5.6. Advanced Algebra
Radical Expressions (Part 2)

DATE: 5/22

Target 10B. Perform operations on radical expressions with various indices.

Product Property of Radicals

For any real numbers a and b and any integer $n > 1$,

1. If n is even and a and b are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
2. If n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Simplify each expression.

1. $\sqrt[3]{16p^8q^7}$

$\sqrt[3]{2^4p^8q^6q} = 2^2p^4q^3\sqrt{q} = 4p^4q^3\sqrt{q}$ ✓

2. $\sqrt[3]{75a^6b^{13}}$

$\sqrt[3]{5^2 \cdot 3a^6b^{12}b} = a^2b^4\sqrt{75b}$ ✓

Quotient Property of Radicals

For any real numbers a and $b \neq 0$, and any integer $n > 1$,

Quotient Property of Radicals

For any real numbers a and $b \neq 0$, and any integer $n > 1$,

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

Simplify each expression and rationalize the denominator.

3. $\sqrt{\frac{x^4}{y^5}} = \frac{\sqrt[2]{x^4}}{\sqrt[2]{y^4 y}} = \frac{x^2}{y^2\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}} = \frac{x^2\sqrt{y}}{y^2 \cdot y}$

$= \frac{x^2\sqrt{y}}{y^3}$ ✓

4. $\sqrt[5]{\frac{5}{4a}} = \frac{\sqrt[5]{5}}{\sqrt[5]{2^2 a}} \cdot \frac{\sqrt[5]{2^3 \cdot a^4}}{\sqrt[5]{2^3 a^4}} = \frac{\sqrt[5]{40a^4}}{2a}$ ✓

You can use the Product and Quotient Properties to multiply and divide some radicals, respectively.

Multiply Radicals

5. $6\sqrt[3]{9n^2} \cdot 3\sqrt[3]{24n}$

6. $5\sqrt[3]{100a^2} \cdot \sqrt[3]{10a}$

You can use the Product and Quotient Properties to multiply and divide some radicals, respectively.

Multiply Radicals

5. $6\sqrt[3]{9n^2} \cdot 3\sqrt[3]{24n}$

$18\sqrt[3]{216\,n^3}$

$18 \cdot \sqrt[3]{2^3 \cdot 3^3 n^3}$

$18 \cdot 2 \cdot 3 \cdot n = 108n$ ✓

6. $5\sqrt[3]{100a^2} \cdot 1\sqrt[3]{10a}$

$5 \cdot \sqrt[3]{1000\,a^3}$

$5 \cdot \sqrt[3]{10^3\,a^3}$

$5 \cdot 10 \cdot a = 50a$ ✓

You cannot add radicals in the same manner in which you multiply them. You can only add/subtract radicals if they are "like radical expressions". This means that both the indices and the radicands are alike.

Add and Subtract Radicals

7. $2\sqrt{12} - 3\sqrt{27} + 2\sqrt{48}$

$2\sqrt[2]{2^2 \cdot 3} - 3\sqrt[2]{3^2 \cdot 3} + 2 \cdot \sqrt[2]{2^4 \cdot 3}$

$2 \cdot 2\sqrt{3} - 3 \cdot 3\sqrt{3} + 2 \cdot 2^2 \cdot \sqrt{3}$

$4\sqrt{3} - 9\sqrt{3} + 8\sqrt{3}$

$3\sqrt{3}$ ✓

8. $3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$

$3\sqrt[2]{3^2 \cdot 5} - 5 \cdot \sqrt[2]{2^4 \cdot 5} + 4\sqrt[2]{2^2 \cdot 5}$

$3 \cdot 3\sqrt{5} - 5 \cdot 2^2 \cdot \sqrt{5} + 4 \cdot 2\sqrt{5}$

$9\sqrt{5} - 20\sqrt{5} + 8\sqrt{5}$

$-3\sqrt{5}$ ✓

CHECK UNDERSTANDING:

$1\sqrt{5} + 3\sqrt{7} + 2\sqrt{5} = ?$

$3\sqrt{5} + 3\sqrt{7}$!

Multiply Radicals Using the Distributive Property

9. $(3\sqrt{5} - 2\sqrt{3})(2 + \sqrt{3})$

10. $(5\sqrt{3} - 6)(5\sqrt{3} + 6)$