Key Concept #1 – Special Functions and Linear Systems

A. Graph, transform, and identify key features of absolute value functions.[F.BF.3, F.IF.4,F.IF.7b] (Unit 2)
  - Key features: domain and range (interval notation), axis of symmetry, vertex.

B. Graph piecewise and step functions [F.IF.7b] (Lesson Lab 2.5)
  - Apply graphs of piecewise, and step functions to a problem in context. [A.CED.2]

C. Solve linear systems in 3 variables [A.REI.6] (2.9)

Target 1A: Graph, transform, and identify key features of absolute value functions.[F.BF.3, F.IF.4,F.IF.7b] (Unit 2)

There is a family of functions related to the one you represented in the Solve It!

Essential Understanding Just as the absolute value of x is its distance from 0, the absolute value of $f(x)$, or $|f(x)|$, gives the distance from the line $y = 0$ for each value of $f(x)$.

The simplest example of an **absolute value function** is $f(x) = |x|$. The graph of the absolute value of a linear function in two variables is V-shaped and symmetric about a vertical line called the **axis of symmetry**. Such a graph has either a single maximum point or a single minimum point, called the **vertex**.

take note

Key Concept Absolute Value Parent Function $f(x) = |x|$

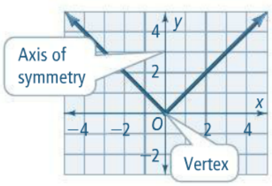
Table

x	$y = \|x\|$
−3	3
−2	2
−1	1
0	0
1	1
2	2
3	3

Function

$$f(x) = \begin{cases} |x| = x, \text{ when } x \geq 0 \\ |x| = -x, \text{ when } x < 0 \end{cases}$$

Graph



The transformations you studied in Lesson 2-4 also apply to absolute value functions.

take note

Key Concept The Family of Absolute Value Functions

Parent Function $y = |x|$

Vertical Translation

Translation up k units, $k > 0$
$y = |x| + k$

Translation down k units, $k > 0$
$y = |x| - k$

Horizontal Translation

Translation right h units, $h > 0$
$y = |x - h|$

Translation left h units, $h > 0$
$y = |x + h|$

Vertical Stretch and Compression

Vertical stretch, $a > 1$
$y = a|x|$

Vertical compression, $0 < a < 1$
$y = a|x|$

Reflection

In the x-axis
$y = -|x|$

In the y-axis
$y = |-x|$

The right branch of the graph of $y = |x|$ has slope 1. The graph of $y = a|x|$, $a > 0$, is a stretch or compression of the graph of $y = |x|$. Its right branch has slope a. The graph of $y = -a|x|$ is a reflection of $y = a|x|$ in the x-axis and its right branch has slope $-a$.

Problem 1 Graphing an Absolute Value Function

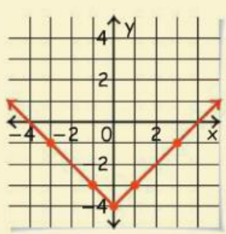
What is the graph of the absolute value function $y = |x| - 4$? How is this graph different from the graph of the parent function $f(x) = |x|$?

Think

Make a table of values and graph the function.

Write

x	y
-3	-1
-1	-3
0	-4
1	-3
3	-1



Use the location of the vertex to see how the graph has been translated. The parent function was not multiplied by a number, so the graph wasn't stretched,

Since the vertex is at (0, −4), you translated the graph of $y = |x|$ down 4 units.

take note

Key Concept General Form of the Absolute Value Function

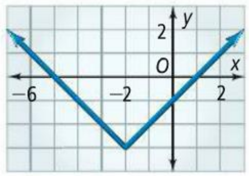
$y = a|x - h| + k$

The stretch or compression factor is $|a|$, the vertex is located at (h, k), and the axis of symmetry is the line $x = h$.

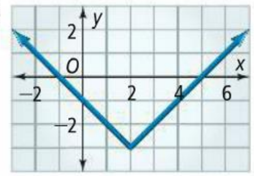
Problem 2 Combining Translations

Multiple Choice Which of the following is the graph of $y = |x + 2| + 3$?

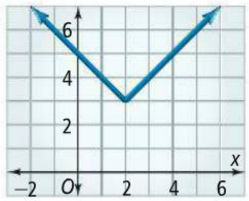
(A)



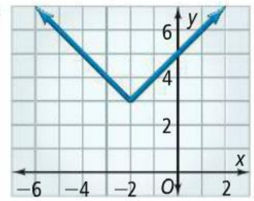
(C)



(B)



(D)



Compare $y = |x + 2| + 3$ to each form, $y = |x + h|$ and $y = |x| + k$.

$y = |x + h|$ The parent function, $y = |x|$, is translated left 2 units.

