is $\frac{x}{y} \times \frac{a}{b}$ equivalent to $\frac{(x-2y)}{y} = \frac{(a-2b)}{b}$? \Rightarrow $b(x-2y) = y(a-2b)$

\downarrow
 $bx = ay$

$$bx - 2by = ay - 2by$$

$$+ 2by \quad + 2by$$

Yes,
equivalent!

$\rightarrow bx = ay$

5. Find the arithmetic and geometric means between 3 and 27.

AM $\Rightarrow \frac{3+27}{2} = 15$

\therefore AM = 15 ✓

GM $\Rightarrow \frac{3}{x} = \frac{x}{27} \Rightarrow \sqrt{x^2} = \sqrt{81}$

$\Rightarrow x = \pm 9$

\therefore GM = ± 9 ✓