Key Concept 3 Review (Exponential & Log Functions) Name: Solution Key

Non-Calculator

1) Evaluate: $-7\log 10^3 - 3$

$-7\cancel{\log} \cancel{10}^3 - 3$

$-7(3) - 3$

$-21 - 3$

$\boxed{-24}$

2) Evaluate: $\log_{17} 17^{\frac{9}{14}}$

$\cancel{\log_{17}} \cancel{17}^{9/14}$

$\boxed{9/14}$

3) Solve for m: $\log_{\frac{1}{5}}\left(\sqrt[3]{25}\right)^5 = m$

$\log_{\frac{1}{5}} \sqrt[3]{25}^{\,5} = m$

$\left(\frac{1}{5}\right)^m = \sqrt[3]{25}^{\,5}$

$\left(\frac{1}{5}\right)^m = 25^{5/3}$

$(5^{-1})^m = (5^2)^{5/3}$

$5^{-m} = 5^{10/3}$

$-m = 10/3$

$\boxed{m = -10/3}$

4) Solve for q: $\frac{1}{16} = 2^{q-3}$

$\frac{1}{16} = 2^{q-3}$

$\frac{1}{2^4} = 2^{q-3}$

$2^{-4} = 2^{q-3}$

$-4 = q - 3$

$\boxed{-1 = q}$

5) Condense the expression: $2\,[5\log(x+2) + \log x] - \log(x+4)$

$10\log(x+2) + 2\log x - \log(x+4)$

$\log(x+2)^{10} + \log x^2 - \log(x+4)$

$\log((x+2)^{10} \cdot x^2) - \log(x+4)$

$\boxed{\log\left(\frac{(x+2)^{10} \cdot x^2}{x+4}\right)}$

6) Condense: $2\log_3 y + \log_3 z - \frac{1}{3}\log_3 x$

$\log_3 y^2 + \log_3 z - \log_3 x^{1/3} \rightarrow \log_3(y^2 z) - \log_3 \sqrt[3]{x}$

$\boxed{\log_3\left(\frac{y^2 z}{\sqrt[3]{x}}\right)}$

7) Solve for w: $\log_5(2w-3) = 2$

Exponentiation the left hand side of the equation and right hand side of equation with base 5 to undo log base 5 (Anti-log!). You can also convert the equation to exponential form.

$5^{\log_5(2w-3)} = 5^2$

$\log_5(2w-3) = 2$

$2w - 3 = 5^2$

$2w - 3 = 25$

$2w = 28$

$\boxed{w = 14}$

*Don't forget about extraneous solutions, so check your answer in the original equation.

8) Solve for x: $\ln 15 - \ln x = \ln 3$

Use quotient property (minus means divide the arguments). Then Anti-log with base *e*.

$\ln\frac{15}{x} = \ln 3$

$e^{\ln\frac{15}{x}} = e^{\ln 3}$

$\frac{15}{x} = 3$, multiply both sides of equation by *x*

$15 = 3x$

$\boxed{x = 5}$

*Don't forget about extraneous solutions, so check your answer in the original equation.

9) Solve for a: $-4 = \log_a \frac{1}{16}$

$a^{-4} = \log_{\cancel{a}} \frac{1}{16}$

$a^{-4} = \frac{1}{16}$

$a^{-4} = \frac{1}{2^4}$

$a^{-4} = 2^{-4}$

$\boxed{a = 2}$ Notice exponents are equal, so base *a* must equal base 2.

10) Solve: $\frac{e^x - 4e^{-x}}{3} = 1$

$e^x - 4e^{-x} = 3$

$(e^x)e^x - \left(\frac{4}{e^x}\right)e^x = (3)e^x$

$(e^x)^2 - 4 = 3e^x$

$(e^x)^2 - 3e^x - 4 = 0$

let $u = e^x$

$u^2 - 3u - 4 = 0$

$(u-4)(u+1) = 0$

$u = 4, u = -1$

$u = 4, \quad u = -1$

$e^x = 4 \quad e^x = -1$

$\cancel{\ln e}^x = \ln 4 \quad \ln e^x = \ln(-1)$

$\boxed{x = \ln 4}$ can't ln "–" #s

11) Solve for x: $\log(x-6)^2 = 4$

Solve: $\log_{10}(x-6)^2 = 4$

$(x-6)^2 = 10^4$

$\sqrt{(x-6)^2} = \sqrt{10^4}$

$x - 6 = \pm 10^2$

$x - 6 = 100 \quad \boxed{x = 106}$

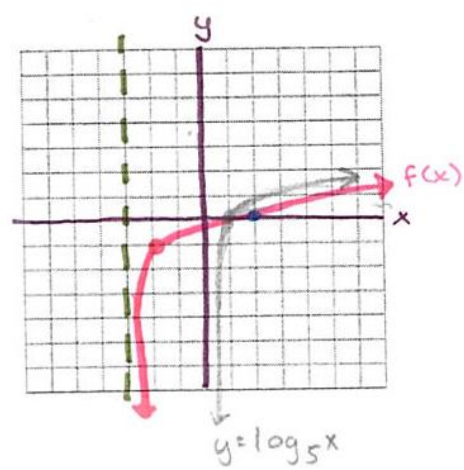
$x - 6 = -100 \quad \boxed{x = -94}$

both #s check ☺

12) Find the domain, range, x & y – intercept, and asymptote(s) of:

$$f(x) = \underbrace{-1}_{\text{translate down 1 unit}} + \log_5(\underbrace{x+3}_{\text{translate Left 3 units}})$$

Graph the function and label all parts.



