PreCalculus KC 1 Review (Functions) Name: ______________________________

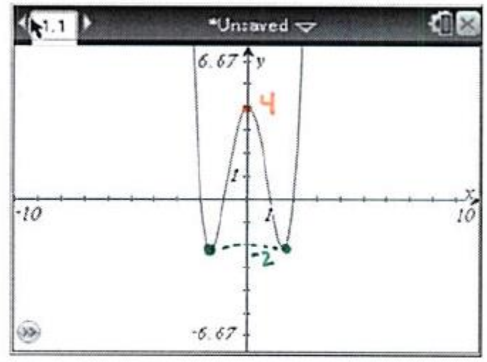
Non-Calculator

1) Using the graph of $f(x)$ on the right:

a) estimate the value(s) of the relative max and min.

relative max : 4

relative min : -2



b) write the end behavior using limit notation.

$\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$

2) Find the domain and range of the following:

a) $f(x) = \sqrt{x-1} + 3$

Domain: $x - 1 \geq 0$
$x \geq 1$

Range: $\sqrt{x-1}$ is ≥ 0
so, $\sqrt{x-1} + 3$ is ≥ 3

Domain: $[1, \infty)$

Range: $[3, \infty)$

b) $k(x) = \sqrt{-2x+1}$

Domain: $-2x + 1 \geq 0$
$-2x \geq -1$
$x \leq 1/2$

Range: $\sqrt{-2x+1}$ is ≥ 0

Domain: $(-\infty, \frac{1}{2}]$

Range: $[0, \infty)$

3) Find $(f \text{ o } g)(x)$ and $(g \text{ o } f)(x)$ if $f(x) = x^2 - 7$ and $g(x) = \sqrt{x+3}$. Then, write the domain of each.

$(f \circ g)(x) = f(g(x))$
$= (\sqrt{x+3})^2 - 7$
$= x + 3 - 7$
$\boxed{(f \circ g)(x) = x - 4}$

Domain: $[-3, \infty)$

$(g \circ f)(x) = \sqrt{x^2 - 7 + 3}$
$\boxed{(g \circ f)(x) = \sqrt{x^2 - 4}}$

Domain: $(-\infty, -2] \cup [2, \infty)$

4) Find $f(g(2))$ and $g(f(-1))$ if $f(x) = 2x - 5$ and $g(x) = x^2 + 3x - 1$.

$f(g(2)) = f(9) = 2(9) - 5 = \boxed{13}$

$g(2) = (2)^2 + 3(2) - 1 = 4 + 6 - 1 = 9$

$g(f(-1)) = g(-7) = (-7)^2 + 3(-7) - 1 = 49 - 21 - 1 = \boxed{27}$

$f(-1) = 2(-1) - 5 = -2 - 5 = -7$

5) Find the inverse of:

a) $f(x) = 4x - 8$

$y = 4x - 8$

$x = 4y - 8$

$x + 8 = 4y$

$\frac{x+8}{4} = y$

$f^{-1}(x) = \frac{x+8}{4} = \frac{1}{4}x + 2$

b) $h(x) = (x-3)^2 + 9$

$y = (x-3)^2 + 9$
$x = (y-3)^2 + 9$
$x - 9 = (y-3)^2$
$\sqrt{x-9} = \sqrt{(y-3)^2}$

$\pm\sqrt{x-9} = y - 3$

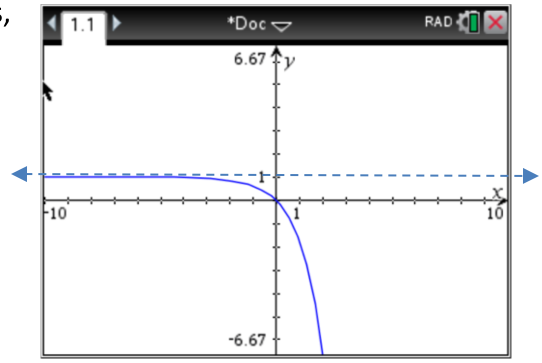
$\boxed{\pm\sqrt{x-9} + 3 = y}$

Note the equation of above is NOT a function, so I don't write $h^{-1}(x)$.

