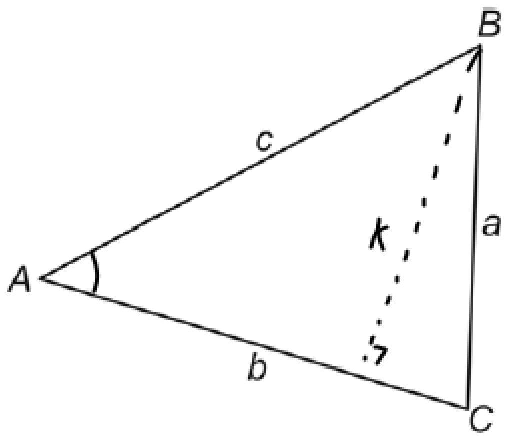
The Law of Sines

NAME ______________________________

> Right triangle trigonometry can be used to solve problems involving right triangles. However, many interesting problems involve non-right triangles. In this lesson, you will use right triangle trigonometry to develop the *Law of Sines*. The law of sines is important because it can be used to solve problems involving non-right triangles as well as right triangles.

Consider oblique ΔABC shown to the right.



1. Sketch an altitude from vertex B.

2. Label the altitude k.

3. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles, and $\angle C$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin A$, and one involving $\sin C$.

   $\sin A = \frac{K}{C}$ $\qquad\qquad$ $\sin C = \frac{K}{a}$

4. Notice that each of the equations in Question 3 involves k. (Why does this happen?) Solve each equation for k. They share the same altitude. $K = C \cdot \sin A$, $K = a \cdot \sin C$

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$\sin A =$ ______ $\sin C =$ ______

4. Notice that each of the equations in Question 3 involves k. (Why does this happen?) Solve each equation for k.

$c \sin A = k$, $a \cdot \sin C = k$

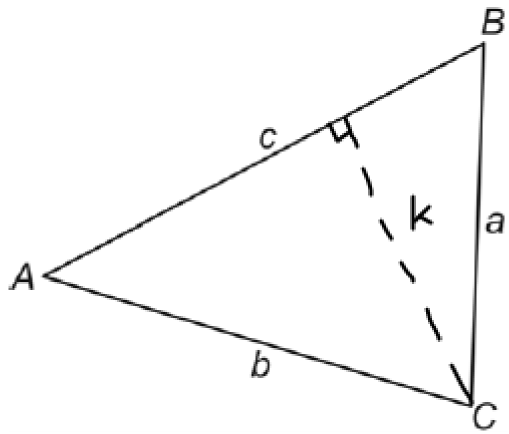
5. Since both equations in Question 4 are equal to k, they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.

Transitive property: New → $c \cdot \sin A = a \cdot \sin C$

6. Notice that the equation in Question 5 no longer involves k. (Why not?) Write an equation equivalent to the equation in Question 5, regrouping a with $\sin A$ and c with $\sin C$.

$$\frac{\cancel{c} \cdot \sin A}{a \cdot \cancel{c}} = \frac{\cancel{a} \cdot \sin c}{\cancel{a} \cdot c} \Rightarrow \boxed{\frac{\sin A}{a} = \frac{\sin c}{c}} \quad *$$

Again, consider oblique ΔABC.



7. This time, sketch an altitude from vertex C.

8. Label the altitude k.

9. The altitude creates two right triangles inside ΔABC. Notice that $\angle A$ is contained in one of the right triangles and $\angle B$ is contained in the other. Using right triangle trigonometry, write two equations, one involving $\sin A$ and one involving $\sin B$.

$\sin A = \dfrac{K}{b}$ $\qquad\qquad$ $\sin B = \dfrac{K}{a}$

10. Notice that each of the equations in Question 9 involves k. (Why does this happen?) Solve each equation for k.

$b \cdot \sin A = K$, $\qquad$ $a \cdot \sin B = K$

11. Since both equations in Question 10 are equal to k, they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.

$b \cdot \sin A = a \cdot \sin B$

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11. Since both equations in Question 10 are equal to k, they can be set equal to each other. (Why is this possible?) Set the equations equal to each other to form a new equation.

12. Notice that the equation in Question 11 no longer involves k. (Why not?) Write an equation equivalent to the equation in Question 11, regrouping a with sin A and b with sin B.

$$b \cdot \sin A = a \cdot \sin B \Rightarrow \boxed{\frac{\sin A}{a} = \frac{\sin B}{b}} \ **$$

13. Use the equations in Question 6 and Question 12 to write a third equation involving b, c, sin B, and sin C.

If $\frac{\sin C}{c} = \frac{\sin A}{a}$ and $\frac{\sin A}{a} = \frac{\sin B}{b}$, then $\frac{\sin C}{c} = \frac{\sin B}{b}$ (trans. prop.)

