

PreCalculus Unit 4 Review (Practice Test)

Name: \_\_\_\_\_

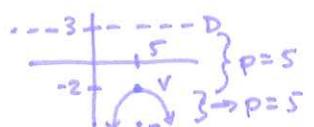
Directions: Practice problems 8, 10, 12, and 22 with and without a calculator.

- 1) Find the focus for a parabola with vertex  $(5, -2)$  & directrix  $y = 3$ .

$$\text{focus} = (h, k-p)$$

$$= (5, -2-5)$$

$$= (5, -7)$$



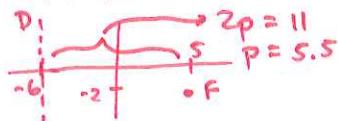
vertex:  $(5, -2)$

focus:  $(5, -7)$

- 2) Find the vertex for a parabola with focus  $(5, -2)$  & directrix  $x = -6$ .

$$\text{vertex} = (5 - 5.5, -2)$$

$$= (-0.5, -2)$$



focus:  $(5, -2)$

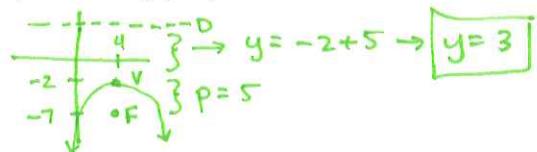
vertex:  $(-0.5, -2)$

- 3) Find the directrix for a parabola with vertex  $(4, -2)$  & focus  $(4, -7)$ .

$$\text{directrix: } y = k+p$$

$$y = -2+5$$

$$y = 3$$



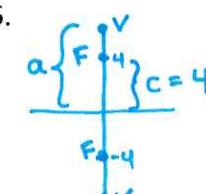
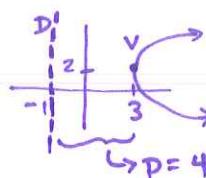
$$y = -2+5 \rightarrow y = 3$$

- 4) Write the equation for a parabola with vertex  $(3, 2)$  & directrix  $x = -1$ .

$$(y-k)^2 = 4p(x-h)$$

$$(y-2)^2 = 4(4)(x-3)$$

$$(y-2)^2 = 16(x-3)$$



- 5) Find the vertices of an ellipse with foci  $(0, 4)$ ,  $(0, -4)$  & minor axis of 6.

$$a^2 = b^2 + c^2$$

$$a^2 = 3^2 + 4^2$$

$$a^2 = 9+16$$

$$a^2 = 25$$

$$a = 5$$

$$\text{vertices: } (0, 5) \\ (0, -5)$$

- 6) Write the equation for an ellipse with vertices  $(13, 3)$ ,  $(-13, 3)$  & foci  $(12, 3)$ ,  $(-12, 3)$ .

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-0)^2}{13^2} + \frac{(y-3)^2}{b^2} = 1$$

$$\frac{x^2}{169} + \frac{(y-3)^2}{b^2} = 1$$

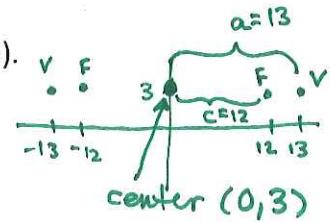
$$a^2 = b^2 + c^2$$

$$13^2 = b^2 + 12^2$$

$$169 = b^2 + 144$$

$$25 = b^2$$

$$5 = b$$



- 7) Draw the graph and write the equation of an ellipse with a major axis of 12, minor axis of 10 & center at the origin.