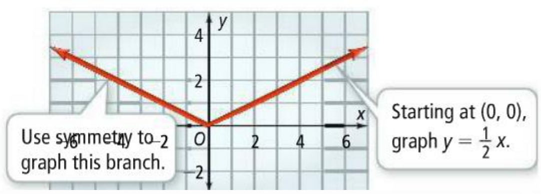
$y = |x| + k$ The parent function, $y = |x|$, is translated up 3 units.

The parent function $y = |x|$ is translated left 2 units and up 3 units. The vertex will be at $(-2, 3)$. The correct choice is D.

Problem 3 Vertical Stretch and Compression

What is the graph of $y = \frac{1}{2}|x|$?

The graph is a vertical compression of the graph of $f(x) = |x|$ by the factor $\frac{1}{2}$. Graph the right branch and use symmetry to graph the left branch.



Problem 4 Identifying Transformations

Without graphing, what are the vertex and axis of symmetry of the graph of $y = 3|x - 2| + 4$? How is the parent function $y = |x|$ transformed?

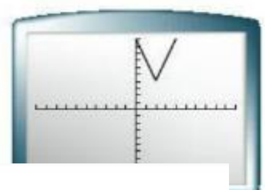
Compare $y = 3|x - 2| + 4$ with the general form $y = a|x - h| + k$.

$a = 3, h = 2,$ and $k = 4$.

The vertex is $(2, 4)$ and the axis of symmetry is $x = 2$.

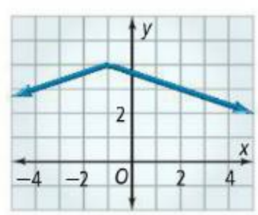
The parent function $y = |x|$ is translated 2 units to the right, vertically stretched by the factor 3, and translated 4 units up.

Check Check by graphing the equation on a graphing calculator.



Problem 5 Writing an Absolute Value Function

What is the equation of the absolute value function?



Step 1 Identify the vertex.

The vertex is at $(-1, 4)$, so $h = -1$ and $k = 4$.

Step 2 Identify a.

The slope of the branch to the right of the vertex is $-\frac{1}{3}$, so $a = -\frac{1}{3}$.

Step 3 Write the equation.

Substitute the values of a, h, and k into the general form $y = a|x - h| + k$. The equation that describes the graph is $y = -\frac{1}{3}|x + 1| + 4$.

Target 1B: Graph piecewise and step functions [F.IF.7b] (Lesson Lab 2.5)

A **piecewise function** is a function that has different rules for different parts of its domain.

Essential Understanding You can graph a piecewise function by graphing each part of the function.

Problem 3 Graphing a Piecewise Function

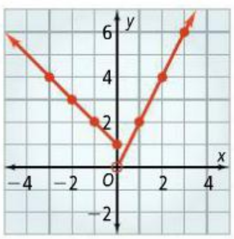
Graph the function $f(x) = \begin{cases} 2x, \text{ for } x > 0 \\ -x + 1, \text{ for } x \leq 0 \end{cases}$

Step 1 Make two tables.

x	$f(x) = 2x$
0	$2 \cdot 0 = 0$
1	$2 \cdot 1 = 2$
2	$2 \cdot 2 = 4$
3	$2 \cdot 3 = 6$

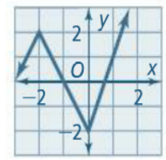
x	$f(x) = -x + 1$
−3	$-(-3) + 1 = 3 + 1 = 4$
−2	$-(-2) + 1 = 2 + 1 = 3$
−1	$-(-1) + 1 = 1 + 1 = 2$
0	$-(0) + 1 = 0 + 1 = 1$

Step 2 Plot the points on a graph.
Notice the graph has an open circle at $x = 0$ for the function $f(x) = 2x$ and a closed circle at $x = 0$ for the function $f(x) = -x + 1$.



Example 5

What piecewise function represents the graph?



Piece 1 When $x \leq -2$, the rule is $f(x) = 2x + 6$.

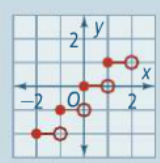
Piece 2 When $-2 < x \leq 0$, the rule is $f(x) = -2x - 2$.

Piece 3 When $x > 0$, the rule is $f(x) = 3x - 2$.

$$f(x) = \begin{cases} 2x + 6, \text{ for } x \leq -2 \\ -2x - 2, \text{ for } -2 < x \leq 0 \\ 3x - 2, \text{ for } x > 0 \end{cases}$$

Given a piecewise function in one form, you can represent it in each of the other forms, including a table, graph, algebraic function, or verbal statement.

Another example of a piecewise function is a **step function**. A step function is a function that pairs every number in an interval with a single value. The graph of a step function can look like the steps of a staircase. Each piece of the graph is a horizontal segment with any missing points indicated by open circles.



