Names: ___________________________________

Swinging Ellipses: Plotting an Ellipse

Any ellipse centered at the origin can be expressed in the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where $\pm a$ and $\pm b$ represent the *x*- and *y*-intercepts of the ellipse.

To graph an ellipse on Logger *Pro*, the expression above must first be solved for *y* to obtain

$$y = \pm b\sqrt{1 - \frac{x^2}{a^2}}$$

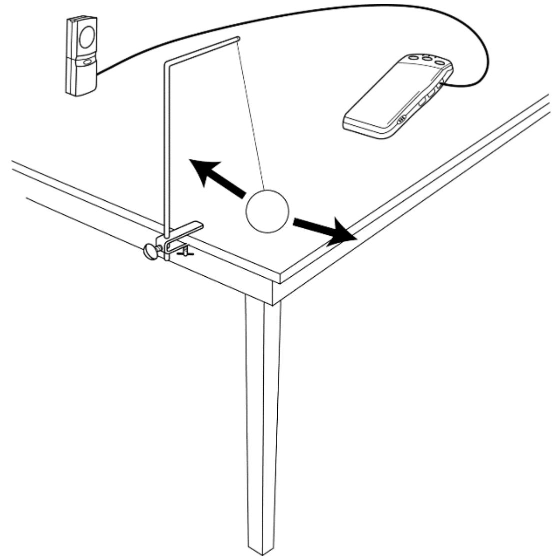
This equation is entered into the computer in two parts, one expression for the positive part (upper half of the ellipse) and one for the negative part (lower half of the ellipse).

In this activity, you will use the Motion Detector to record the position and velocity of a swinging pendulum. You'll find that the plot of velocity *vs*. position is elliptical, and that you can model it with the standard equation of an ellipse.

GROUP ROLES

- Computer User (**CU**) – runs the logger pro software
- Supervisor/Data Recorder (**S**) – ensures accurate data is collected and records results
- Motion Device Operator (**M**) – connects and operates the motion detector
- Pendulum Setup & Operator (**P**) – responsible for setting up and swinging the pendulum

ALL GROUP MEMBERS MUST EQUALLY CONTRIBUTE TO THE ANALYSIS PART OF THE LAB



OBJECTIVES

- Record position and velocity *vs*. time data for a swinging pendulum.
- Plot data as a velocity *vs*. position phase plot.
- Determine an ellipse that fits the phase plot.

MATERIALS

computer	pendulum weight
Vernier computer interface	string
Logger *Pro*	meter stick
Motion Detector	

PROCEDURE

1. Connect the pendulum weight to a 60 cm long string and suspend it from a support. The bottom of the weight should be no more than 1 cm above the flat surface. (**P, S**)

2. Place the Motion Detector 30 cm from the pendulum weight center. (**M, S**)

3. Connect the Motion Detector to the DIG/SONIC 1 channel of the interface. (**M**)

4. Launch Logger *Pro* and open the file “20 Swinging Ellipses” from the *Real-World Math w Vernier* folder. (**CU**)

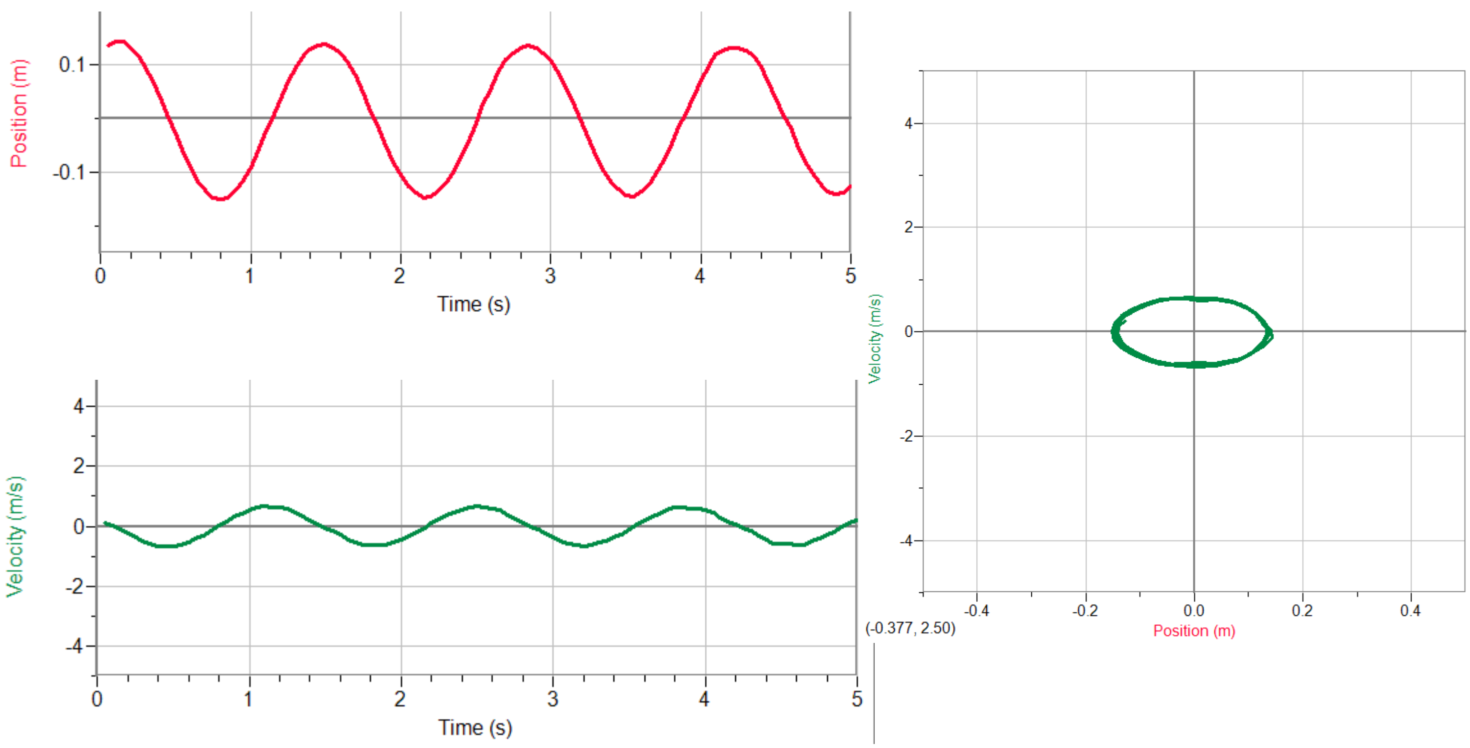
5. So that the position data are measured from the pendulum equilibrium position, you need to zero the Motion Detector. Do this by allowing the weight to hang straight downward, with no motion. Click Ø to zero the Motion Detector. (**S**, **CU**, **M**)

