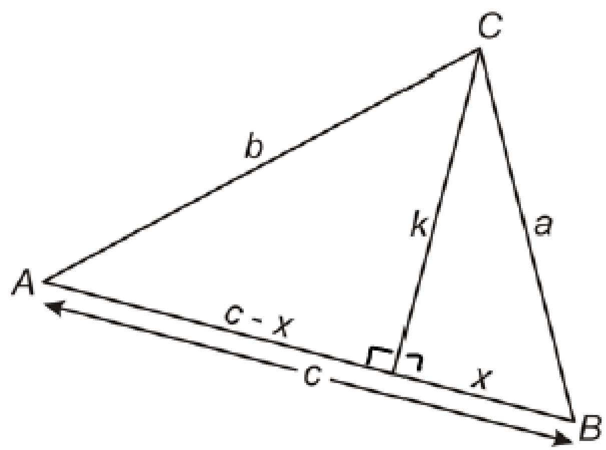
The Law of Cosines

NAME Key

> The law of sines can be used to determine the measures of missing angles and sides of triangles when the measures of two angles and a side (AAS or ASA) or the measures of two sides and a non-included angle (SSA) are known. However, the law of sines cannot be used to determine the measures of missing angles and sides of triangles when the measures of two sides and an included angle (SAS) or the measures of three sides (SSS) are known. Since the law of sines can only be used in certain situations, we need to develop another method to address the other possible cases. This new method is called the **Law of Cosines.**

To develop the **law of cosines**, begin with $\triangle ABC$. From vertex C, altitude k is drawn and separates side c into segments x and $c-x$. (Why can the segments be represented in this way?)

Subtraction Property



1. The altitude separates $\triangle ABC$ into two right triangles. Use the Pythagorean theorem to write two equations,

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3. Since both of the equations in Question 2 are equal to k^2, they can be set equal to each other. (Why is this true?) Set the equations equal to each other to form a new equation.

By transitive property: $a^2 - x^2 = b^2 - (c-x)^2$.

4. Notice that the equation in Question 3 involves x. However, x is not a side of ΔABC. As a result, we will attempt to rewrite the equation in Question 3 so that it does not include x. Begin by expanding the quantity $(c-x)^2$.

$$a^2 - x^2 = b^2 - [(c-x)(c-x)]$$
$$a^2 - x^2 = b^2 - (c^2 - 2cx + x^2)$$
$$a^2 - x^2 = b^2 - c^2 + 2cx - x^2$$

$(c-x)(c-x)$
$c^2 - cx - cx + x^2$
$c^2 - 2cx + x^2$

5. Solve the equation in Question 4 for b^2.

$$a^2 - \cancel{x^2} = b^2 - c^2 + 2cx - \cancel{x^2}$$
$$+\cancel{x^2} \qquad\qquad +\cancel{x^2}$$

$$-2cx + a^2 + c^2 = b^2 - \cancel{c^2} + \cancel{2cx}$$
$$+\cancel{c^2} - \cancel{2cx}$$

$$a^2 + c^2 - 2cx = b^2$$

6. The equation in Question 5 still involves x. To eliminate x from the equation, we will attempt to substitute an equivalent expression for x. Write an equation involving both $\cos B$ and x. (Why use $\cos B$?)

$$a^2 + c^2 - 2cx = b^2$$

$$\cos B = \frac{x}{a}$$

Since cos B has x and you want it out.

7. Solve the equation from Question 6 for x. (What value for x?)

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7. Solve the equation from Question 6 for x. (Why solve for x?)

$$\cos B = \frac{x}{a} \Longrightarrow a \cdot \cos B = x$$

8. Substitute the equivalent expression for x into the equation from Question 5. The resulting equation contains only sides and angles of ΔABC. This equation is called the **Law of Cosines.**

$$b^2 = a^2 + c^2 - 2cx$$

- $b^2 = a^2 + c^2 - 2ac\cos B$ "Law of Cosines"

9. Using a similar method, two other forms of this law could be developed for a^2 and c^2. Based on your work for Questions 1–8, write the two other forms of the law of cosines for ΔABC.

- $a^2 = b^2 + c^2 - 2bc\cos A$ "Law of Cosines"
- $c^2 = a^2 + b^2 - 2ab\cos C$

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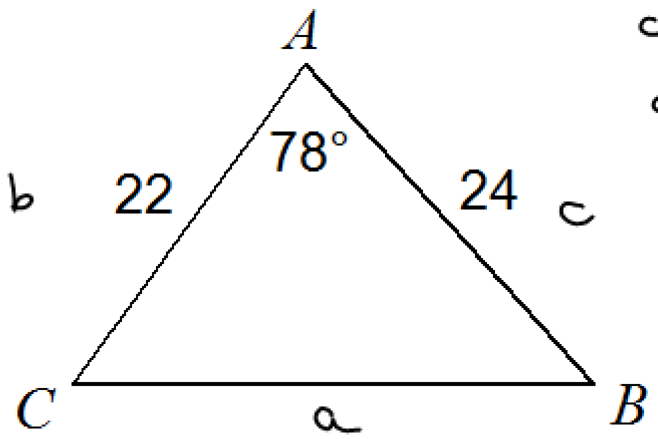
Question number   Directions   Lines   Reveal answer   Zoom: Jump   1-up   Close

Find each measurement indicated. Round your answers to the nearest tenth.