Domain: $(-3, \infty)$

Range: $(-\infty, \infty)$

Vertical Asymptote: $x = -3$

y-int → when x = 0

$f(0) = -1 + \log_5(0+3)$

$= -1 + \log_5 3$

$\boxed{\text{y-int}: (0, -1 + \log_5 3)}$

x-int → when y = 0

$0 = -1 + \log_5(x+3)$

$5^1 = 5^{\log_5(x+3)}$ → $5 = x + 3$

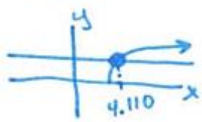
$\boxed{\text{x-int}: (2, 0)}$

Calculator

13) Solve for x: $\ln(x+4)+\ln(x-3)=2\ln 3$

Graph + get intersection

$\boxed{x = 4.110}$



OR

$\ln(x+4)(x-3) = \ln 3^2$

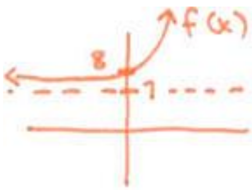
$(x+4)(x-3) = 9$

$x^2+4x-3x-12=9$

$x^2+x-21=0 \rightarrow$ graph + get zeroes

$\boxed{x = 4.120}$ $x = -5.110$ ← extraneous

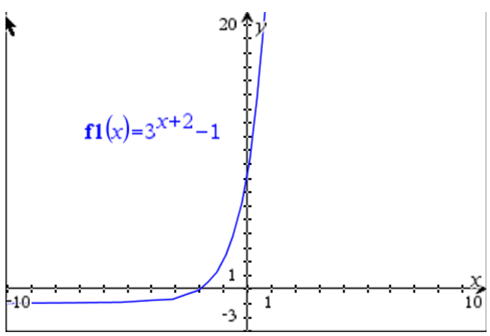
14) Find the domain & range of: $f(x) = e^x + 7$



Domain: $(-\infty, \infty)$

Range: $(7, \infty)$

15) Identify the domain, range, *x* & *y*-intercept, and asympotote(s) for:
$f(x) = 3^{x+2} - 1$



Domain: $(-\infty, \infty)$

Range: $(-1, \infty)$

x-int: $(-2, 0)$

y-int: $(0, 8)$

no V.A

H.A. @ $y = -1$

Note $f(x)$ is shifted 2 units right and 1 unit down from the parent function.

16) The number of bacteria in a petri dish after *t* hours is $B = 100e^{kt}$, where $t = 0$ represents the time at 12 pm. At 6 am, the number of bacteria was 42.

$\hookrightarrow t = -6$ B

a) Find *k*.

b) Using *k*, find the number of bacteria at 8 pm.

$t = 8$

$B = 100e^{.145t}$

$B = 100e^{.145(8)}$

$B = 317.932$

$\boxed{317 \text{ bacteria}}$

a) $B = 100e^{kt}$

$42 = 100e^{k(-6)}$

$42 = 100e^{-6k}$

$\frac{42}{100} = e^{-6k}$

$\ln\left(\frac{42}{100}\right) = \ln e^{-6k}$

$\ln\left(\frac{42}{100}\right) = -6k$

$k = \frac{\ln\left(\frac{42}{100}\right)}{-6}$

$\boxed{k = .145}$ ← store in calculator

17) The population of Wellsville can be represented by $P = 1500e^{kt}$, where *t = 0* is 2010. In 1990, the population was 1400. Find *k* and use this to predict the population in 2020.

$t = -20$ $t = 10$

$P = 1500e^{kt}$

$1400 = 1500e^{k(-20)}$

$\frac{1400}{1500} = e^{-20k}$

$\ln\left(\frac{14}{15}\right) = \ln e^{-20k}$

$\ln\left(\frac{14}{15}\right) = -20k$

$k = \frac{\ln\left(\frac{14}{15}\right)}{-20}$

$\boxed{k = .003}$ ← "sto" in calc.

$P = 1500e^{.003t}$

$P = 1500e^{.003(10)}$

$P = 1552.648$

$\boxed{1552 \text{ people}}$

18) You invest $1300 at Peter Venkman's savings and take a loan at 8% interest compounded continuously. How long will it take for the balance to double?

$A = Pe^{rt}$

$P = 1300$

$r = .08$

$A = 2600$ (double $1300)

$2600 = 1300e^{.08t}$

$2 = e^{.08t}$

$\ln 2 = .08t$

$\frac{\ln 2}{.08} = t \rightarrow \boxed{t = 8.664 \text{ years}}$