6. Practice swinging the weight by pulling it back about 15 cm, and then releasing it so that the weight swings in a line directly away from the Motion Detector. (**P**, **M**)

7. With the weight swinging properly, click ▶ Collect to begin data collection. Data collection will run for five seconds. (**CU**, **M**, **P**)

8. Examine the position and velocity *vs*. time graphs. Each time graph should be sinusoidal and centered on zero. Check with your instructor if you are not sure whether you need to repeat the data collection. Return to Step 4 to repeat data collection. (**S**, **CU**)

Your graphs should look similar to these:



ANALYSIS

NOTE: To replay the data collection, select Replay from the Analyze menu.

1. While the position and velocity *vs*. time graphs are sinusoidal in shape, a graph of velocity *vs*. distance is elliptical. The graph of velocity *vs*. distance is known as a *phase plot*.
Your phase plot has multiple *x*- and *y*-intercepts. To determine the parameters for your ellipse model, you need to determine some average values of these intercepts. One way to do this is by choosing a typical intercept among those available.
Click the Examine button, , and move the cursor along your velocity *vs.* position plot and record a typical positive *x*-intercept. Record the value in your data table as the first *x*-intercept, rounded to 0.001. Then, determine a typical negative *x*-intercept. Do the same for positive and negative *y*-intercepts. Record these values in your data table.

First *x*-intercept	Second *x*-intercept

First *y*-intercept	Second *y*-intercept

2. Find the average of the absolute values of the *x*-intercepts and record this value as a in your data table. Find the average of the absolute values of the *y*-intercepts and record this value as b in your data table

Absolute Value of First *x*-intercept	Absolute Value of Second *x*-intercept	***a***

Absolute Value of First *y*-intercept	Absolute Value of Second *y*-intercept	***b***

3. What do these values of a and b represent in the ellipse?

4. Turn off the analysis mode by clicking the Examine button, , again.

5. Enter the two halves of your ellipse equation for graphing using the manual curve fit option of Logger *Pro (directions below)*. Logger *Pro* uses the notation sqrt( ) for square root. One half is the positive function $b\sqrt{1-\frac{x^2}{a^2}}$ while the other half has a minus sign preceding it.

a. Click once on your phase plot to make it the active graph.

b. Select Curve Fit from the Analyze menu. Choose the Manual Fit Type option.

c. Click the Define Function button.

d. Enter –b*sqrt(1 – x^2/a^2) in the Define Custom Function field. Click OK.

e. Enter your values of a and b in the Parameters fields.

f. Click OK to see half of your ellipse plotted on the phase plot.

g. Repeat for the positive half of the ellipse. [Enter b*sqrt(1 – x^2/a^2) in the Define Custom Function field]

ANALYSIS QUESTIONS

1. How well do the curves of the model ellipses and the data points agree?

2. **a)** At what location (or place) in the pendulum's swing is its velocity the largest?

b) At what location (or place) in the pendulum's swing is the velocity zero?

c) How are these values related to the x- and y-intercepts?

3. **a)** How would the data change if the amplitude (the largest distance from the middle position) of the pendulum's swing were increased?

b) Would this change affect the values of the constants a and b? If so, how?

4. What do the vertices represent in context to the data collected? How about the co-vertices? (Select Replay from the Analyze menu to replay the data collection)

5. What is the equation of this ellipse in standard form?

EXTENSION

Show how the algebraic expression $y = \pm b\sqrt{1 - \frac{x^2}{a^2}}$ used in this activity can be derived from the ellipse equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.