Honors Pre-Calculus Limits DATE: ________________

Unit 9 Review

1. Figure 5.2 shows the graph of $g(x)$. Find $\lim_{x\to 3} g(x)$

$\boxed{\lim_{x\to 3} g(x) = 3}$

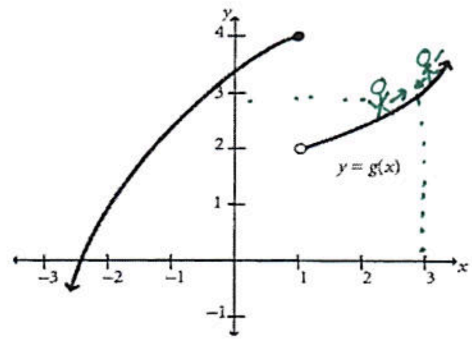


Figure 5.2

2. Figure 5.2 shows the graph of $g(x)$. Find $g(1)$ ← y-value of g(x) @ x = 1

$\boxed{g(1) = 4}$

3. Graph $f(x) = sin\frac{\pi}{x}$. Use the graph to help you find $\lim_{x\to 0}\sin\frac{\pi}{x}$

$\boxed{\lim_{x\to 0} \sin\frac{\pi}{x} = \text{DNE}}$

4. Use the graph from #~~5~~ #3 to help you find $\lim_{x\to\infty}\sin\frac{\pi}{x}$

$\boxed{\lim_{x\to\infty} \sin\frac{\pi}{x} = 0}$

5. Find $\lim_{x\to 0}\sin\frac{x}{x-1}$

$\boxed{\lim_{x\to 0} \sin\frac{x}{x-1} = 0}$

6. Find $\lim_{x\to\infty}\sin\frac{x}{x-1}$

$\lim_{x\to\infty} \sin\left(\frac{x}{x-1}\right) = \sin 1 = \boxed{.841}$

deg Num = deg Denom,
∴ Look @ coefficients ☺

7. Find $\lim_{x\to 0}\sin(\sqrt{x}-2)$

$\lim_{x\to 0^-} \sin(\sqrt{x}-2)$ DNE
$\lim_{x\to 0^+} \sin(\sqrt{x}-2) = -2$
$\rightarrow \lim_{x\to 0} \sin(\sqrt{x}-2)$ $\boxed{\text{DNE}}$ b/c $\lim_{x\to 0^-} \sin(\sqrt{x}-2) \neq \lim_{x\to 0^+} \sin(\sqrt{x}-2)$

8. Find $\lim_{x\to 0}\frac{x^2+x-12}{x+4} = \frac{0^2+0-12}{0+4} = \frac{-12}{4} = \boxed{-3}$

9. Find $\lim_{x\to -4}\frac{x^2+x-12}{x+4} = \lim_{x\to -4}\frac{(x+4)(x-3)}{x+4} = \lim_{x\to -4}(x-3) = -4-3 = \boxed{-7}$

10. Find $\lim_{x\to 5} f(x)$, where $f(x) = \begin{cases} x+2, x<5 \\ 2-2x, x \geq 5 \end{cases}$

$$\left.\begin{aligned} &\lim_{x\to 5^-}(x+2) = 5+2 = 7 \\ &\lim_{x\to 5^+}(2-2x) = 2-2(5) = -8 \end{aligned}\right\} \quad \lim_{x\to 5} f(x)\ \boxed{DNE}\ \text{b/c}\ \lim_{x\to 5^-} f(x) \neq \lim_{x\to 5^+} f(x)$$

11. If $\lim_{x\to 2} f(x) = -4$ and $\lim_{x\to 2} g(x) = 9$ Find $\lim_{x\to 2}\frac{\sqrt{g(x)}}{(f(x))^2} = \frac{\sqrt{\lim_{x\to 2} g(x)}}{\left(\lim_{x\to 2} f(x)\right)^2} = \frac{\sqrt{9}}{(-4)^2} = \boxed{\frac{3}{16}}$

12. Find $\lim_{x\to -5^+}\frac{2|x+5|}{x+5} = \boxed{2}$

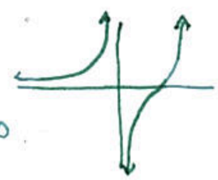
13. Find $\lim_{x\to 0}\frac{\sin 3x}{x} = \lim_{x\to 0}\frac{3\cdot\sin 3x}{3\cdot x} = \lim_{x\to 0} 3\left(\frac{\sin 3x}{3x}\right)$

