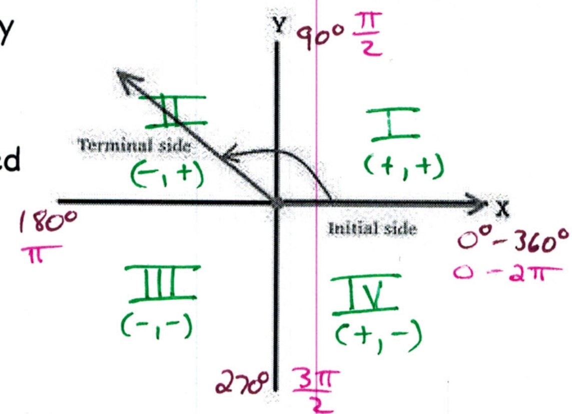
Honors Advanced Algebra DATE:_______________

Trig Extended: Circular Functions Cont'd

Target 6C. Use the Pythagorean identity to find sine of theta, cosine of theta, or tangent of theta, and the quadrant of the angle.

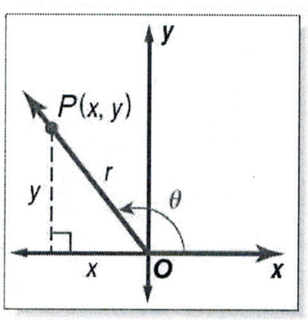
Recall that on a coordinate plane, an angle may be generated by the rotation of two rays that share a fixed endpoint at the origin. One ray, called the **initial side** of the angle is fixed along the positive x-axis (it doesn't move). The other ray, called the **terminal side** of the angle, can rotate about the center.



The measure of an angle is determined by the amount and direction of rotation from the initial side to the terminal side.

Trigonometric Functions of θ in Standard Position

Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of θ. Using the Pythagorean Theorem, the distance r from the origin to P is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows:



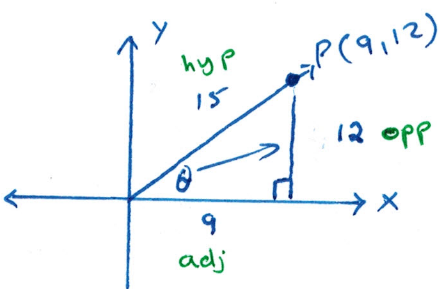
$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x},\ x \neq 0$$

$$\csc\theta = \frac{r}{y},\ y \neq 0 \qquad \sec\theta = \frac{r}{x},\ x \neq 0 \qquad \cot\theta = \frac{x}{y},\ y \neq 0$$

Evaluate Trigonometric Functions for a Given Point

Find the exact values of the six trigonometric functions of θ if the terminal side of θ contains the given point. Think like this: "SOH CAH TOA" Like (3, 4, 5)

1. (9, 12)



$r = \sqrt{9^2 + 12^2} = 15$

$\sin\theta = \frac{12}{15} = \frac{4}{5}$

$\cos\theta = \frac{9}{15} = \frac{3}{5}$

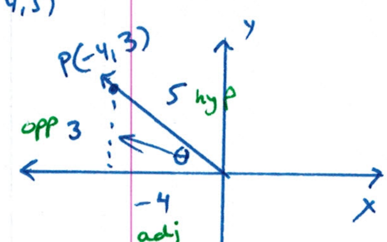
$\tan\theta = \frac{12}{9} = \frac{4}{3}$

$\csc\theta = \frac{5}{4}$ "reciprocal of $\sin\theta$"

$\sec\theta = \frac{5}{3}$ "reciprocal of $\cos\theta$"

$\cot\theta = \frac{3}{4}$ "reciprocal of $\tan\theta$"

2. (-4, 3)



