Name: Key Period:______

Honors Advanced Algebra

Key Concept 4 Review

Rewrite using the properties of exponents.

1) $n^{2/9} \cdot n^{5/9} \cdot n^{1/9}$

$= n^{\frac{2}{9}+\frac{5}{9}+\frac{1}{9}}$

$= n^{\frac{8}{9}}$

2) $a^{5/4} \cdot a^{3/10} \cdot a^{2/5}$

→ NEED COMMON DENOMINATOR

$\frac{5\cdot 5}{5\cdot 4} + \frac{2\cdot 3}{2\cdot 10} + \frac{4\cdot 2}{4\cdot 5}$

$= \frac{25}{20} + \frac{6}{20} + \frac{8}{20}$

$= \frac{39}{20}$

$\therefore a^{\frac{39}{20}}$

3) $\left(\dfrac{8w^{12}}{343}\right)^{1/3}$

Distribute rational exponent

$\left(\dfrac{2^3 w^{12}}{7^3}\right)^{\frac{1}{3}}$

$= \dfrac{2^{3\cdot\frac{1}{3}} w^{12\cdot\frac{1}{3}}}{7^{3\cdot\frac{1}{3}}}$

$= \dfrac{2w^4}{7}$

Switch forms. (radical <—> exponential)

4) $\sqrt[2]{x^3}$

$= x^{\frac{3}{2}}$

5) $18^{\frac{2}{3}}$

$= \sqrt[3]{(18)^2}$

Simplify.

6) $\sqrt[2]{128x^{14}y^9}$

$= \sqrt[2]{2^7 x^{14} \cdot y^8 \cdot y}$

$= \sqrt[2]{2^6 \cdot 2 \cdot x^{14} \cdot y^8 \cdot y}$

$= 2^3 |x^7| y^4 \sqrt[2]{2y}$

$= 8|x^7|y^4\sqrt{2y}$

7) $\sqrt[4]{16a^{20}b^{14}}$

$= \sqrt[4]{2^4 a^{20} b^{12} \cdot b^2}$

$= 2|a^5||b^3|\sqrt[4]{b^2}$

What rational exponent must equal "y" for each equation to be true.

8) $\sqrt[3]{\sqrt[2]{x^7}} = x^y$

$\sqrt[3]{x^{\frac{7}{2}}} = x^y$

$\left(\left(x^{\frac{7}{2}}\right)^{\frac{1}{3}}\right) = x^y$

$x^{\frac{7}{6}} = x^y$

$\therefore y = \frac{7}{6}$

9) $\sqrt[2]{\sqrt[3]{(x-1)^{24}}} = (x-1)^y$

$= \sqrt[2]{(x-1)^{\frac{24}{3}}} = (x-1)^y$

$= \sqrt[2]{(x-1)^8} = (x-1)^y$

$= (x-1)^{\frac{8}{2}} = (x-1)^y$

$\therefore y = \frac{8}{2} = 4$

Simplify. Is the answer rational or irrational?

10) $-4\sqrt{10} - 13\sqrt{10}$

$= -17\sqrt{10}$

Irrational

11) $-3\sqrt[2]{2} * 4\sqrt[2]{32}$

$-3 \cdot 4 \cdot \sqrt[2]{2 \cdot 2^5}$

$= -12 \cdot \sqrt[2]{2^6}$

$= -12 \cdot 2^3$

$= -12 \cdot 8$

$= -96$ Rational

Solve the equation. Check for extraneous solutions.

12) $\left(\sqrt[4]{3x^2-48}\right)^4 = \left(\sqrt[4]{2x^2+2x}\right)^4$

Take it to 4th power both sides

$3x^2 - 48 = 2x^2 + 2x$

$-2x^2 \quad -2x^2$

$x^2 - 48 = 2x$

$-2x \quad -2x$

$x^2 - 2x - 48 = 0$

$(x-8)(x+6) = 0$

$x = 8$ or $x = -6$

Both solutions

CHECK: $x = 8$

$\sqrt[4]{3(8)^2 - 48} = \sqrt[4]{2(8)^2 + 2(8)}$

$\sqrt[4]{144} = \sqrt[4]{144}$ ✓

CHECK: $x = -6$

$\sqrt[4]{3(-6)^2 - 48} = \sqrt[4]{2(-6)^2 + 2(-6)}$

$\sqrt[4]{60} = \sqrt[4]{60}$ ✓

13) $(x+1)^{\frac{3}{2}} - 2 = 25$

$+2 \quad +2$

$\left((x+1)^{\frac{3}{2}}\right)^{\frac{2}{3}} = (27)^{\frac{2}{3}}$

$x + 1 = 27^{\frac{2}{3}}$

$x + 1 = (3^3)^{\frac{2}{3}}$

$x + 1 = 3^2$

$x + 1 = 9$

$x = 8$

CHECK:

$(8+1)^{\frac{3}{2}} - 2 = 25$

$9^{\frac{3}{2}} - 2 = 25$

$(3^2)^{\frac{3}{2}} - 2 = 25$

$3^3 - 2 = 25$

$27 - 2 = 25$

$25 = 25$ ✓

CHECK: $x=-9$

$-9-8 \neq \sqrt{-25\cdot -9} - 2$

$-17 \neq \sqrt{225} - 2$

$-17 \neq 15-2$

$-17 \neq 13$ FALSE

Solve the equation. Check for extraneous solutions.