6) Using your knowledge of parent functions and transformations, write the function that produces the graph on the right.

exponential function $(\mathrm{H.A.}\ @\ y = 1)$

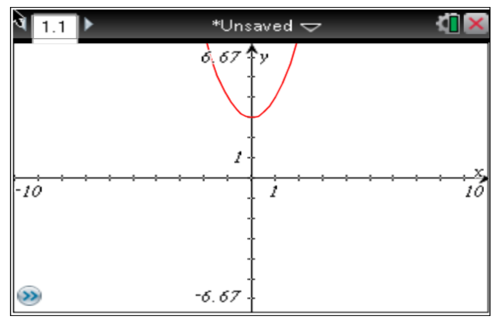
$y = -e^{x} + 1$



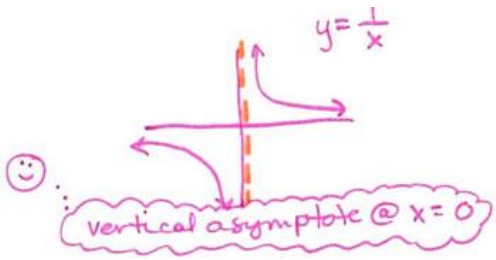
7) Using your knowledge of parent functions and transformations, write the function that produces the graph on the right.

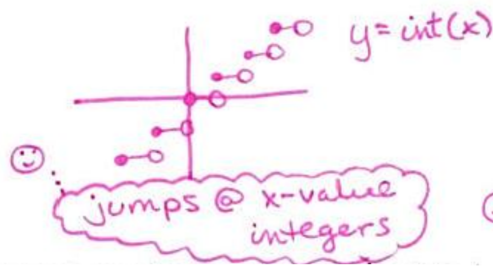
Squaring/quadratic function

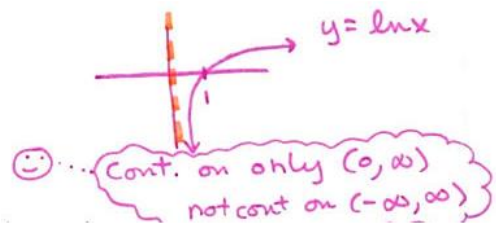
$y = x^2 + 3$



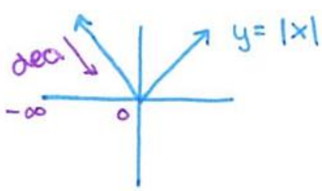
8) Sketch a graph of 3 functions that are NOT continuous over the Real Numbers. → from $-\infty$ to ∞

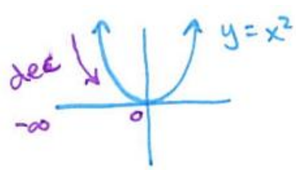


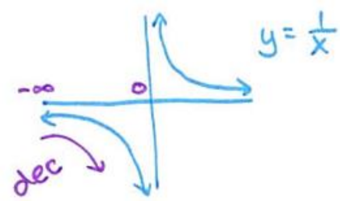




9) Sketch a graph of 3 functions that are decreasing on the interval $(-\infty, 0)$.



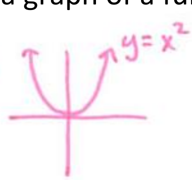


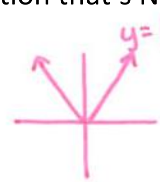


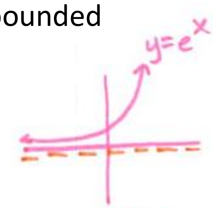
10) Sketch the following:

a) graphs of 3 functions that are bounded below
b) graphs of 2 bounded functions
c) a graph of a function that's NOT bounded

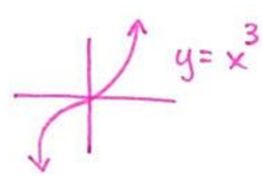
a)



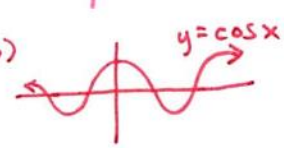


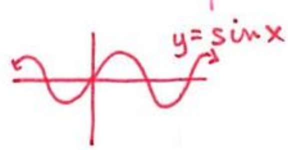


c)



b)



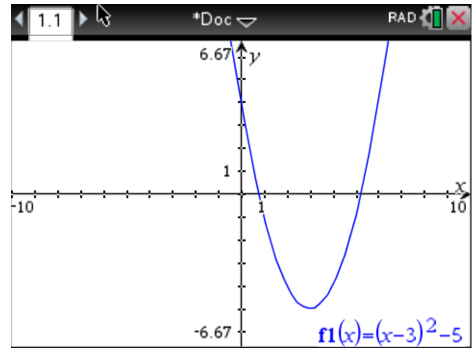


11) Describe the transformation of $q(x) = (x-3)^2 - 5$ from its parent function.

right 3 down 5

$\boxed{\text{Translate (shift) down 5 units and translate (shift) right 3 units}}$

Then, sketch and label the graph of the function.