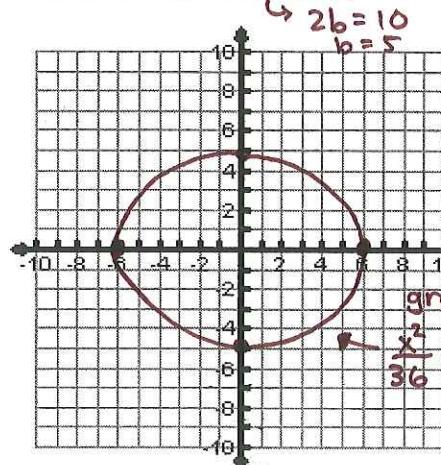
$$\text{center } (0, 0)$$

$$2a = 12 \\ a = 6$$

$$\frac{(x-0)^2}{6^2} + \frac{(y-0)^2}{5^2} = 1 \quad \text{OR}$$

$$\frac{(y-0)^2}{6^2} + \frac{(x-0)^2}{5^2} = 1$$

$$\frac{x^2}{36} + \frac{y^2}{25} = 1 \quad \text{OR} \quad \frac{y^2}{36} + \frac{x^2}{25} = 1$$



$$\text{graph of: } \frac{x^2}{36} + \frac{y^2}{25} = 1$$

8) Draw & label the graph of  $12x^2 + 4y^2 = 48$ .

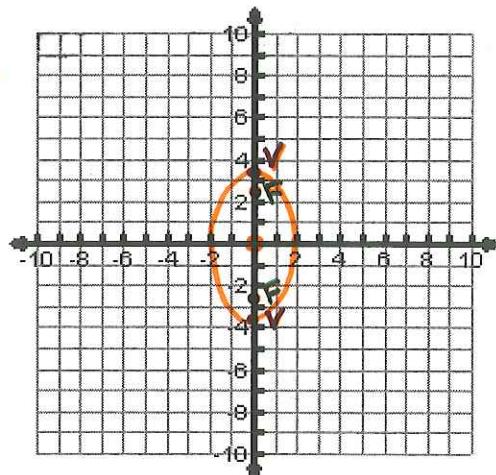
$$\begin{aligned} \text{center: } &(0,0) \\ \text{vertices: } &(0, \pm\sqrt{2}) \\ \text{foci: } &(0, \pm\sqrt{8}) \end{aligned}$$

$$\begin{aligned} &\frac{48}{4} + \frac{48}{12} = 1 \\ &\frac{x^2}{4} + \frac{y^2}{12} = 1 \\ &b^2 = 4 \quad \downarrow \quad a^2 = 12 \\ &b = 2 \quad \quad \quad a = \sqrt{12} \end{aligned}$$

$\sqrt{9} < \sqrt{12} < \sqrt{16}$   
 $3 < \sqrt{12} < 4$

9) Find the eccentricity of #8.

$$\begin{aligned} e &= \frac{c}{a} \\ &= \frac{\sqrt{8}}{\sqrt{12}} \\ &= \frac{\sqrt{8}}{\sqrt{12}} \\ e &= \sqrt{\frac{2}{3}} \end{aligned}$$



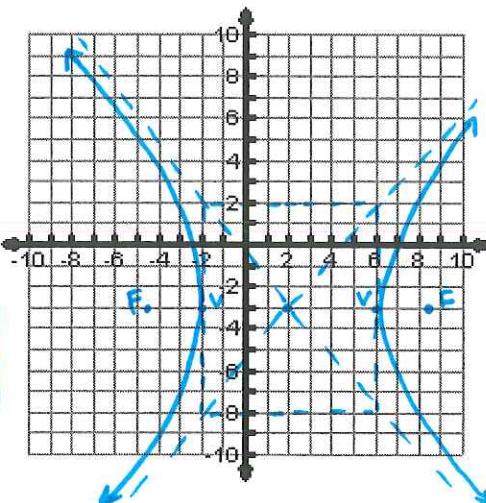
10) Draw & label the graph of  $25(x-2)^2 - 16(y+3)^2 = 400$ .