$\sin\theta = \frac{3}{5}$ $\quad\csc\theta = \frac{5}{3}$

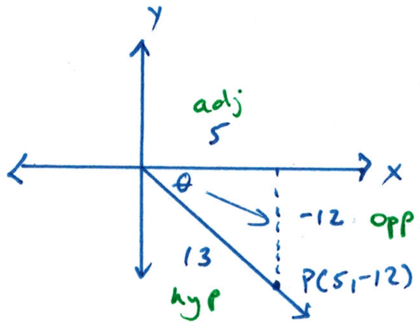
$\cos\theta = -\frac{4}{5}$ $\quad\sec\theta = -\frac{5}{4}$

$\tan\theta = -\frac{3}{4}$ $\quad\cot\theta = -\frac{4}{3}$

☺

like (5, 12, 13)

3. (5, -12)



$\sin\theta = -\frac{12}{13}$

$\cos\theta = \frac{5}{13}$

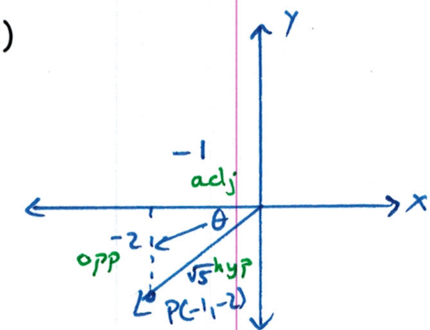
$\tan\theta = -\frac{12}{5}$

$\csc\theta = -\frac{13}{12}$

$\sec\theta = \frac{13}{5}$

$\cot\theta = -\frac{5}{12}$

4. (-1, -2)



$\sin\theta = \frac{-2}{\sqrt{5}}$

$\cos\theta = \frac{-1}{\sqrt{5}}$

$\tan\theta = \frac{-2}{-1} = 2$

$\csc\theta = -\frac{\sqrt{5}}{2}$

$\sec\theta = -\frac{\sqrt{5}}{1} = -\sqrt{5}$

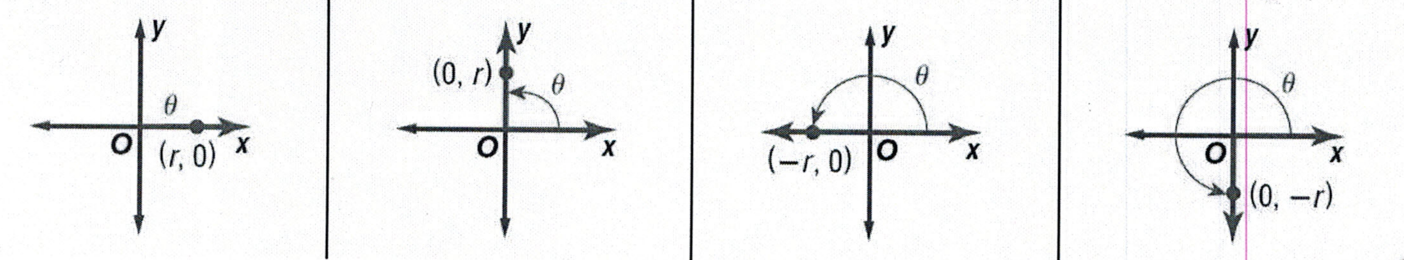
$\cot\theta = \frac{1}{2}$

$r = \sqrt{x^2+y^2}$
$= \sqrt{(-1)^2+(-2)^2}$
$= \sqrt{1+4}$
$= \sqrt{5}$

Quadrantal Angles

If the terminal side of angle θ lies on one of the axes, θ is called a quadrantal angle. The quadrantal angles are $0°$, $90°$, $180°$, and $270°$. Notice that for these angles, either x or y is equal to zero. Since division by zero is undefined, two of the trigonometric values are undefined for each quadrant.