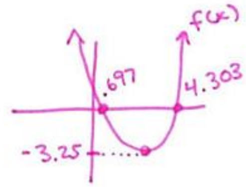
Calculator

12) Find the zeroes of $f(x) = x^2 - 5x + 3$. What is the domain and range of this function?

$x = 0.697$, $x = 4.303$

Domain : $(-\infty, \infty)$

Range : $[-3.25, \infty)$



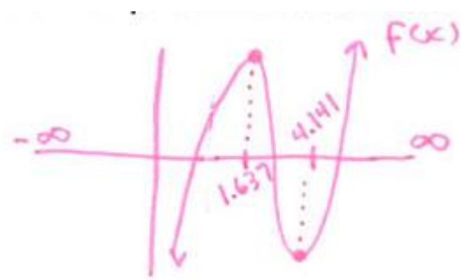
13) Using limit notation, write the end behavior of $f(x) = 3x^3 - 26x^2 + 61x - 30$.

$\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = \infty$

14) Determine to 3 decimal places the interval(s) on which the function in problem 13 is decreasing and increasing.

f(x) inc on $(-\infty, 1.637) \cup (4.141, \infty)$

f(x) dec on $(1.637, 4.141)$



15) Perform the following transformation: Reflect $q(x)$ across the x-axis if $q(x) = (x-3)^2 - 5$. Write the new function and call it $p(x)$.

$p(x) = -q(x)$

$= -[(x-3)^2 - 5]$

$\boxed{p(x) = -(x-3)^2 + 5}$

16) What is the best fit regression curve, given the data on the right? Write the regression model.

# of minutes	3	4	5	6	8
# of cars	8	15	24	35	63

Quadratic function

$y = x^2 - 1$

17) Graphite Inc. makes tennis racquets. If each racquet costs \$53 to make with fixed overhead costs of \$567,000, what is the best fit regression curve? Write a function that models the cost of producing *x* rackets.

C = cost

x = # of racquets

$C(x) = 53x + 567000$

$\boxed{\text{Linear function}}$

18) Is the function below continuous over the real numbers? If not, state the x-value(s) where the discontinuity occurs and tell whether the discontinuity is removable or non-removable.

$f(x) = \frac{x(x^2-4)}{x^3-2x^2-8x}$

$f(x) = \frac{x(x-2)(x+2)}{x(x^2-2x-8)}$

$= \frac{x(x-2)(x+2)}{x(x-4)(x+2)}$ → discontinuities when denominator = 0

$x(x-4)(x+2) = 0$

$\boxed{x=0,\ x=4,\ x=-2}$

notice:

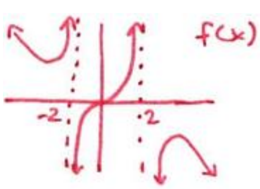
$f(x) = \frac{\cancel{x}(x-2)\cancel{(x+2)}}{\cancel{x}(x-4)\cancel{(x+2)}}$

$x=0,\ x=-2$ removable

$x=4$ nonremovable

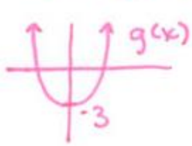
19) Tell whether each of the following functions is odd, even, or neither (support your answers graphically and algebraically):

a) $f(x) = \frac{x^3}{4-x^2}$



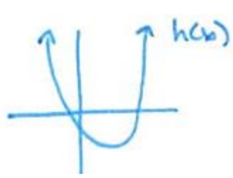
$\boxed{\text{ODD}}$

b) $g(x) = x^2 - 3$



$\boxed{\text{EVEN}}$

c) $h(x) = x^2 - 2x - 2$



$\boxed{\text{NEITHER}}$

$h(-x) = (-x)^2 - 2(-x) - 2$

$= x^2 + 2x - 2$

$\neq h(x)$

∴, not even

$h(-x) = (-x)^2 - 2(-x) - 2$

$= x^2 + 2x - 2$, factor " − "

$= -(-x^2 - 2x + 2)$

$\neq -h(x)$

∴, not odd