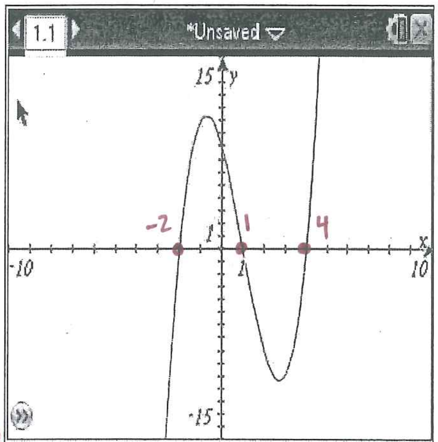
PreCalculus Unit 2 Review (Practice Test) DATE: ________________

Non-Calculator

Write a function that fits each graph in problems 1-3.

1)



factored form $\boxed{f(x) = (x+2)(x-1)(x-4)}$

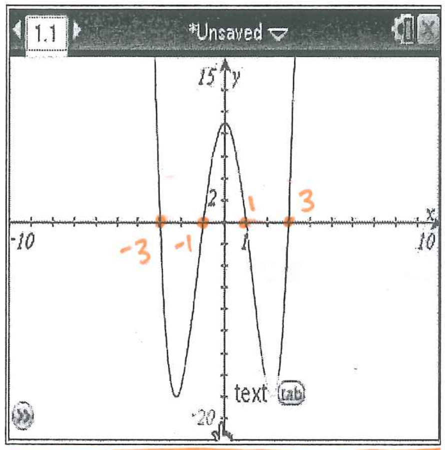
OR

Standard form:

$f(x) = (x^2+x-2)(x-4)$

$\boxed{f(x) = x^3 - 3x^2 - 6x + 8}$

2)



$\boxed{g(x) = (x-3)(x+3)(x-1)(x+1)}$

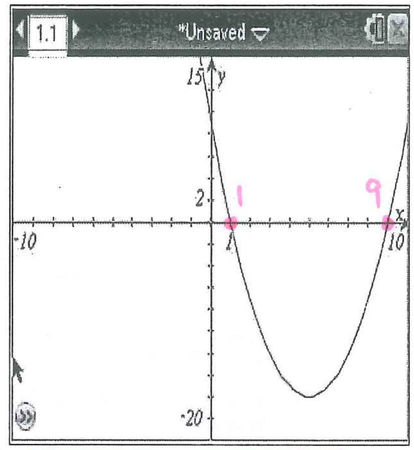
OR

Standard form:

$g(x) = (x^2-9)(x^2-1)$

$\boxed{g(x) = x^4 - 10x^2 + 9}$

3)



$\boxed{h(x) = (x-1)(x-9)}$

OR

Standard form:

$\boxed{h(x) = x^2 - 10x + 9}$

4) Solve the quadratic equation two different ways: $5x^2 - 2x - 3 = 0$

Factoring

by grouping

$5x^2 - 2x - 3 = 0$

$5x^2 - 5x + 3x - 3 = 0$

$5x(x-1) + 3(x-1) = 0$

$(x-1)(5x+3) = 0$

$\boxed{x = 1,\ x = -3/5}$

$5x^2 \cdot -3$

$-15x^2$

~~$-1x, 15x$~~

~~$-3x, 5x$~~

$-5x, 3x$

Quadratic Formula

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-3)}}{2(5)}$

$x = \dfrac{2 \pm \sqrt{64}}{10}$

$x = \dfrac{2 \pm 8}{10}$

$x = \dfrac{2+8}{10} = 1$

$x = \dfrac{2-8}{10} = -\dfrac{6}{10} = -3/5$

$\boxed{x = 1,\ x = -3/5}$

5) Given $x = 4$ is a root, find the rest of the zeros for $f(x) = x^3 + x^2 - 16x - 16$.

$$\begin{array}{r|rrrr} 4 & 1 & 1 & -16 & -16 \\ & \downarrow & 4 & 20 & 16 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$

$\underbrace{1 \quad 5 \quad 4}_{x^2+5x+4}$

$x^2 + 5x + 4 = 0$

$(x+4)(x+1) = 0$

$x = -4, x = -1$

$\boxed{\text{zeroes: } -4, -1, 4}$

6) Determine the polynomial of least degree given the zeros $3 - i$ and -2.

conjugate pair is also a zero. $3+i$

$x = 3 - i,\quad x = 3 + i,\quad x = -2$

$x - 3 + i = 0 \quad x - 3 - i = 0 \quad x + 2 = 0$

$f(x) = (x - 3 + i)(x - 3 - i)(x + 2)$

$= (x^2 - 6x + 10)(x + 2)$

$\boxed{f(x) = x^3 - 4x^2 - 2x + 20}$

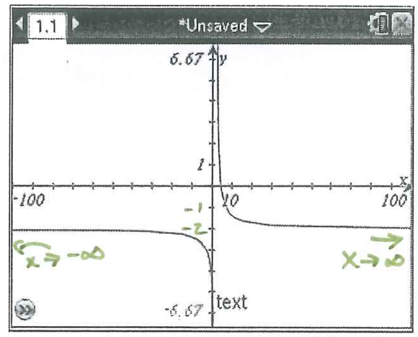
7) Determine how many complex zeros there are for $g(x) = 3x^4 - 6x^2 + 5x - 11$ and explain your reasoning.

degree

There are 4 complex zeros for g(x).

The # of complex zeros is equal to the degree of the polynomial.

8) Given the graph, determine the $\lim_{x \to -\infty} g(x)$ and $\lim_{x \to \infty} g(x)$.



