DATE: _____

4.1 Angles and Their Measures (Target 5A)

Review of Prior Concepts

The wheels on the bus go round and round, round and round, round and round.
The wheels on the bus go round and round, all through the town.
If the radius of the wheel of the bus is 70 cm, what is the circumference of the wheel?

$C = 2\pi r$

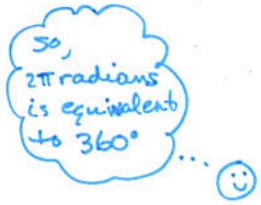
$C = 2\pi(70) \rightarrow 140\pi = \boxed{439.823\text{ cm}}$

Vocabulary

- Degree - measure of an angle

  Degree of ⊙ = 360°

  - unit of angular measure
- Radian - ratio of arc length to length of radius



$$\text{Radian of } \odot = \frac{\text{Length of } \odot}{\text{Lenth of radius of } \odot} = \frac{2\pi r}{r} = 2\pi \text{ radians}$$

Convert from Degrees to Radians

Multiply degrees by $\frac{2\pi \text{ radians}}{360^\circ} \Rightarrow \frac{\pi \text{ radians}}{180^\circ}$

Example: Convert 36° to radians

$36^\circ \cdot \frac{\pi \text{ radians}}{180^\circ}$

$36^{\cancel{\circ}} \cdot \frac{\pi \text{ radians}}{180^{\cancel{\circ}}}$ ← degree symbols "cancel" & left with radians

$\frac{36\pi}{180}$ radians $= \frac{1}{5}\pi$ radians or $\boxed{\frac{\pi}{5} \text{ radians}}$

Convert from Radians to Degrees

Multiply degrees by $\frac{360^\circ}{2\pi \text{ radians}} \Rightarrow \frac{180^\circ}{\pi \text{ radians}}$

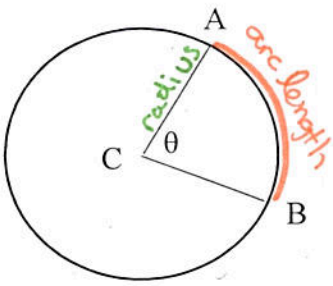
Example: Convert $\frac{2\pi}{3}$ radians to degrees

$\frac{2\pi}{3} \text{ radians} \left(\frac{180^\circ}{\pi \text{ radians}}\right)$

$\frac{2\pi}{3} \cancel{\text{radians}} \left(\frac{180^\circ}{\pi \cancel{\text{radians}}}\right)$ ← radian measurements "cancel" & left with degrees

$\frac{2\pi(180^\circ)}{3(\pi)} = \frac{360^{\cancel{\circ}}}{3\cancel{\pi}} = \frac{360^\circ}{3} = \boxed{120^\circ}$

Arc Length



$\widehat{AB} = \frac{\angle ACB}{360^\circ}$ (circumference)

$= \frac{\theta}{360^\circ}$ ($2\pi r$)

$= \frac{2\pi r\theta}{360^\circ}$

$= \frac{\pi}{180^\circ} \cdot r\theta$ where θ is measured in degrees

*What if θ is measured in radians?

$S = \frac{\pi}{180^\circ} r\theta \left(\frac{180^\circ}{\pi \text{ radians}}\right)$ ← convert to radians

$S = r\theta$ where θ is measured in radians

Examples:

radius

p.358 #28

$S = 2.5$ cm, $\theta = \frac{\pi}{3}$ radians, $r = ?$

$S = r\theta$

$2.5 = r\left(\frac{\pi}{3}\right)$

$\frac{3}{\pi} \cdot 2.5 = r\left(\frac{\pi}{3}\right) \cdot \frac{3}{\pi}$

$\frac{7.5}{\pi}$ cm $= r$

p.358 #32

$S = ?$, $r = 5$ ft, $\theta = 18^\circ$

convert to radians

$18^\circ\left(\frac{\pi \text{ radians}}{180^\circ}\right)$

$\frac{18\pi}{180}$ radians

$\frac{\pi}{10}$ radians

$S = r\theta$

$S = 5\left(\frac{\pi}{10}\right)$

$= \frac{\pi}{2}$ ft

p.358 #34

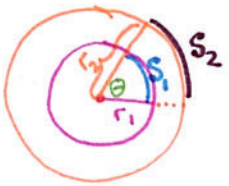
concentric circles – circles w/ same center

with center

$\theta = ?$ $r_1 = 8$ km

$S_1 = 36$ km

$r_2 = ?$ $S_2 = 72$ km



$S_1 = r_1\theta$

$36 = 8\theta$

$4.5 = \theta$ radians

$S_2 = r_2\theta$

$72 = r_2(4.5)$

$16 \text{ km} = r_2$

$\theta = 4.5$ radians

$r_2 = 16$ km

p.358 #36

$S = 7$cm, $\theta = 100^\circ$, $r = ?$

convert 100° to radians

$100^\circ\left(\frac{\pi \text{ radians}}{180^\circ}\right)$

$\frac{100\pi}{180}$ radians

$\frac{5\pi}{9}$ radians

$S = r\theta$

$7 = r\left(\frac{5\pi}{9}\right)$

$\left(\frac{9}{5\pi}\right)7 = r\left(\frac{5\pi}{9}\right)\left(\frac{9}{5\pi}\right)$

$\frac{63}{5\pi} = r \rightarrow r = 4.011$ cm

4cm