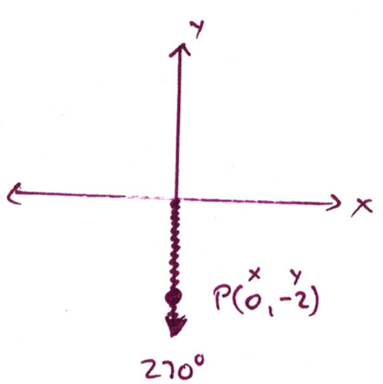
$\theta = 0°$ or 0 radians | $\theta = 90°$ or $\frac{\pi}{2}$ radians | $\theta = 180°$ or π radians | $\theta = 270°$ or $\frac{3\pi}{2}$ radians



Quadrantal Angles Examples

Find the values of the six trigonometric functions for an angle in standard position with the given measure.

5. $270°$ at $(0, -2)$



$r = \sqrt{x^2+y^2}$
$= \sqrt{0^2+(-2)^2}$
$= 2$

$\sin\theta = \frac{y}{r} = \frac{-2}{2} = -1$

$\cos\theta = \frac{x}{r} = \frac{0}{2} = 0$

$\tan\theta = \frac{y}{x} = \frac{-2}{0} \Rightarrow$ undefined or "D.N.E." Does not exist

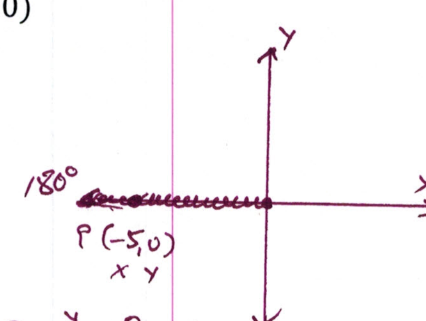
$\csc\theta = \frac{r}{y} = \frac{2}{-2} = -1$

$\sec\theta = \frac{r}{x} = \frac{2}{0} \Rightarrow$ D.N.E

$\cot\theta = \frac{x}{y} = \frac{0}{2} = 0$

6. $180°$ at $(-5, 0)$

$r = \sqrt{(-5)^2+(0)^2} = 5$



$\sin\theta = \frac{y}{r} = \frac{0}{5} = 0$

$\cos\theta = \frac{x}{r} = \frac{-5}{5} = -1$

$\tan\theta = \frac{y}{x} = \frac{0}{-5} = 0$

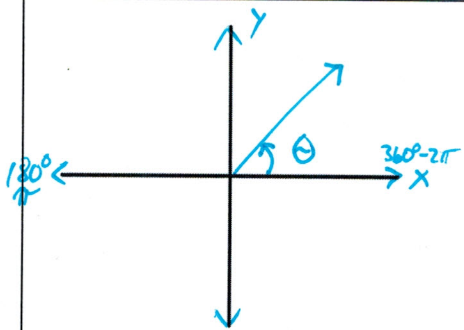
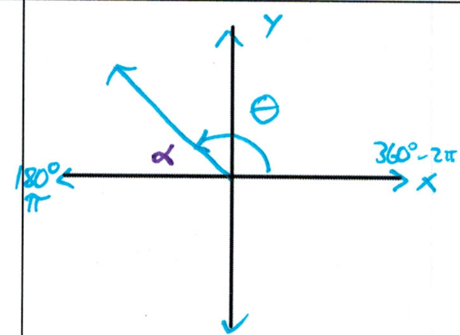
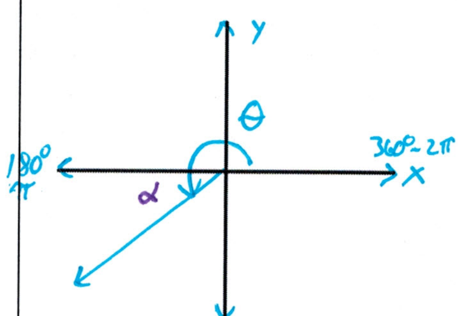
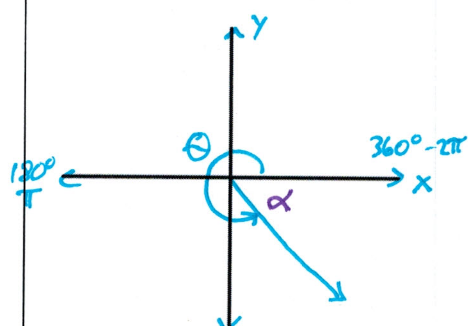
$\csc\theta = \frac{r}{y} = \frac{5}{0} \Rightarrow$ D.N.E.