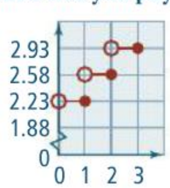
Example 4

Media Postage **You want to mail a book that weighs 2.5 lb. The table lists postage for a book weighing up to 3 lb. Define and graph the media-postage function. How much will you pay in postage?**

Media Postage

Weight (lb)	Price ($)
$x \leq 1$	2.23
$1 < x \leq 2$	2.58
$2 < x \leq 3$	2.93

$$f(x) = \begin{cases} 2.23, \text{ for } 0 < x \leq 1 \\ 2.58, \text{ for } 1 < x \leq 2 \\ 2.93, \text{ for } 2 < x \leq 3 \end{cases}$$

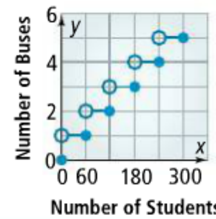


Since $2 < 2.5 \leq 3$, you will pay \$2.93.

Problem 5 Graphing a Step Function

Transportation **A school will charter buses so that the student body can attend a football game. Each bus holds a maximum of 60 students. Make a graph that models the relationship between the number of students x that go to the game by bus and the number of buses y that are needed.**

You will need 0 buses for 0 students. As the number of students increases, the number of buses goes up by 1 every time the number of students exceeds a multiple of 60. Draw a closed circle when the endpoints are part of the graph, and then draw an open point when they are not.



Target 1C: Solve linear systems in 3 variables [A.REI.6] (2.9)

Essential Understanding To solve systems of three equations in three variables, you can use some of the same algebraic methods you used to solve systems of two equations in two variables.

You can use the elimination and substitution methods to solve a system of three equations in three variables by working with the equations in pairs. You will use one of the equations *twice*. When one point represents the solution of a system of equations in three variables, write it as an ordered triple (x, y, z).

You can apply the method in Problem 1 to most systems of three equations in three variables. You may need to multiply in one, two, or all three equations by one, two, or three nonzero numbers. Your goal is to obtain a system—equivalent to the original system—with coefficients that allow for the easy elimination of variables.

Problem 1 Solving a System Using Elimination

What is the solution of the system? Use elimination. The equations are numbered to make the procedure easy to follow.

$$\begin{cases} ① \; 2x - y + z = 4 \\ ② \; x + 3y - z = 11 \\ ③ \; 4x + y - z = 14 \end{cases}$$

Step 1 Pair the equations to eliminate z. Then you will have two equations in x and y.

Add.

$$\begin{aligned} ① \quad & 2x - y + z = 4 \\ ② \quad & x + 3y - z = 11 \\ \hline ④ \quad & 3x + 2y = 15 \end{aligned}$$

Subtract.

$$\begin{aligned} ② \quad & x + 3y - z = 11 \\ ③ \quad & 4x + y - z = 14 \\ \hline ⑤ \quad & -3x + 2y = -3 \end{aligned}$$

Step 2 Write the two new equations as a system. Solve for x and y.

Add and solve for y.

$$\begin{aligned} ④ \quad & 3x + 2y = 15 \\ ⑤ \quad & -3x + 2y = -3 \\ \hline & 4y = 12 \\ & y = 3 \end{aligned}$$

Substitute $y = 3$ and solve for x.

$$\begin{aligned} ④ \quad 3x + 2y &= 15 \\ 3x + 2(3) &= 15 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

Step 3 Solve for z. Substitute the values of x and y into one of the original equations.

① $2x - y + z = 4$	Use equation ①.
$2(3) - 3 + z = 4$	Substitute.
$6 - 3 + z = 4$	Simplify.
$z = 1$	Solve for z.

Step 4 Write the solution as an ordered triple. The solution is (3, 3, 1).

Problem 2 Solving an Equivalent System

What is the solution of the system? Use elimination.

$$\begin{cases} x + y + 2z = 3 & (1) \\ 2x + y + 3z = 7 & (2) \\ -x - 2y + z = 10 & (3) \end{cases}$$

Think	Write
You are trying to get two equations in x and z. Multiply ① so you can add it to ② and eliminate y. Do the same with ② and ③.	① $x + y + 2z = 3$ → $-x - y - 2z = -3$; ② $2x + y + 3z = 7$ → $2x + y + 3z = 7$; ④ $x + z = 4$ ② $2x + y + 3z = 7$ → $4x + 2y + 6z = 14$; ③ $-x - 2y + z = 10$ → $-x - 2y + z = 10$; ⑤ $3x + 7z = 24$
Multiply ④ so you can add it to ⑤ and eliminate x.	④ $x + z = 4$, ⑤ $3x + 7z = 24$ → $-3x - 3z = -12$; $3x + 7z = 24$; $4z = 12$; $z = 3$
Substitute $z = 3$ into ④. Solve for x.	$x + 3 = 4$; $x = 1$
Substitute the values for x and z into ① to find y. Check the answer in the three original equations.	$x + y + 2z = 3$; $1 + y + 2(3) = 3$; $y = -4$ Check: $1 + (-4) + 2(3) = 3$ ✔; $2(1) + (-4) + 3(3) = 7$ ✔; $-(1) - 2(-4) + 3 = 10$ ✔ The solution is $(1, -4, 3)$.