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 16 + 25 \\ c^2 &= 41 \\ c &= \sqrt{41} \end{aligned}$$

$$\begin{aligned} \sqrt{36} < \sqrt{41} < \sqrt{49} \\ 6 < \sqrt{41} < 7 \end{aligned}$$

$$\frac{(x-2)^2}{16} - \frac{(y+3)^2}{25} = 1$$

$$\begin{aligned} \text{center: } &(2, -3) \\ \text{vertices: } &(-2, -3) \text{ and } (6, -3) \\ \text{foci: } &(2+\sqrt{41}, -3) \text{ and } (2-\sqrt{41}, -3) \\ \text{asymptotes: } &y = \pm \frac{5}{4}(x-2) - 3 \end{aligned}$$



11) Find the eccentricity of #10.

$$\begin{aligned} e &= \frac{c}{a} \\ e &= \frac{\sqrt{41}}{4} \\ a^2 &= 16 \\ a &= 4 \end{aligned}$$

12) Find the vertices & foci of  $4y^2 - 6x^2 = 36$ .

$$\frac{36}{36} \quad \frac{36}{36} \quad \frac{36}{36}$$

$$\frac{y^2}{9} - \frac{x^2}{6} = 1$$

center:  $(0, 0)$

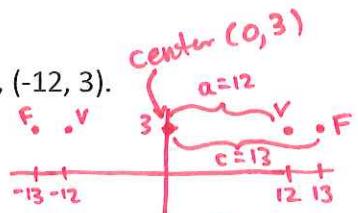
$$\begin{aligned} a^2 &= 9 \\ a &= 3 \\ c^2 &= a^2 + b^2 \\ c^2 &= 9 + 6 \\ c^2 &= 15 \\ c &= \sqrt{15} \end{aligned}$$

$$\begin{aligned} \text{vertices: } &(0, 3) \text{ and } (0, -3) \\ \text{foci: } &(0, \sqrt{15}) \text{ and } (0, -\sqrt{15}) \end{aligned}$$

- 13) Write the equation for a hyperbola with foci  $(13, 3), (-13, 3)$  & vertices  $(12, 3), (-12, 3)$ .

$$\frac{x^2}{144} - \frac{(y-3)^2}{25} = 1$$

$$\frac{(x-0)^2}{12^2} - \frac{(y-3)^2}{5^2} = 1$$



- 14) Find the equation of the asymptotes of #13.

$$y = \pm \frac{5}{12}(x-0) + 3$$

$$y = \pm \frac{5}{12}x + 3$$

$$c^2 = a^2 + b^2$$

$$13^2 = 12^2 + b^2$$

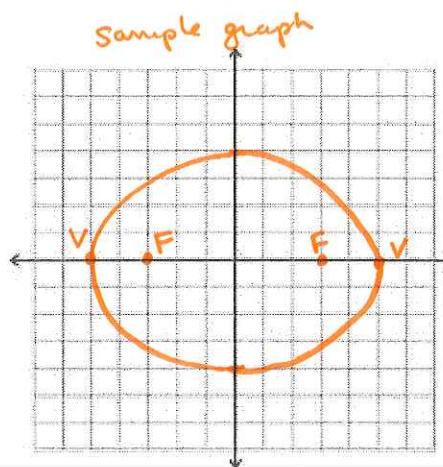
$$169 = 144 + b^2$$

$$25 = b^2$$

$$5 = b$$

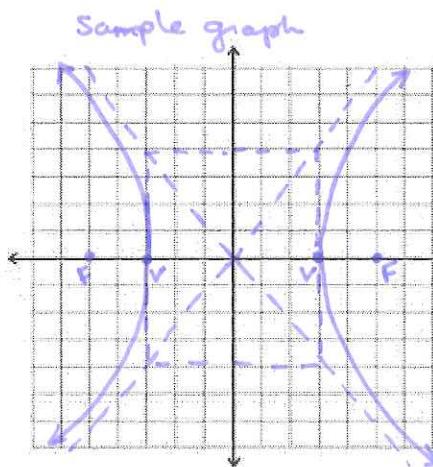
- 15) Draw and Label all parts of an ellipse.