$= 3\cdot 1$

$= \boxed{3}$

14. Find $\lim_{x\to 0}\frac{e^x - 3}{x}$ $\boxed{DNE}$

$\lim_{x\to 0^-}\frac{e^x-3}{x} = \infty \neq \lim_{x\to 0^+}\frac{e^x-3}{x} = -\infty$



15. Find $\lim_{x\to 4}\frac{\frac{1}{4}-\frac{1}{x}}{x-4} = \lim_{x\to 4}\frac{\frac{x}{x}\cdot\frac{1}{4}-\frac{1}{x}\cdot\frac{4}{4}}{x-4} = \lim_{x\to 4}\frac{\frac{x-4}{4x}}{\frac{x-4}{1}} = \lim_{x\to 4}\frac{\cancel{x-4}}{4x}\cdot\frac{1}{\cancel{x-4}} = \lim_{x\to 4}\frac{1}{4x} = \boxed{\frac{1}{16}}$

16. Find $\lim_{x\to\infty}\frac{7x^2}{2x^2+7} = \frac{7}{2}$

deg Num = deg Denom
∴, look @ leading coefficients ...

17. Find $\lim_{x\to\infty}\frac{x+3}{x^2-9} = 0$

deg Num < deg Denom
∴ $\lim_{x\to\infty} f(x) = 0$ ...

18. If $f(x) = 1 - \frac{4}{x}$, then $f(x+h) = 1 - \frac{4}{x+h}$

19. If $f(x) = 3x^2 - 2x + 1$, then $\frac{f(x+h)-f(x)}{h} = \frac{3(x+h)^2 - 2(x+h) + 1 - (3x^2 - 2x + 1)}{h}$

$$= \frac{3(x^2 + 2xh + h^2) - 2x - 2h + 1 - 3x^2 + 2x - 1}{h}$$

$$= \frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 1 - 3x^2 + 2x - 1}{h}$$

$$= \frac{6xh + 3h^2 - 2h}{h} = \frac{h(6x + 3h - 2)}{h} = \boxed{6x + 3h - 2}$$

20. Find $\lim_{h \to 0} \frac{4x^2h + 2xh^2 + h^3}{h}$

$$= \lim_{h \to 0} \frac{h(4x^2 + 2xh + h^2)}{h}$$

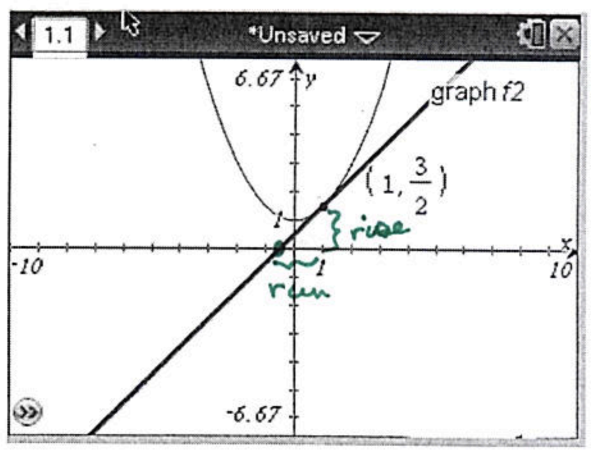
$$= \lim_{h \to 0} (4x^2 + 2xh + h^2) = \boxed{4x^2}$$

21. Find $\lim_{h \to 0} \frac{\sqrt{2x+h} - \sqrt{2x}}{h} = \lim_{h \to 0} \frac{\sqrt{2x+h} - \sqrt{2x}}{h} \cdot \frac{\sqrt{2x+h} + \sqrt{2x}}{\sqrt{2x+h} + \sqrt{2x}} = \lim_{h \to 0} \frac{2x + h - 2x}{h(\sqrt{2x+h} + \sqrt{2x})}$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{2x+h} + \sqrt{2x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{2x+h} + \sqrt{2x}} = \frac{1}{\sqrt{2x} + \sqrt{2x}} = \boxed{\frac{1}{2\sqrt{2x}}}$$

22. In Figure 5, approximate the slope of the curve at $x = 1$



$$\frac{\text{rise}}{\text{run}} = \frac{3/2}{1.5} = \frac{3/2}{3/2}$$

$$= \boxed{1}$$

23. Find $\lim_{h \to -12} \frac{h^2 - 144}{h + 12}$ numerically. Fill in the given table.

h	-12.1	-12.01	-12.001	-12	-11.999	-11.99	-11.9
f (h)	-24.1	-24.01	-24.001	?	-23.999	-23.99	-23.9

$$\lim_{h \to -12} \frac{(h-12)(h+12)}{h+12} = \lim_{x \to -12} (h - 12) = -24$$