Problem 3 Solving a System Using Substitution

Multiple Choice What is the x-value in the solution of the system?

$$\begin{cases} 2x + 3y - 2z = -1 & (1) \\ x + 5y = 9 & (2) \\ 4z - 5x = 4 & (3) \end{cases}$$

Ⓐ 1　　Ⓑ 4　　Ⓒ 6　　Ⓓ 10

Step 1 Choose equation ②. Solve for x.

② $x + 5y = 9$

$x = 9 - 5y$

Step 2 Substitute the expression for x into equations ① and ③ and simplify.

①
$$\begin{aligned} 2x + 3y - 2z &= -1 \\ 2(9 - 5y) + 3y - 2z &= -1 \\ 18 - 10y + 3y - 2z &= -1 \\ 18 - 7y - 2z &= -1 \\ (4)\quad -7y - 2z &= -19 \end{aligned}$$

③
$$\begin{aligned} 4z - 5x &= 4 \\ 4z - 5(9 - 5y) &= 4 \\ 4z - 45 + 25y &= 4 \\ 4z + 25y &= 49 \\ (5)\quad 25y + 4z &= 49 \end{aligned}$$

Step 3 Write the two new equations as a system. Solve for y and z.

$$\begin{cases} -7y - 2z = -19 & (4) \\ 25y + 4z = 49 & (5) \end{cases}$$

$-14y - 4z = -38$　Multiply by 2.

$25y + 4z = 49$　Then add.

$11y = 11$

$y = 1$

④
$$\begin{aligned} -7y - 2z &= -19 \\ -7(1) - 2z &= -19 \quad \text{Substitute the value of } y \text{ into (4).} \\ -2z &= -12 \\ z &= 6 \end{aligned}$$

Step 4 Use one of the original equations to solve for x.

②
$$\begin{aligned} x + 5y &= 9 \\ x + 5(1) &= 9 \quad \text{Substitute the value of } y \text{ into (2).} \\ x &= 4 \end{aligned}$$

The solution of the system is $(4, 1, 6)$, and $x = 4$.

Problem 4 Solving a Real-World Problem

Business **You manage a clothing store and budget $6000 to restock 200 shirts. You can buy T-shirts for $12 each, polo shirts for $24 each, and rugby shirts for $36 each. If you want to have twice as many rugby shirts as polo shirts, how many of each type of shirt should you buy?**

Relate T-shirts + polo shirts + rugby shirts = 200

rugby shirts = 2 · polo shirts

12 · T-shirts + 24 · polo shirts + 36 · rugby shirts = 6000

Define Let x = the number of T-shirts.

Let y = the number of polo shirts.

Let z = the number of rugby shirts.

Write

$$\begin{cases} ① & x + y + z = 200 \\ ② & z = 2 \cdot y \\ ③ & 12 \cdot x + 24 \cdot y + 36 \cdot z = 6000 \end{cases}$$

Step 1 Since 12 is a common factor of all the terms in equation ③, write a simpler equivalent equation.

$$\begin{cases} ③ & 12x + 24y + 36z = 6000 \\ ④ & x + 2y + 3z = 500 \end{cases} \quad \text{Divide by 12.}$$

Step 2 Substitute $2y$ for z in equations ① and ④. Simplify to find equations ⑤ and ⑥.

① $x + y + z = 200$

$x + y + (2y) = 200$

⑤ $x + 3y = 200$

④ $x + 2y + 3z = 500$

$x + 2y + 3(2y) = 500$

⑥ $x + 8y = 500$

Step 3 Write ⑤ and ⑥ as a system. Solve for x and y.

$$\begin{cases} ⑤ & x + 3y = 200 \\ ⑥ & x + 8y = 500 \end{cases}$$

$-x - 3y = -200$ Multiply by -1.

$x + 8y = 500$ Then add.

$5y = 300$

$y = 60$

⑤ $x + 3y = 200$

$x + 3(60) = 200$ Substitute the value of y into ⑤.

$x = 20$

Step 4 Substitute the value of y in ② and solve for z.

② $z = 2y$

$z = 2(60) = 120$

You should buy 20 T-shirts, 60 polo shirts, and 120 rugby shirts.