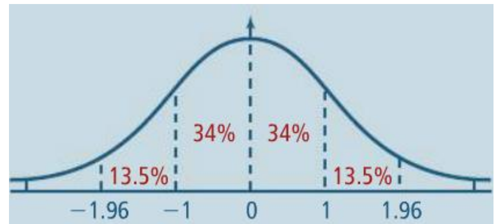
9.F – Margin of Error and Confidence Intervals

The mean of a sample may or may not be the mean of the population the sample was drawn from. The **margin of error** helps you find the interval in which the mean of the population is likely to be. The margin of error is based on the size of the sample and the confidence level desired.

A 95% confidence level means that the probability is 95% that the true population mean is within a range of values called a **confidence interval**. It also means that when you select many different large samples from the same population, 95% of the confidence intervals will actually contain the population mean.

The means of all the samples follow a normal distribution. The normal distribution shown here shows that 95% of values are between -1.96 and 1.96 standard deviations from the mean. To find the margin of error based on the mean of a large set of data at a 95% confidence level, you use the formula $ME = 1.96 \cdot \frac{s}{\sqrt{n}}$ where ME is the margin of error, s is the standard deviation of the sample data, and n is the number of value in the sample.



The confidence interval for the population mean μ (pronounced myoo) is $\bar{x} \pm ME$ which is also written as $\bar{x} - ME \leq \mu \leq \bar{x} + ME$, where $\bar{x}$ is the sample mean.

You can also find the margin of error and confidence interval for a sample proportion. A sample proportion $\hat{p}$ is the ratio $\frac{x}{n}$ where x is the number of times an event occurs in a sample of size n.

To find the margin of error for a **sample proportion** at a 95% confidence level, use the formula $ME = 1.96 \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ where ME is the margin of error, $\hat{p}$ is the sample proportion, and n is the same size. The confidence interval for the population proportion p is $\hat{p} \pm ME$ which is also written as $\hat{p} - ME \leq p \leq \hat{p} + ME$.

Activity #1

A grocery store manager wanted to determine the wait times for customers in the express lines. He times customers chose at random.

Waiting Time (minutes)				
3.3	5.1	5.2	6.7	7.3
7.5	4.6	6.2	5.5	3.6
3.4	3.5	8.2	4.2	3.8
4.7	4.6	4.7	4.5	9.7
5.4	5.9	6.7	6.5	8.2
3.1	3.2	8.2	2.5	4.8

1) What is the mean and standard deviation of the sample? Round to the nearest tenth of a minute. Used "1 variable stats" to get $\bar{x} \approx 5.4$ minutes and $s \approx 1.8$ minutes

2) At a 95% confidence level, what is the approximate margin of error? Round to the nearest tenth of a minute.

$$ME = 1.96 \cdot \frac{s}{\sqrt{n}} = 1.96 \cdot \frac{1.8}{\sqrt{30}} = \boxed{0.6}$$

3) What is the confidence interval for a 95% confidence level?

$\bar{x} \pm ME = 5.4 \pm 0.6$ $\quad 5.4 - 0.6 = 4.8$ $\quad 5.4 + 0.6 = 6.0$

$\hookrightarrow$ $\boxed{4.8 \text{ to } 6.0}$

4) What is the meaning of the interval in terms of wait times for customers?

We are 95% confident that our population mean will fall between 4.8 and 6.0

Activity #2

What is the sample proportion for each situation? Write the ratios as percents rounded to the nearest tenth of a percent.

5) In a poll of 1085 voters selected randomly, 564 favor Candidate A.

$\frac{564}{1085} \approx 0.520 = \hat{p}$

6) A coin is tossed 40 times, and it comes up heads 25 times.

$\frac{25}{40} \approx 0.625 = \hat{p}$

Activity #3

Find (a) the sample proportion, (b) the margin of error, and (c) the 95% confidence interval for the population proportion.

7) In a survey of 530 randomly selected high school students, 280 preferred watching football to watching basketball.

