Honors Advanced Algebra DATE: ___________

5-6 Rational Functions and Their Graphs

Target 3B. Graph transform and identify the key features of the graph of a rational function.

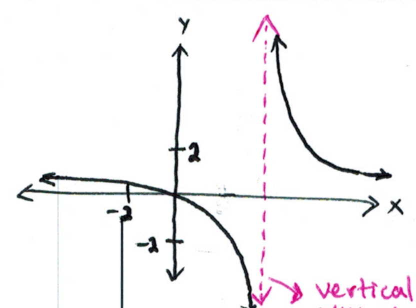
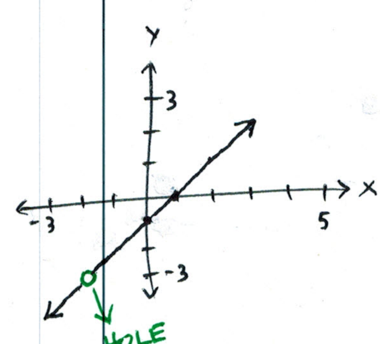
Vertical Asymptotes and Point Discontinuity

Examples of rational functions:

$$f(x) = \frac{x}{x+3} \qquad g(x) = \frac{5}{x-6} \qquad h(x) = \frac{x+4}{(x-1)(x+4)}$$

No denominator in a rational function can be zero because division by zero is not defined. In the examples above the functions are not defined at $x =$ <u>-3</u> $x =$ <u>6</u> and $x =$ <u>1</u> and $x =$ <u>-4</u> respectively.

The graphs of rational functions may have breaks in ***continuity***. This means that unlike polynomial functions which can be traced with a pencil never leaving the paper not all rational functions are traceable. Breaks in continuity can appear as a ***vertical asymptote*** or as ***point discontinuity (hole)***.

Point of Discontinuity			
Key Concept	**Words**	**Example**	**Model**
Vertical Asymptote	If the rational expression of a function is written in simplest form and the function is undefined for $x = a$ then $x = a$ is a <u>non-removable discontinuity</u>.	For $f(x) = \frac{x}{x-3}$, $x =$ <u>3</u> is a vertical asymptote. f(x) is in simplest form. Non-removable	
Hole	If the original function is undefined for $x = a$ but the rational expression in simplest form is defined for $x = a$ then there is a <u>removable discontinuity</u> in the graph at $x = a$.	removable $f(x) = \frac{\cancel{(x+2)}(x-1)}{\cancel{x+2}}$ Can be simplified to $f(x) = x - 1$. So $x =$ <u>-2</u> represents a hole in the graph.	

Find the domain and points of discontinuity of each rational function. Determine whether the discontinuities are removable or non-removable. Then find the vertical asymptotes and holes for the graph of each rational function.

We are just analyzing the denominator to find discontinuities

undefined (not included)

1. $f(x) = \frac{x^2-1}{x^2-6x+5} = \frac{(x+1)\cancel{(x-1)}}{(x-5)\cancel{(x-1)}} = \frac{x+1}{x-5}$

$x-5=0$, $x=5$; $x-1=0$, $x=1$

Domain: not included

$(-\infty,1)\cup(1,5)\cup(5,\infty)$

- Removable @ $x=1$, so we have a HOLE @ $x=1$.
- Non-removable @ $x=5$, so we have a V.A. @ $x=5$.

2. $f(x) = \frac{x^2-4}{x^2+5x+6} = \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x+3)} = \frac{x-2}{x+3}$

$x+2=0$, $x=-2$; $x+3=0$, $x=-3$

Domain:

$(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$

- Removable @ $x=-2$, so HOLE @ $x=-2$.
- Non-removable @ $x=-3$, so V.A. @ $x=-3$.

3. $f(x) = \frac{x^2-9}{x+3} = \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}} = \frac{x-3}{1}$ → No discontinuity in denominator

$x+3=0$, $x=-3$

Domain:

$(-\infty,-3)\cup(-3,\infty)$

- Removable @ $x=-3$, so HOLE @ $x=-3$.
- No non-removable, so NO V.A.

4. $f(x) = \frac{x-1}{x^2+4x-5} = \frac{\cancel{x-1}}{(x+5)\cancel{(x-1)}} = \frac{1}{x+5}$

$x+5=0$, $x=-5$; $x-1=0$, $x=1$

Domain:

$(-\infty,-5)\cup(-5,1)\cup(1,\infty)$

- Removable @ $x=1$, so Hole @ $x=1$
- Non-removable @ $x=-5$, so V.A. @ $x=-5$.

5. $f(x) = \frac{2}{x^2-5x+6} = \frac{2}{(x-2)(x-3)}$

$x-2=0$, $x=2$; $x-3=0$, $x=3$

Domain:

$(-\infty,2)\cup(2,3)\cup(3,\infty)$

- No removable discontinuity, so No HOLE.
- Non-removable @ $x=2$ and @ $x=3$, so V.A. @ $x=2$ and V.A. @ $x=3$.

6. $f(x) = \frac{x^2-8x+16}{x-4} = \frac{\cancel{(x-4)}(x-4)}{\cancel{(x-4)}} = \frac{x-4}{1}$ → No discontinuity in denominator

$x-4=0$, $x=4$

Domain:

$(-\infty,4)\cup(4,\infty)$

- Removable @ $x=4$, so HOLE @ $x=4$
- No non-removable, so NO V.A.