9.B – Standard Deviation

- **Sample:** consists of one or more observations from a population. Measures that describe a sample are "statistics."
- **Population:** includes all elements from a set of data. Measures that describe a population are "parameters."
- **Standard Deviation:** a measure of how far the numbers in a data set deviate from the mean. The formula below is for the standard deviation of a sample.

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}}$$

- **Variance:** another measure of variation, like standard deviation, that shows how much data values deviate from the mean. It's the average of the squared differences from the mean. The formula below is for the variance of a sample.

Sample Notation
s = std dev.
$\bar{x}$ = "x bar" = mean
n = size

statistics

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$$

parameters

Population Notation
σ = "sigma" = std. dev.
μ = "mu" = mean
N = size

Finding Variance and Standard Deviation

What are the mean, variance, and standard deviation of these values?
6.9 8.7 7.6 4.8 9.0

$\bar{x} = \frac{6.9 + 8.7 + 7.6 + 4.8 + 9.0}{5} = 7.4$ Find the mean.

Make a table.

x	$\bar{x}$	$x - \bar{x}$	$(x - \bar{x})^2$
6.9	7.4	−0.5	0.25
8.7	7.4	1.3	1.69
7.6	7.4	0.2	0.04
4.8	7.4	−2.6	6.76
9.0	7.4	1.6	2.56
		Sum	11.30

Find difference between each value and the mean. Square the differences.

Add the squares of the differences.

$s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{11.30}{5-1} = 2.825$ = variance

$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{11.30}{5-1}} \approx 1.68$ = std. dev.

Using a Calculator to Find Standard Deviation STEM

Meteorology **The table displays the number of U.S. hurricane strikes by decade from the years 1851 to 2000. What are the mean and standard deviation for this data set?**

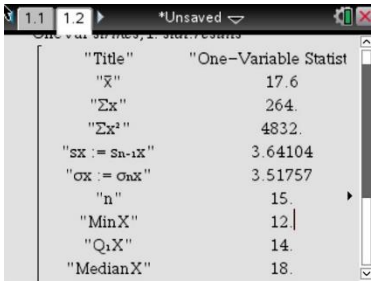
Decade	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Strikes	19	15	20	22	21	18	21	13	19	24	17	14	12	15	14

Source: National Hurricane Center

① plug in strikes data to a "lists and spreadsheets" page.

② add a "calculator" page and run "1 var stats"
↳ menu → 6 → 1 → 1

③



④ $\bar{x} \approx 17.6$ = sample mean

$s_x \approx 3.64$ = sample std. dev.

Using a Calculator to Find Standard Deviation STEM

Meteorology The table displays the number of U.S. hurricane strikes by decade from the years 1851 to 2000. What are the mean and standard deviation for this data set?

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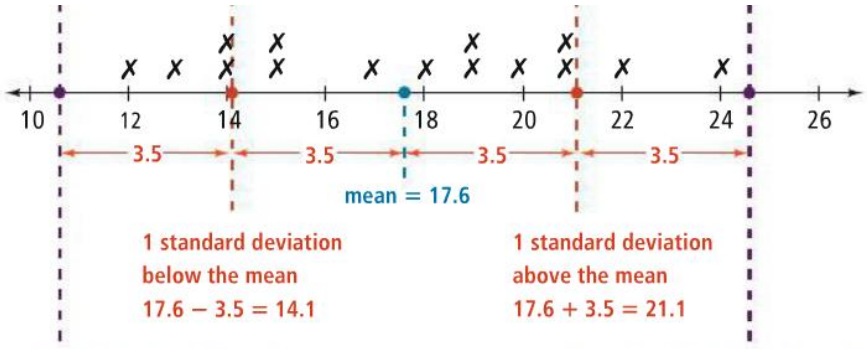
Source: National Hurricane Center

Using Standard Deviation to Describe Data STEM

Meteorology Use the U.S. hurricane-strike data from Problem 2. Within how many standard deviations from the mean do all of the values fall?

This problem uses σ for std dev. which is for the population.

$\sigma = 3.5$



2 standard deviations below the mean
$17.6 - 2(3.5) = 10.6$

2 standard deviations above the mean
$17.6 + 2(3.5) = 24.6$

All of the values fall within two standard deviations of the mean. Hurricane watchers can expect that the number of U.S. hurricane strikes in a decade will probably fall within two standard deviations of the 15-decade mean.

Practice

1) Find the mean, variance, and standard deviation for each data set.

a. 78 90 456 673 111 381 21

x	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
78	258.571	-180.571	32605.9
90	258.571	168.571	28416.2
456	258.571	197.429	38978.2
673	258.571	414.429	171751
111	258.571	-147.571	21777.2
381	258.571	122.429	14988.9
21	258.571	-237.571	56440

$\sum = 364958$

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{364958}{7-1} \approx 60826.3 = s^2$$

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{364958}{7-1}} \approx 246.63 = s$$

b. 13 15 17 18 12 21 10

x	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
13	15.14	-2.14	4.58
15	15.14	-0.14	0.02
17	15.14	1.86	3.46
18	15.14	2.86	8.18
12	15.14	-3.14	9.86
21	15.14	5.86	34.34
10	15.14	-5.14	26.42

$\sum = 86.86$

$$s^2 = \frac{\sum (x-\bar{x})^2}{n-1} = \frac{86.86}{7-1} \approx 14.48 = s^2$$

$$s = \sqrt{\frac{\sum (x-\bar{x})^2}{n-1}} = \sqrt{\frac{86.86}{7-1}} \approx 3.81 = s$$

2) The Dow Jones Industrial average for the first 12 weeks of 1988 are listed below. Find the mean and standard deviation.

1911.31	1956.07	1903.51	1958.22	1910.48	1983.26
2014.59	2023.21	2057.86	2034.98	2087.37	2067.14

$\bar{x} \approx 1992.33$ $S \approx 64.60$

used calculator by entering the data into a column of a "lists and spreadsheets" page. Then I used the "1 variable stats" tool.

3) The mean price of the nonfiction books on a bestsellers list is $25.07; the standard deviation is $2.62. The data values are: $26.95, $22.95, $24.00, $24.95, $29.95, $19.95, $24.95, $24.00, $27.95, and $25.00. Determine the whole number of standard deviations from the mean that include all data values.

2 standard deviations contain all 10 values.

4) The mean length of Beethoven's nine symphonies is 37 minutes; the standard deviation is 12 minutes. The data values are the following minutes: 23, 30, 47, 35, 30, 40, 35, 22, and 65. Determine the whole number of standard deviations from the mean that include all data values.

3 standard deviations include all 9 values.

5) Find the standard deviation for each data set. Use the standard deviation to compare each pair of data sets. Data set A is the fastest recorded speeds of various large wild cats (mph): 70, 50, 30, 40, 35, 30, 30, 40, 15. Data set B is the fastest recorded speeds of various birds in flight (mph): 217, 106, 95, 56, 65, 37, 50, 31, 53, 25, 25, 25.

$S_{cats} \approx 15.43$

$S_{birds} \approx 54.67$

The speeds of the birds are much more spread out compared to the the speeds of cats.

The standard deviations were found with the "1 variable stats" tool in the calculator.