$\frac{280}{530} \approx 0.528 = \hat{p}$

$ME = 1.96 \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$ME = 1.96 \cdot \sqrt{\frac{0.528(1-0.528)}{530}}$

$ME = 0.043$

$\hat{p} \pm ME$

$\hat{p} - ME = 0.528 - 0.043 = 0.485$

$\hat{p} + ME = 0.528 + 0.043 = 0.571$

$\boxed{0.485 \text{ to } 0.571}$

$\hookrightarrow$ 48.5% to 57.1%

8) In a simple random sample of 500 people, 342 reported using social networking sites on the Internet.

$\frac{342}{500} \approx 0.684 = \hat{p}$

$ME = 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$ME = 1.96 \cdot \sqrt{\frac{0.684(1-0.684)}{500}}$

$ME \approx 0.041$

$\hat{p} \pm ME$

$0.684 - 0.041 \approx 0.643$

$0.684 + 0.041 \approx 0.725$

$\boxed{0.643 \text{ to } 0.725}$

$\hookrightarrow$ 64.3% to 72.5%

<u>Practice</u>

For exercises 9-10, find the 95% confidence interval for the population mean or population proportion, and interpret the confidence interval in context.

9) A consumer research group tested the battery life of 36 randomly chosen batteries to establish the likely battery life for the population of that type of battery.

Battery Life (hours)			
63.2	84.6	78.4	85.8
62.1	81.8	63.6	64.2
79.4	75.2	54.1	73.4
66.3	74.5	71.6	60.1
61.2	74.5	72.4	81.3
61.4	83.6	75.6	74.1
68.3	82.2	59.3	47.6
86.2	64.3	72.7	71.8
71.4	63.6	59.6	68.1

$\bar{x} \approx 70.5$

$s \approx 9.5$

$M.E. = 1.96 \cdot \frac{s}{\sqrt{n}} = 1.96 \cdot \frac{9.5}{\sqrt{36}} \approx 3.1$

$\bar{x} \pm M.E.$

$70.5 - 3.1 = 67.4$

$70.5 + 3.1 = 73.6$

$\boxed{67.4 \text{ to } 73.6}$

10) In a poll of 720 likely voters, 358 indicate they plan to vote for Candidate A.

$\hat{p} = \frac{358}{720} \approx 0.497$

$M.E. = 1.96 \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96 \sqrt{\frac{0.497(1-0.497)}{720}} \approx 0.037$

$\hat{p} \pm M.E. \rightarrow 0.497 - 0.037 = 0.46 \rightarrow 0.497 + 0.037 = 0.534$ so $\rightarrow \boxed{0.46 \text{ to } 0.534}$

$\hookrightarrow$ 46.0% to 53.4%

11) Roll a number cube 30 times. Record the results from each roll. In parts (a) and (b), find the sample proportion, the margin of error for a 95% confidence level, and the 95% confidence interval for the population proportion.

a. rolling a 2

Randseed (112233)

Randint(1,6,30)

$\hat{p} = \frac{7}{30} \approx 0.23$

$M.E. = 1.96 \cdot \sqrt{\frac{0.23(1-0.23)}{30}} \approx 0.151$

$\hat{p} \pm M.E. \rightarrow 0.23 - 0.151 = 0.079$

$0.23 + 0.151 = 0.381$

0.079 to 0.381

7.9% to 38.1%

b. rolling a 3

Randseed (112233)

Randint (1,6,30)

$\hat{p} = \frac{6}{30} \simeq 0.2$

$M.E. = 1.96 \cdot \sqrt{\frac{0.2(1-0.2)}{30}} \approx 0.143$

$\tilde{p} \pm M.E. \rightarrow 0.2 - 0.143 = 0.057$

$0.2 + 0.143 = 0.343$

0.057 to 0.343

5.7% to 34.3%

c. Is the 95% confidence interval for the population proportion about the same for rolling a 2 and for rolling a 3?

Yes. They are rather close.

d. Compare your sample proportions to the theoretical proportion for parts (a) and (b). Would you expect the theoretical proportion to be within the confidence intervals you found? Explain.

The theoretical proportion for the situation is $\frac{1}{6} = 0.167$. It is just below the sample proportions found in "part a and part b." Because everything is similar, we can expect to find the theoretical proportion in both intervals..... which was confirmed.