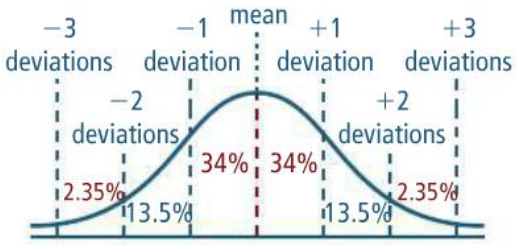
9.E – Normal Distribution

- **Discrete probability distribution:** a distribution with a finite number of possible events, or values.
- **Continuous probability distribution:** a distribution that can be any value in an interval of real numbers.
- **Normal distribution:** a normal, symmetric curve that contains data that vary randomly from the mean.
    - 68% of data fall within one standard deviation, 95 % falls within two, and 99.7% fall within three.
    - See graph below.



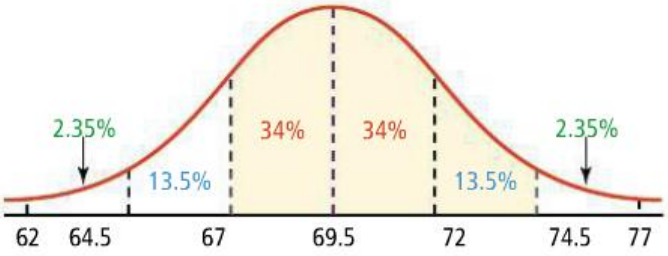
Analyzing a Normal Distribution

The heights of adult American males are approximately normally distributed with mean 69.5 in. and standard deviation 2.5 in.

A What percent of adult American males are between 67 in. and 74.5 in. tall?

Draw a normal curve. Label the mean. Divide the graph into sections of standard-deviation widths. Label the percentages for each section.

Distribution of Heights—Adult American Males



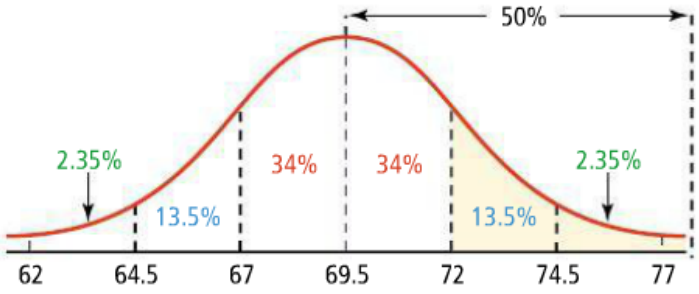
$P(67 < \text{height} < 74.5) = 0.34 + 0.34 + 0.135 = 0.815$

About 82% of adult American males are between 67 in. and 74.5 in. tall.

B In a group of 2000 adult American males, about how many would you expect to be taller than 6 ft (or 72 in.)?

Because the graph is symmetric about the mean, the right half of the distribution contains 50% of the data. If you subtract everything between 69.5 in. and 72 in. from the right half, only the part of the distribution that is greater than 72 in. remains.

Distribution of Heights—Adult American Males

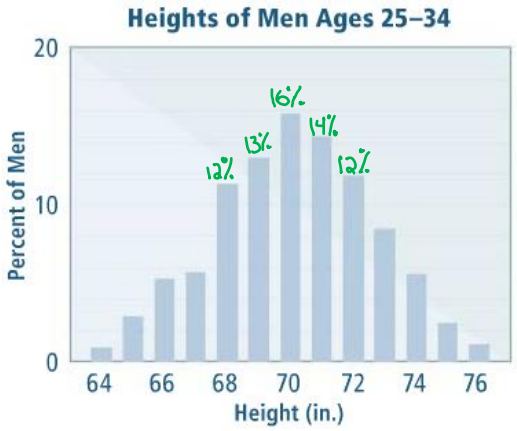


$P(\text{height} > 72) = 0.50 - 0.34 = 0.16$

You would expect about 16% of the 2000 adult American males to be taller than 72 in.
You would expect about $0.16 \cdot 2000 = 320$ to be over 6 ft tall.

Practice

1) The heights of men in a survey are normally distributed about the mean in the graph to the right. Use the graph to answer the following questions:



a) About what percent of men age 25 to 34 are 69-71 inches tall?

Estimate from the graph. 13% + 16% + 14% = 43%

b. Suppose the survey included data on 100 men. About how many would you expect to be 69-71 tall? 100 (.43) = 43 men

c) The mean of the data is 70, and the standard deviation is 2.5. Approximately what percent of men are within one standard deviation of the mean height?