14) $2\cdot\sqrt[3]{4x+16}+21=29$

$-21 \quad -21$

$\frac{2\cdot\sqrt[3]{4x+16}}{2} = \frac{8}{2}$

$\left(\sqrt[3]{4x+16}\right)^3 = (4)^3$ "cube both sides"

$4x+16=64$

$-16 \quad -16$

$\frac{4x}{4} = \frac{48}{4}$

$x=12$

CHECK:

$2\cdot\sqrt[3]{4(12)+16}+21=29$

$2\cdot\sqrt[3]{64}+21=29$

$2\cdot\sqrt[3]{4^3}+21=29$

$2\cdot 4+21=29$

$8+21=29$

$29=29$ ✓

15) $x-8=\sqrt{-25x}-2$

$+2 \quad +2$

$(x-6)^2 = (\sqrt{-25x})^2$ "square both sides"

$(x-6)(x-6) = -25x$

	x	-6
x	x^2	$-6x$
-6	$-6x$	36

$x^2-12x+36 = -25x$

$+25x \quad +25x$

$x^2+13x+36=0$

$(x+9)(x+4)=0$

$x=-9$ or $x=-4$

$\therefore$ NO SOLUTION

CHECK: $x=-4$

$-4-8 \neq \sqrt{-25\cdot -4} - 2$

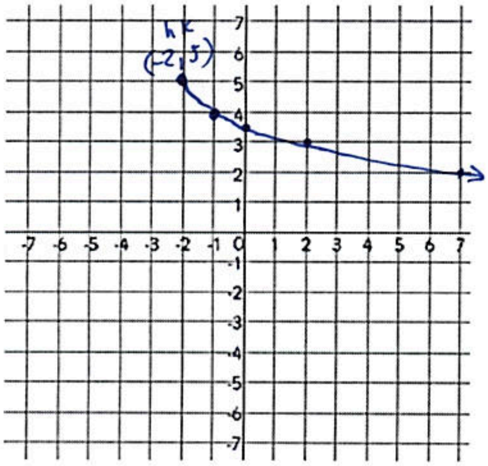
$-12 \neq \sqrt{100} - 2$

$-12 \neq 10-2$

$-12 \neq 8$ FALSE

Graph the function and state the domain and range.

16) $a(x) = -\sqrt{x+2}+5$

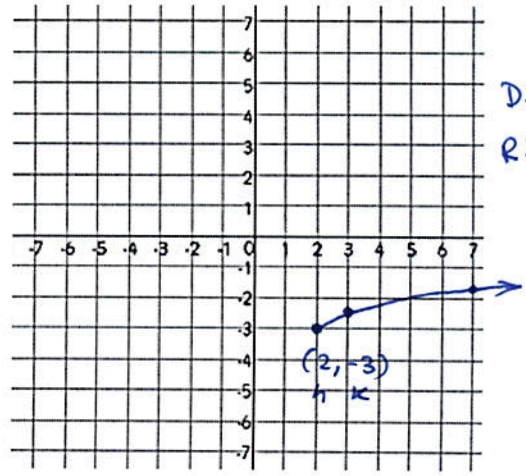


Graph using Nspire & use Table feature to plot points

D: $[-2,\infty)$

R: $(-\infty, 5]$

17) $b(x) = \frac{1}{2}\sqrt{x-2}-3$



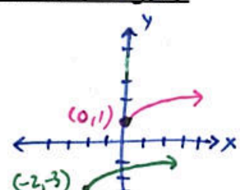
D: $[2,\infty)$

R: $[-3,\infty)$

Describe the transformation from f(x) to g(x).