$\lim_{x \to -\infty} g(x) = -2$

$\lim_{x \to \infty} g(x) = -2$

9) Determine the end behavior in problems 1 and 2.

means same as #8.

$\boxed{\#1}$ end behavior

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = \infty$

$\boxed{\#2}$

$\lim_{x \to -\infty} g(x) = \infty$

$\lim_{x \to \infty} g(x) = \infty$

10) Write a polynomial function of least degree in factored form with the following zeros: $-2, 0, 1$, and $\frac{3}{5}$

$x = -2, \quad x = 0, \quad x = 1, \quad x = 3/5$

$x + 2 = 0, \quad x = 0, \quad x - 1 = 0, \quad x - 3/5 = 0$

$5x - 3 = 0$

this one looks nicer :)

$\boxed{f(x) = (x+2)(x)(x-1)(5x-3)}$ OR $\boxed{f(x) = x(x+2)(x-1)(5x-3)}$

11) Write a polynomial function in factored form that has a zero of 0 with multiplicity of 2, a zero of –3 with multiplicity of 3, and a zero of 1 with multiplicity of 2.

(x) ... :) (x+3) (x-1) ... :)

$\boxed{f(x) = x^2(x+3)^3(x-1)^2}$

12) $P(x) = -2x^4 + ax^3 - 3x^2 + bx - 15$. P(x) is divisible by x – 3. P(x) has a remainder of –32 when divided by x + 1. Find a and b.

x = 3

x = -1

:) ... Using Remainder Theorem

could also solve w/ synthetic division ... :)

$P(3) = 0$

$-2(3)^4 + a(3)^3 - 3(3)^2 + b(3) - 15 = 0$

$-162 + 27a - 27 + 3b - 15 = 0$

$27a + 3b - 204 = 0$

$27a + 3b = 204$

$P(-1) = -32$

$-2(-1)^4 + a(-1)^3 - 3(-1)^2 + b(-1) - 15 = -32$

$-2 - a - 3 - b - 15 = -32$

$-a - b - 20 = -32$

$-a - b = -12$

Solve system of equations ... :)

$27a + 3b = 204$

$3(-a - b = -12)$

$27a + 3b = 204$

$-3a - 3b = -36$

$24a = 168$

$\boxed{a = 7}$

$-7 - b = -12$

$-b = -5$

$\boxed{b = 5}$

Calculator

13) Solve for q: $\underbrace{2q^3 - 10q}$ $= \underbrace{5}$

graph in f1(x), in f2(x) and get intersection

OR $\underbrace{2q^3 - 10q - 5}= 0$ graph f1(x) and get the zeros.

$\boxed{\begin{array}{l} q = -1.924 \\ q = -.530 \\ q = 2.453 \end{array}}$

14) Find the solutions of the following equation: $c^2 + 3 = c$

no real solutions, so use quadratic formula.

$c^2 - c + 3 = 0$

$c = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(3)}}{2(1)}$

$c = \dfrac{1 \pm \sqrt{-11}}{2}$

$\boxed{c = \frac{1}{2} \pm \frac{\sqrt{11}}{2}\, i}$

15) Determine all complex zeros for $w(x) = x^4 - 8x^2 - 9$.

use calculator to get real zeros: $x = -3, x = 3$

3	1	0	-8	0	-9
	↓	3	9	3	9
-3	1	3	1	3	0
	↓	-3	0	-3	
	1	0	1	0	

$\underbrace{1 \quad 0 \quad 1}_{x^2+1}$

$x^2 + 1 = 0$

$x^2 = -1$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

$\boxed{\begin{array}{l} \text{zeros: } x = -3 \\ x = 3 \\ x = i \\ x = -i \end{array}}$

16) How many real zeros are there for $b(x) = 2x^3 + 3x^2 + 3x + 9$? How many are imaginary?

graph to find x-intercepts (real zeros), real zero: -1.930

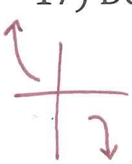
so, $\boxed{\text{one real zero}}$

Since degree of polynomial is 3, there are 3 complex zeros; ∴, $\boxed{\text{2 are imaginary}}$

17) Describe the end behavior of $m(x) = -2x^3 - x + 1$.

degree odd, so opposite end behaviors

leading coefficient negative, so as $x \to \infty$, $m(x) \to -\infty$



$\boxed{\begin{array}{l} \lim\limits_{x \to -\infty} m(x) = \infty \\ \lim\limits_{x \to \infty} m(x) = -\infty \end{array}}$

18) Find the vertical and horizontal asymptotes for:

a) $h(x) = \dfrac{1x-5}{1x+3}$

V.A. → denominator = 0

$x + 3 = 0$

$\boxed{\text{V.A.} @ x = -3}$

H.A. → degree of Num = degree of Denom

∴, y = leading coefficients

$y = \frac{1}{1}$

$\boxed{\text{H.A.} @ y = 1}$

b) $k(x) = \dfrac{x+3}{x^2-5x-24}$

$= \dfrac{\cancel{x+3}}{(x-8)\cancel{(x+3)}}$ removable (hole)

V.A. $x - 8 = 0$

$\boxed{\text{V.A.} @ x = 8}$

H.A. degree of Num < degree of Denom.

∴, $y = 0$

$\boxed{\text{H.A.} @ y = 0}$

c) $n(x) = \dfrac{3x}{x^2-2x-24}$

$= \dfrac{3x}{(x-6)(x+4)}$

V.A. $x - 6 = 0$, $x + 4 = 0$

$\boxed{\text{V.A.} @ x = 6, x = -4}$

H.A. degree of Num < degree of Denom

∴, $y = 0$

$\boxed{\text{H.A.} @ y = 0}$