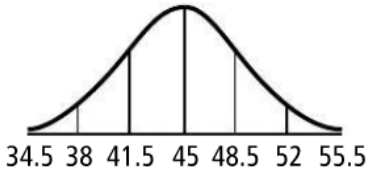
Estimate from the graph

One std dev below and above the mean is 67.5 to 72.5. The graph shows 68 to 72 so just use that range as an estimate

12% + 13% + 16% + 14% + 12% = 67%

70 + 2.5 = 72.5 70 − 2.5 = 67.5

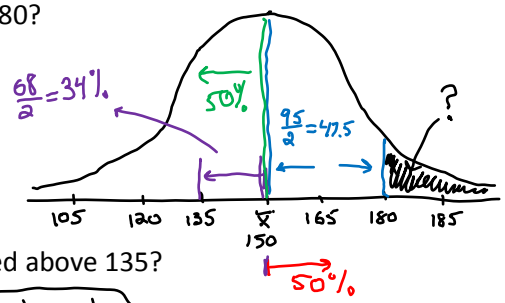
2) Sketch a normal curve of a distribution with a mean of 45 and a standard deviation of 3.5. Label the x-axis values at 1, 2, and 3 standard deviations.



3) The scores on the Algebra 2 final are approximately normally distributed with a mean of 150 and a standard deviation of 15. Use his information to answer the following questions:

a) What percentage of the students who took the test scored above 180?

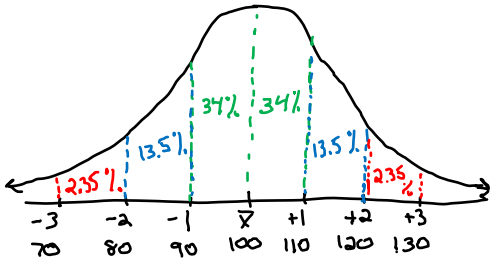
100% − 47.5% − 50% = 2.5%



b) If 250 students took the final exam, approximately how many scored above 135?

34% + 50% = 84% 250(.84) = 210 students

4) A set of data has a normal distribution with a mean of 100 and a standard deviation of 10. Find the percent of data within each interval.



68%/2 = 34% 99.7%/2 = 49.85%

95%/2 = 47.5% 49.85% − 47.5% = 2.35%

47.5% − 34% = 13.5%

a) from 80 to 100

13.5% + 34% = 47.5%

b) from 90 to 120

34% + 47.5% = 81.5%

c) at most 110 50% + 34% = 84%

d) from 70 to 130

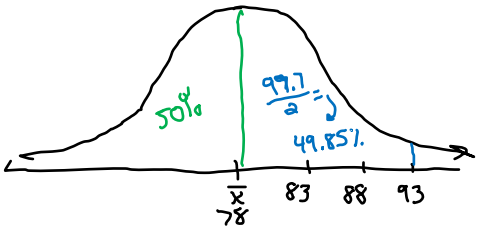
99.7% Just a given...no work necessary

e) at least 100

50% Simply half the area under the curve.

f) at least 80 50% + 47.5% = 97.5%

5) In a class of 25, one student receives a score of 100 on a test. The grades are distributed normally with a mean of 78 and a standard deviation of 5. Do you think the student's score is an outlier? Explain.



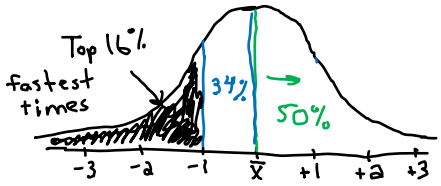
The student's score is more than 4 standard deviations above the mean so it can definitely be considered an outlier.

- There is a 99.85% chance of getting a 93 which is 3 standard deviations above the mean... so 50% + 49.85% = 99.85%

6) To qualify as a contestant in a race, a runner has to be in the fastest 16% of all applicants. The runner times are normally distributed with a mean of 63 minutes and a standard deviation of 4 minutes. To the nearest minute, what is the qualifying time for the race?

100% - (50% + 34%) = 16%

59 minutes because its one standard deviation below the mean and 16% is below this time.

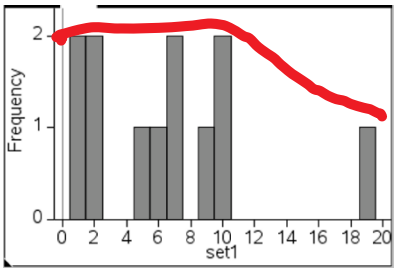


7) From the table below, select the set of values that appears to be distributed normally. Use that set to make a histogram of the values and sketch a normal curve over the graph.

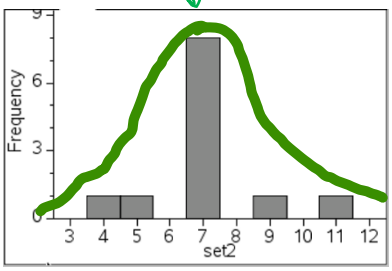
Set 1	1	10	5	19	2	7	1	7	2	10	6	9
Set 2	5	7	7	7	4	11	7	7	7	9	7	7
Set 3	5	6	9	1	1	5	11	1	10	4	2	8

Plug data into a list and spreadsheets page and create three histograms.

Slighty skewed right or uniform with an outlier



Normally distributed "bell curve"



Right skewed