Honors Advanced Algebra
Multiplicity Discovery

Name: Lab filled in

Essential question: How do multiple zeros affect the graph of a polynomial function?

1. Graph $f(x) = x(x-2)(x+2)(x-1)$ on a graphing calculator.
   a. What are the zeros of the function (menu, analyze graph, zero) ?

   Zeros: $x=0$, $x=2$, $x=-2$, $x=1$

   b. For what value(s) of *x* does the graph of the function <u>cross</u> the *x*-axis?

   $x=0$, $x=2$, $x=-2$, $x=1$

   c. For what value(s) of *x* does the graph of the function <u>touch but not cross</u> the *x*-axis?

   None

   d. What degree is the polynomial?

   4 – quartic

2. Graph each function in the table. For each function, answer the questions asked in Question 1. Use the table below to record your results.

#	Function	Zeros	Cross	Touch	Degree
1	$f(x) = (x+1)^2(x-2)^1(x-1)^1$	−1, 2, 1	1, 2	−1	4
2	$f(x) = (x-2)^2(x+1)^1(x-1)^1$	−1, 2, 1	−1, 1	2	4
3	$f(x) = (x+2)^2(x-1)^2$	−2, 1	None	−2, 1	4
4	$f(x) = (x+1)^3(x-1)^1(x-2)^1$	−1, 1, 2	−1, 1, 2	None	5
5	$f(x) = (x-2)^2(x-1)^1(x+1)^2$	−1, 1, 2	1	−1, 2	5

$\underbrace{(x-a)}_{\text{Factor}}$

3. How are the zeros of a polynomial function related to the factors of a polynomial function?
   The zeros of the function are the solutions when the factors are set equal to zero and solved. When the coefficient of x is 1 in the factor, the zero and the constant term in the factor have opposite signs.

4. How do the exponents in each term in the factored form of the polynomial function affect its graph? When does the graph cross the x-axis and when does the graph touch the x-axis?

   $m \geq 2$ multiplicity

   $(x-a)^m$

   ① Exponent ODD ⇒ graph CROSSES x-axis at corresponding zero.

   ② Exponent EVEN ⇒ graph touches x-axis at corresponding zero.

5. Revisit graphs **#1–#5,** and observe the end behavior for the polynomial functions. What does the degree of the polynomial function tell you about its end behavior?

   • Degree EVEN and leading coefficient positive ⇒ $\lim_{x\to-\infty} f(x) = \infty$, $\lim_{x\to\infty} f(x) = \infty$

   • Degree ODD and leading coefficient positive ⇒ $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = \infty$

Zero: $x = -2$ $\quad x = 5 \quad x = 3$ → multiplicity greater than 1

factor: $(x+2)$ $\quad (x-5) \quad (x-3)$

6. When a polynomial has a repeated linear factor, it has a multiple zero. Write the factored form of a polynomial function that crosses the *x*-axis at *x* = –2 and *x* = 5 and touches the x-axis at *x* = 3. Which of the zeros of the function must have a multiplicity greater than 1? Explain your reasoning.

$f(x) = (x+2)(x-5)(x-3)^2$ One possibility

$x = 3$ must have multiplicity greater than 1 since it touches x-axis.

7. Write two additional polynomial functions that meet the same conditions as described in Question 6. Explain what is different from your function in Question 6, and how you determined your polynomial functions.

Some examples:

$f(x) = (x+2)^1(x-5)^1(x-4)^4$ Degree 6 b/c $1+1+4=6$

$f(x) = (x+2)^3(x-5)^1(x-3)^2$ Degree 6 b/c $3+1+2=6$

$f(x) = (x+2)^3(x-5)^5(x-3)^2$ Degree 10 b/c $3+5+2=10$

Examples:

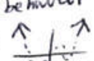
- State the degree and list the zeros of the polynomial function. State the multiplicity of each zero and whether the graph crosses the *x*-axis at the corresponding *x*-intercept. Then sketch the graph of the polynomial function by hand.

1. $f(x) = x^1(x+4)^2(x-6)^1$

Degree 4 b/c $1+2+1=4$

Leading coefficient positive

End behavior

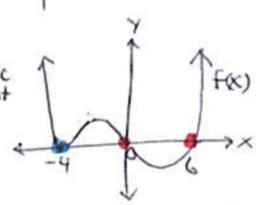


Zeros: $x=0$, $x=-4$, $x=6$

crosses b/c exponent ODD

touches b/c exponent even multiplicity 2

crosses b/c exponent odd



2. $g(x) = (x-5)^3(x+6)^1(x+2)^2$

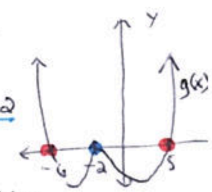
Degree 6 b/c $3+1+2=6$

L.C. positive and so same end behavior as EXAMPLE 1

$x=5$: crosses b/c exponent odd multiplicity 3

$x=-6$: crosses b/c exponent odd

$x=-2$: touches b/c exponent even multiplicity 2



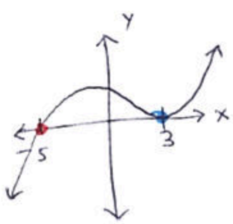
- Write a polynomial function based upon the information below. Then sketch the graph of the polynomial by hand.

3. The function has a zero with multiplicity of 2 at *x* = 3 and a zero with multiplicity of 1 at *x* = -5.

even — touches — $(x-3)$ $\qquad$ odd — crosses — $(x+5)$

$f(x) = (x-3)^2(x+5)^1$

Degree 3, L.C. positive



4. The function has a zero with multiplicity of 2 at *x* = -1 and a zero with multiplicity of 2 at *x* = 3.

even — touches — $(x+1)$ $\qquad$ even — touches — $(x-3)$

$f(x) = (x+1)^2(x-3)^2$

Degree 4, L.C. positive