

DATE: _____

4.2 Trigonometric Functions of Acute Angles (Target 5B)

Review of Prior Concepts

1. Convert each radian measure to degrees:

a) $\frac{\pi}{6}$

$$\left(\frac{\pi}{6} \text{ radian}\right) \times \left(\frac{180^\circ}{\pi \text{ radian}}\right)$$

30°

b) $\frac{\pi}{4}$

$$\left(\frac{\pi}{4} \text{ radian}\right) \times \left(\frac{180^\circ}{\pi \text{ radian}}\right)$$

45°

c) $\frac{\pi}{3}$

$$\left(\frac{\pi}{3} \text{ radian}\right) \times \left(\frac{180^\circ}{\pi \text{ radian}}\right)$$

60°

2. Find the values of $\sin \theta$, $\cos \theta$, $\tan \theta$.

a)

$$\begin{aligned} \text{opp} &= 3 \\ \text{hyp} &= 5 \\ \text{adj} &= 4 \\ \theta & \end{aligned}$$

$$\therefore a^2 + b^2 = c^2$$

$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = \frac{3}{4}$$

b)

$$\begin{aligned} \text{opp} &= 5 \\ \text{hyp} &= \sqrt{39} \\ \text{adj} &= 8 \\ \theta & \end{aligned}$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 25 &= 64 \\ a^2 &= \sqrt{39} \end{aligned}$$

$$\sin \theta = \frac{5}{\sqrt{39}}$$

$$\cos \theta = \frac{8}{\sqrt{39}}$$

$$\tan \theta = \frac{5}{8}$$

\therefore rationalizing optional

Six Trigonometric Ratios

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

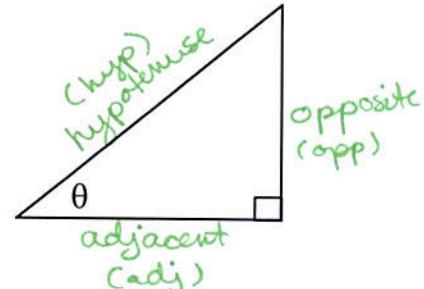
$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos \theta}$$

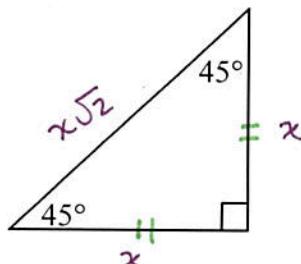
$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\tan \theta}$$



Special Right Triangles

$45^\circ-45^\circ-90^\circ \Delta$

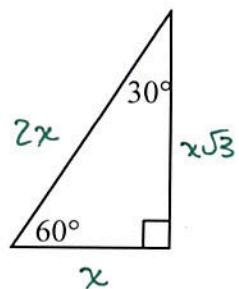


$$\begin{aligned} x^2 + x^2 &= c^2 \\ 2x^2 &= c^2 \\ \sqrt{2x^2} &= c \\ x\sqrt{2} &= c \end{aligned}$$

What do you know about a $45^\circ-45^\circ-90^\circ \Delta$?

Sides are: $x, x, x\sqrt{2}$

$30^\circ-60^\circ-90^\circ \Delta$



$$\begin{aligned} a^2 + x^2 &= (2x)^2 \\ a^2 + x^2 &= 4x^2 \\ a^2 &= 3x^2 \\ a &= \sqrt{3x^2} \\ a &= x\sqrt{3} \end{aligned}$$

What do you know about a $30^\circ-60^\circ-90^\circ \Delta$?

Sides are: $x, x\sqrt{3}, 2x$

Examples:

Find the value of the variables.

1)

$$x \cdot a = 3$$

$$c = x\sqrt{2}$$

$$\rightarrow x = 3$$

2)

$$8 = x\sqrt{2}$$

$$\frac{8}{\sqrt{2}} = x$$

$$\frac{8\sqrt{2}}{2} = x$$

$$4\sqrt{2} = x$$

$$d = 4\sqrt{2}$$

$$f = 4\sqrt{2}$$

3)

$$k = 2(5)$$

$$k = 10$$

$$h = 5\sqrt{3}$$

$$\rightarrow x = 5$$

4)

$$7 = x\sqrt{3}$$

$$\frac{7}{\sqrt{3}} = x$$

$$\frac{7\sqrt{3}}{3} = x$$

$$m = \frac{7\sqrt{3}}{3}$$

$$p = \frac{14\sqrt{3}}{3}$$

Evaluate without using a calculator:

5) $\tan\left(\frac{\pi}{3}\right) \quad \frac{\pi}{3} \cdot \frac{180^\circ}{\pi} = 60^\circ$

$\tan(60^\circ)$

$$\tan 60^\circ = \frac{\text{opp}}{\text{adj}}$$

$$= \frac{x\sqrt{3}}{x}$$

$$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

6) $\csc\left(\frac{\pi}{4}\right) \quad \frac{\pi}{4} \cdot \frac{180^\circ}{\pi} = 45^\circ$

$\csc(45^\circ)$

$$\csc(45^\circ) = \frac{\text{hyp}}{\text{opp}}$$

$$= \frac{x\sqrt{2}}{x}$$

$$\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$$

Find the acute angle θ , in both degrees and radians, without using a calculator.

7) $\tan \theta = \sqrt{3}$ which Δ uses $\sqrt{3}$?
 $\frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{1}$
 $30^\circ-60^\circ-90^\circ \Delta$

$$\theta = 60^\circ$$

$$\text{or } \frac{\pi}{3}$$

8) $\cos \theta = \frac{\sqrt{3}}{2}$ which Δ uses $\sqrt{3}$?
 $\frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$
 $30^\circ-60^\circ-90^\circ \Delta$

$$\theta = 30^\circ$$

$$\text{or } \frac{\pi}{6}$$

Find the value of x in the triangle.

9)

$$\sin 35^\circ = \frac{12}{x}$$

$$x \sin 35^\circ = 12$$

$$x = \frac{12}{\sin 35^\circ}$$

$$x = 20.921$$

10)

$$\cos 35^\circ = \frac{10}{x}$$

$$x \cos 35^\circ = 10$$

$$x = \frac{10}{\cos 35^\circ}$$

$$x = 12.208$$