$\sec\theta = \frac{r}{x} = \frac{5}{-5} = -1$

$\cot\theta = \frac{x}{y} = \frac{-5}{0} \Rightarrow$ D.N.E.

Reference Angles

To find the values of trig functions greater than 90° (or less than 0°), you need to know how to find the measures of reference angles. If θ is a non-quadrantal angle in standard position, its reference angle, alpha (α), is defined as the acute angle formed by the terminal side of θ and the x-axis. Below we'll write the reference angle rules on the interval (0°, 360°) or (0, 2π). If the measure of θ is greater than 360° or less than 0°, its reference angle can be found by associating it with a co-terminal angle of positive measure between 0° and 360°.

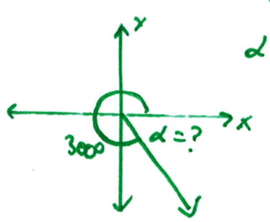
Quadrant I	Quadrant II
 $\alpha = \theta$ (in degrees or radians)	 Degrees: $\alpha = 180° - \theta$ Radians: $\alpha = \pi - \theta$
Quadrant III	**Quadrant IV**
 Degrees: $\alpha = \theta - 180°$ Radians: $\alpha = \theta - \pi$	 Degrees: $\alpha = 360° - \theta$ Radians: $\alpha = 2\pi - \theta$

Reference Angles Examples

Sketch each angle. The find its reference angle.

7. 300°

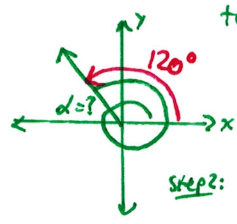
QIV



$\alpha = 360° - \theta$
$= 360° - 300°$
$= \boxed{60°}$

8. 480°

Step 1: $\boxed{\text{Co-terminal}}$ to 480° where $0° < \theta < 360°$:



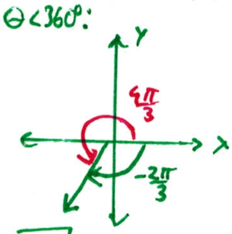
$480° - 360 = \boxed{120°}$

Step 2: Q II

$\alpha = 180° - \theta$
$= 180 - \boxed{120°} = \boxed{60°}$

9. $-2\pi/3$

Step 1: Co-terminal to $-\frac{2\pi}{3}$ where $0 < \theta < 2\pi$:



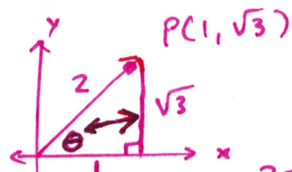
$-\frac{2\pi}{3} + 2\pi = -\frac{2\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{4\pi}{3}}$

Step 2: Q III

$\alpha = \theta - \pi$
$= \boxed{\frac{4\pi}{3}} - \pi = \frac{4\pi}{3} - \frac{3\pi}{3} = \boxed{\frac{\pi}{3}}$

Find the angle that passes through the given point. Give your answer in radians and degrees.

10. $(1, \sqrt{3})$

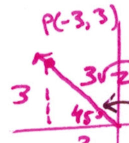


$a^2 + b^2 = c^2$
$1^2 + (\sqrt{3})^2 = c^2$
$1 + 3 = c^2$
$4 = c^2$
$\boxed{2 = c}$

30°	60°	90°
x	$x\sqrt{3}$	$2x$
1	$\sqrt{3}$	2

$\boxed{\theta = 60°}$ or $\boxed{\theta = \frac{\pi}{3}}$

11. (-3, 3)



$180° - 45° = 135° = \theta$

45°	45°	90°
x	x	$x\sqrt{2}$
3	3	$3\sqrt{2}$

$\boxed{\theta = 135°}$ or $\boxed{\theta = \frac{3\pi}{4}}$