1) Find BC



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 22^2 + 24^2 - 2(22)(24)\cos 78°$$

$$a = \sqrt{22^2 + 24^2 - 2(22)(24)\cos 78°}$$

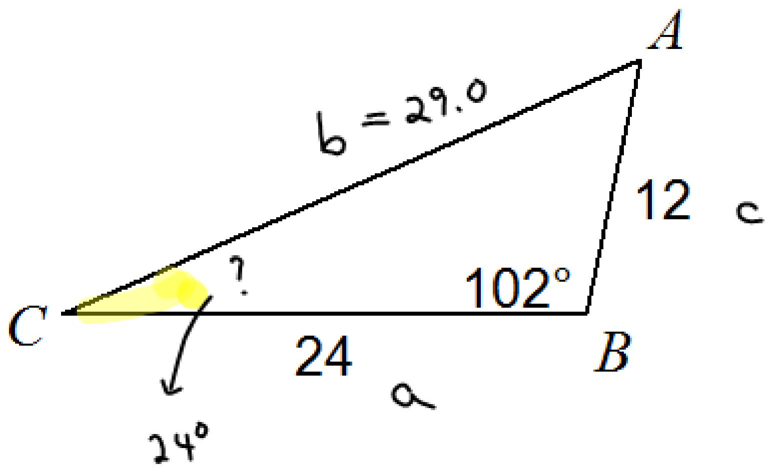
$$a \approx 29.0 \quad \text{(tenth)}$$

Infinite Algebra 2

Question number   Directions   Lines   Reveal answer   Zoom: ← Jump →   1-up   Close

Find each measurement indicated. Round your answers to the nearest tenth.

3) Find $m\angle C$



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 24^2 + 12^2 - 2(24)(12)\cos 102^\circ$$

$$b = \sqrt{24^2 + 12^2 - 2(24)(12)\cos 102^\circ}$$

$$b \approx 29.0$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$12^2 = 24^2 + 29^2 - 2(24)(29)\cdot\cos C$$

$$144 = 576 + 841 - \underbrace{1392\cdot\cos C}$$

$$144 = \cancel{1417} - 1392\cdot\cos C$$

$$-1417 \quad -\cancel{1417}$$

$$\frac{-1273}{-1392} = \frac{-1392\cdot\cos C}{-1392}$$

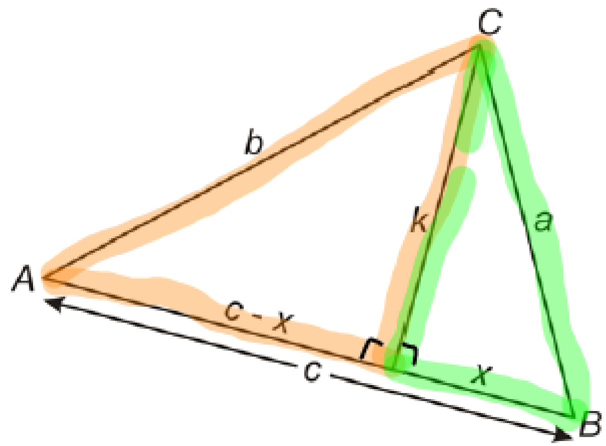
$$\frac{1273}{1392} = \cos C$$

$$\cos^{-1}\left(\frac{1273}{1392}\right) = \cancel{\cos^{-1}\cos}\ C$$

$$24^\circ = C$$

of Cosines.

To develop the **law of cosines**, begin with $\triangle ABC$. From vertex C, altitude k is drawn and separates side c into segments x and $c - x$. (Why can the segments be represented in this way?)



1. The altitude separates $\triangle ABC$ into two right triangles. Use the Pythagorean theorem to write two equations, one relating k, b, and $c - x$, and another relating a, k, and x.

$b^2 = k^2 + (c-x)^2$ $a^2 = k^2 + x^2$

2. Notice that both equations contain k^2. (Why?) Solve each equation for k^2.

Share same altitude.

$$b^2 = k^2 + (c-x)^2$$
$$-(c-x)^2 \qquad -(c-x)^2$$
$$b^2 - (c-x)^2 = k^2$$

$$a^2 = k^2 + x^2$$
$$-x^2 \qquad -x^2$$
$$a^2 - x^2 = k^2$$