18) $f(x)=\sqrt{x+2}-3$ h k $(-2,-3)$

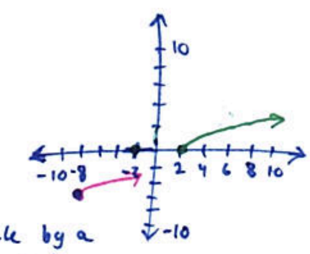
$g(x)=\sqrt{x}+1$ h k $(0,1)$



$\therefore$ Shift 2 units right and 4 units up

19) $f(x)=\sqrt{x-2}$ h k $(2,0)$

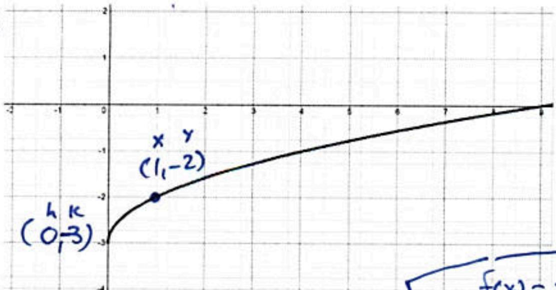
$g(x)=\frac{1}{3}\sqrt{x+8}-5$ h k $(-8,-5)$



$\therefore$ Shift 10 units left and 5 units down and vertically shrink by a factor of $\frac{1}{3}$

Determine the function that matches each graph. State the domain and range too.

20)



$\therefore f(x)=\sqrt{x}-3$

D: $[0,\infty)$

R: $[-3,\infty)$

$y=a\sqrt{x-h}+k$

$-2=a\sqrt{1-0}-3$

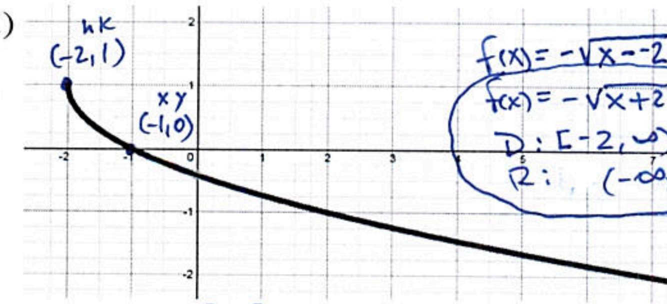
$-2=a\cdot\sqrt{1}-3$

$+3 \quad +3$

$1=a\cdot 1$

$1=a$

21)



$f(x)=-\sqrt{x--2}+1$

$f(x)=-\sqrt{x+2}+1$

D: $[-2,\infty)$

R: $(-\infty,1]$

$y=a\sqrt{x-h}+k$

$0=a\sqrt{-1-(-2)}+1$

$0=a\sqrt{-1+2}+1$

$0=a\cdot\sqrt{1}+1$

$-1=a\cdot 1$

$-1=a$

22) The population of a small town can be modeled by the function $P(x) = 15{,}750\sqrt[3]{x - 1999}$, where 'x' is the year and 2000 is the first year that applies to this model. Answer these questions based upon the model:

A) Rewrite the model $P(x) = 15{,}750\sqrt[3]{x - 1999}$ in rational exponent form.

$$P(x) = 15{,}750(x - 1999)^{\frac{1}{3}}$$

B) State the domain and range, in interval notation, of the model.

D: [2000, ∞)

R: [15,750, ∞)

$$P(2000) = 15{,}750\sqrt[3]{2000-1999} = 15{,}750 \cdot \sqrt[3]{1} = 15{,}750 \cdot 1 = \boxed{15{,}750}$$

C) Based on the model, what will be the population of the small town in 2030?

$$P(2030) = 15{,}750 \cdot \sqrt[3]{2030-1999} = \underbrace{15{,}750 \cdot \sqrt[3]{31}}_{\text{evaluate using calculator}} \approx 49476.745$$

∴ The pop. of small town in 2030 is ≈ 49,476.

D) In what year would the population of the small town reach 50,000?

Solve Algebraically: — OR — Solve Graphically:

$$P(x) = 15{,}750\sqrt[3]{x-1999}$$

Reduce $\left\{ \dfrac{50{,}000}{15{,}750} = \dfrac{15{,}750\sqrt[3]{x-1999}}{15{,}750} \right.$

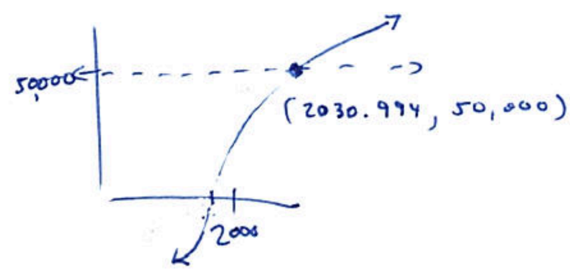
$$\left(\frac{200}{63}\right)^3 = \left(\sqrt[3]{x-1999}\right)^3$$ Cube both sides

$$\left(\frac{200}{63}\right)^3 = x - 1999$$
$$+1999 \quad +1999$$

$$\underbrace{\left(\frac{200}{63}\right)^3 + 1999}_{\text{evaluate on calc.}} = x$$

$$2030.994 \approx x$$

The pop. will be 50,000 in 2031, the year 2031.



The pop. will be 50,000 in 2031.