answers vary



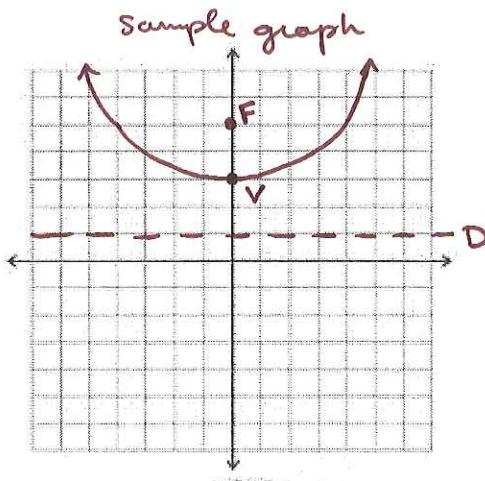
- 16) Draw and Label all parts of a hyperbola.

answers  
vary



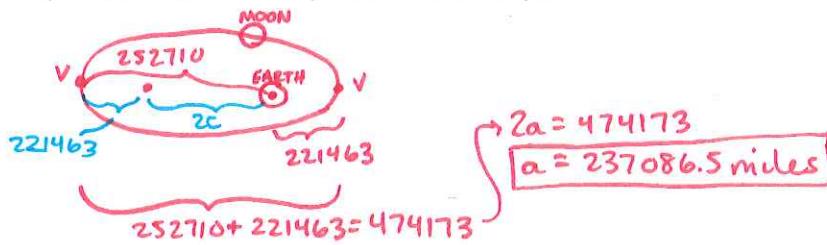
- 17) Draw and Label all parts of a parabola.

answers  
vary



$$\text{focal width} = 14\pi  
= 4(2)  
= 8$$

18) In the textbook do problem #53 on p. 653.



$$2c = 474173 - (2 \cdot 221463)$$

$$2c = 31247$$

$$c = 15623.5 \text{ miles}$$

$$a^2 + b^2 = c^2$$

$$b = 236571.162 \text{ miles}$$

$$e = c/a$$

$$e = .066$$

19) In the textbook do problem #75 on p. 699.

$$18y = x^2$$

$$\text{vertex: } (0, 0)$$

$$4p = 18$$

$$p = \frac{9}{2}$$

$$p = 4.5$$

$$\boxed{\text{focus: } (0, 4.5)}$$

20) In the textbook, do problem #53 on p. 641.

$$\begin{aligned} \text{vertex: } & (0, 2) \\ \text{pt: } & (-6, -4) \end{aligned}$$

*sub in vertex*

$$(y-2)^2 = 4p(x-h)$$

$$(y-2)^2 = 4p(x-0)$$

$$(y-2)^2 = 4px$$

*sub in pt to solve for p*

$$(-4-2)^2 = 4p(-6)$$

$$(-6)^2 = -24p$$

$$36 = -24p$$

$$-\frac{3}{2} = p$$

$$\begin{aligned} (y-2)^2 &= 4(-\frac{3}{2})x \\ (y-2)^2 &= -6x \end{aligned}$$

21) In the textbook, do problem #40 on p. 698.

Parabola

$$\begin{aligned} \text{vertex: } & (1, -2) \\ \text{opens left} & \rightarrow (y-k)^2 = 4p(x-h) \\ \therefore, p < 0 & (y-(-2))^2 = 4p(x-1) \\ & (y+2)^2 = 4p(x-1) \end{aligned}$$

$$\text{focal length} = 2$$

$$\text{so, } p = -2$$

$$\begin{aligned} & \rightarrow (y+2)^2 = 4(-2)(x-1) \\ & \boxed{(y+2)^2 = -8(x-1)} \end{aligned}$$

22) Find the vertex, focus, directrix, and focal width of  $(x+2)^2 = -4(y-1)$ .

$$\boxed{\text{vertex: } (-2, 1)}$$

$$\text{focus: } (-2, 0)$$

$$\text{directrix: } y = 2$$

$$\text{focal width: } 4$$

$$-4 = 4p$$

$$-1 = p$$

