5.1. Advanced Algebra

DATE: _____

Properties of Exponents (Monomials)

Target 6A. Simplify expressions using properties of exponents

Monomials - a number, a variable, or a product of a number and one or more variables

Examples: $4x$, $20x^2yw^3$, -3, a^2b^3, $3yz$ are all monomials.

Note: Monomials cannot contain variables in denominators, variables with negative exponents, or variables under radicals.

Constant - a monomial that is a number without a variable.

Examples: 6, 201, $1{,}000{,}001$, -3^2, π, e, ...

Coefficient - the numerical factor of a monomial.

Base - In an expression of the form x^n, the base is x.

Exponent/Power - In an expression of the form x^n, the exponent/power is n.

The **degree** of a monomial is the sum of the exponents of its variables. The degree of a constant is always 0.

Examples: $3m^6 \rightarrow$ degree $= 6$, $5t^1g^1h^{12} \rightarrow$ degree $= 14$

Writing Monomials Using Exponents

Rewrite the following expressions using exponents.

1. $x \cdot x \cdot x \cdot x \cdot y \cdot y$

$x^4 y^2$

2. $5 \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot c \cdot c \cdot c$

$5a^2b^4c^3$

Writing Expressions WITHOUT Exponents

Write out each expression without exponents (as multiplication).

3. $8a^3b^2$

$8 \cdot a \cdot a \cdot a \cdot b \cdot b$

4. $(xy)^4$

$(xy)(xy)(xy)(xy)$

$x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$

PRODUCT OF POWERS

PRODUCT OF POWERS

5. Simplify the following expression: *(First write out in expanded form, and then rewrite using exponents)*
$(5a^2)(a^5)$

$5(a\cdot a)(a\cdot a\cdot a\cdot a\cdot a) = 5a^7$ So add exponents

PRODUCT OF POWERS RULE

For any real number a, and integers m and n,

$$a^m \cdot a^n = a^{m+n}$$

6. $a^9 \cdot a^4$ a^{13}

7. $w^2 \cdot w^{10}$ w^{12}

8. $r \cdot r^5$ r^6

9. $(k^5)(k^3)$ k^8

10. $x^2 \cdot y^2$ x^2y^2

Multiplying Monomials

If the monomials have coefficients, multiply those, but still add the powers.

11. $(4a^9)(2a^4)$ $8a^{13}$

12. $(7w^2)(10w^{10})$ $70w^{12}$

13. $(2r)(-3r^5)$ $-6r^6$

14. $(3k^5)(7k^3)$ $21k^8$

15. $(12x^2)(2y^2)$ $24x^2y^2$

If the monomials have a mixture of different variables, only add powers of like variables.

16. $(4a^9b^3)(2a^4b)$ $8a^{13}b^4$

17. $(2rt^3)(-3r^5)$ You try it!

18. $(3k^5mn^4)(7k^3m^3n^3)$ $21k^8m^4n^7$

19. $(12x^2y^3)(2xy^2)$ You try it!

QUOTIENT OF POWRS RULE

For any real number $a \neq 0$, and integers m and n,

$$\frac{a^m}{a^n} = a^{m-n}$$

Dividing Monomials

20. $\frac{-f^3g^2h^2}{f^1g^1h^1} = -f^2gh$

3-1 for f
2-1 for g
2-1 for h

METHOD 2:

$\frac{-f \cdot f \cdot f \cdot g \cdot g \cdot h \cdot h}{f \cdot g \cdot h} = -f^2gh$

21. $\frac{24x^3y^5}{-6xy^4} = -4x^2y$

$\frac{24}{-6} = -4$
3-1 for x
5-4 for y

NEGATIVE EXPONENT RULE

For any real number $a \neq 0$ and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

"flip" negative exponents ONLY

22. $\frac{x^{-9}y^{-3}}{xw^6u^2} = \frac{1}{x^1w^6u^2x^9y^3} = \frac{1}{x^{10}w^6u^2y^3}$

23. $\frac{x^{-6}}{x^{-8}} = \frac{x^8}{x^6} = x^2$

8-6=2

PROPERTIES OF POWERS

Suppose a and b are real numbers and m and n are integers. Then the following properties hold:

Power of Zero: $a^0 = 1$ *(any number raised to the power of zero is always one)*	Ex: $2^0 = 1$
Power of a Power: $(a^m)^n = a^{mn}$	Ex: $(3^5)^4 = 3^{5\cdot 4}$
Power of a Product: $(ab)^m = a^m b^m$	Ex: $(2\cdot 7)^3 = 2^3 \cdot 7^3$
Power of a Quotient: $\left(\frac{a}{b}\right)^n = \left(\frac{a^n}{b^n}\right)$, $b \neq 0$ and $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \left(\frac{b^n}{a^n}\right)$	Ex: $\left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5}$ $\left(\frac{2}{3}\right)^{-5} = \frac{2^{-5}}{3^{-5}} = \frac{3^5}{2^5}$

Simplify each expression.

24. $x^0 \cdot y^2$ $= 1 \cdot y^2 = y^2$

25. $(ab)^0$ $= a^0 \cdot b^0 = 1 \cdot 1 = 1$

26. $(b^9)^{10}$ $= b^{9\cdot 10} = b^{90}$

27. $(c^3)^3$ You try it!

28. $(2b^9)^3$ $= 2^3 b^{27} = 8b^{27}$

$2^3 = 2 \cdot 2 \cdot 2$

29. $(7w^{12})^2$ You try it!

30. $(5w^3xy^2)^4$ $= 5^4 w^{12} x^4 y^8$

31. $(7w^{12}y^4z)^2$ You try it!

32. $\frac{5^0 t^4 w u^2}{t^2 w^3 u^2} = \frac{1 \cdot t^4 \cdot w^1 \cdot u^2}{t^2 \cdot w^3 \cdot u^2} = t^2 w^{-2} u^0 = t^2 w^{-2} \cdot 1 = \frac{t^2}{w^2}$

$4-2$ $1-3$ $2-2$

33. $\frac{-27x^4y^5}{-9x^2y^6} = 3x^2y^{-1} = \frac{3x^2}{y^1} = \frac{3x^2}{y}$

4-2 5-6

$\frac{-27}{-9} = 3$

You try it!

34. $\frac{30x^{-10}y^2z^{16}}{5x^{-5}y^6z^{-4}}$

35. $\frac{(x^2y^6)(x^5y)}{(x^8y)^3} = \frac{x^7 \cdot y^7}{x^{24} \cdot y^3} = x^{-17}y^4 = \frac{y^4}{x^{17}}$

7-24 7-3

multiply

36. $\left(\frac{-7a^5b^4c^0}{5a^4b^3c^3}\right)^{-4} = \frac{(-7)^{-4}a^{-20}b^{-16}c^0}{(5)^{-4}a^{-16}b^{-12}c^{-12}} = \frac{5^4a^{16}b^{12}c^{12}}{(-7)^4a^{20}b^{16}} = \frac{5^4}{(-7)^4} \cdot a^{-4}b^{-4}c^{12} = \frac{5^4c^{12}}{(-7)^4a^4b^4}$

16-20 12-16

$c^0 = 1$

36. $\left(\frac{-7a^5b^4c^0}{5a^4b^3c^3}\right)^{-4}$

37. $\frac{(4^3x^5y^4)^{-2}}{(4^5x^{-1}y^5)^{-4}}$

You try it!