Together, the equations in Questions 6, 12, and 13 form the *Law of Sines*. The law of sines is important, because it can be used to solve problems involving both right and non-right triangles, because it involves only the sides and angles of a triangle.

$$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \text{ or } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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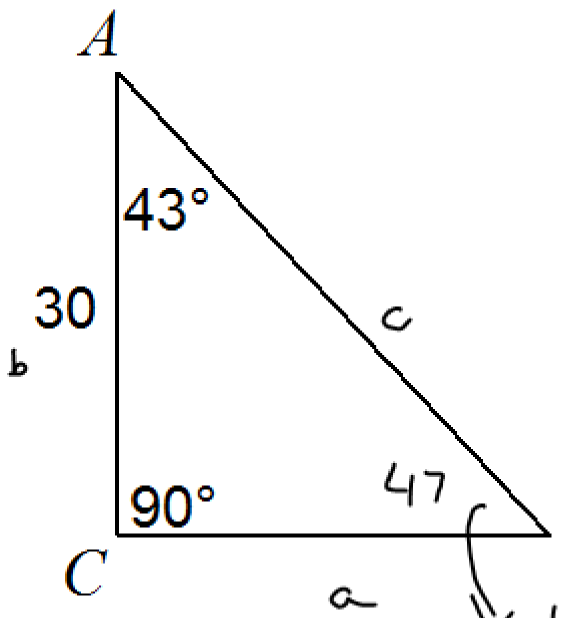
Question number Directions Lines Reveal answer Zoom: Jump 1-up Close

Find each measurement indicated. Round your answers to the nearest tenth.

1) Find BC



$180 - 90 - 43$

$$\tan 43° = \frac{x}{30}$$

$$\underbrace{30 \cdot \tan 43°}_{} = x$$

$$28.0 \approx x$$

LAW OF SINES
- works on any △ that exists

SOH CAH TOA
- works only on right △s

LAW SINES

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

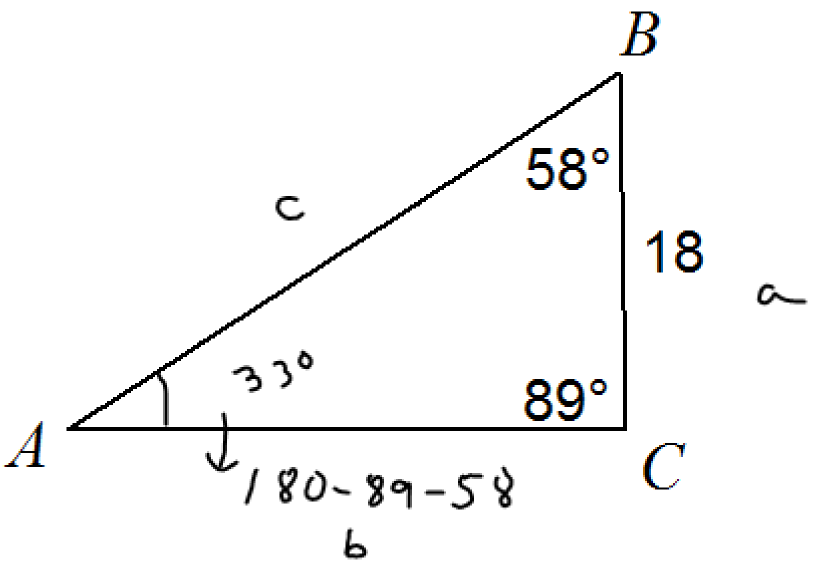
$$\frac{\sin 43°}{a} = \frac{\sin 47°}{30}$$

$$\underbrace{\frac{30 \cdot \sin 43°}{\sin 47°}}_{} = \frac{a \cdot \cancel{\sin 47°}}{\cancel{\sin 47°}}$$

$$28.0 \approx a$$

Find each measurement indicated. Round your answers to the nearest tenth.

4) Find AB



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 33}{18} = \frac{\sin 89}{c}$$

$$\frac{c \cdot \sin \cancel{33}}{\sin \cancel{33}} = \underbrace{\frac{18 \cdot \sin 89}{\sin 33}}$$

$$c \approx 33.0$$

Infinite Algebra 2

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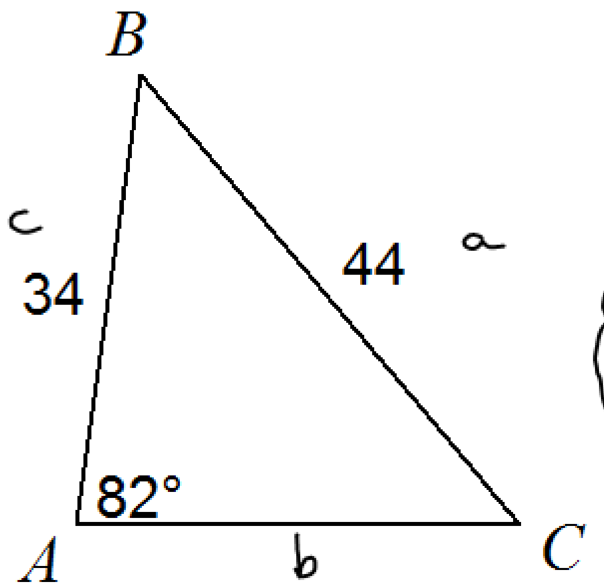
Find each measurement indicated. Round your answers to the nearest tenth.

5) Find $m\angle C$



Use $\sin^{-1}$ to find angle

$$\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin 82}{44} = \frac{\sin C}{34}$$

$$\frac{34 \cdot \sin 82}{44} = \frac{44 \cdot \sin C}{44}$$

$$\frac{34 \cdot \sin 82}{44} = \sin C$$

$$\sin^{-1} \cancel{\sin} C = \underbrace{\sin^{-1}\left(\frac{34 \cdot \sin 82}{44}\right)}$$

$$C \approx 49.9^\circ$$