

5.6 Rational Functions and Their Graphs (Target 3B)

Review of Prior Concepts

Find the domain, vertical asymptotes, and holes for each rational function:

Factor: $(3x^2 - 11x + 10)$

a) $f(x) = \frac{1}{x-5}$

$$\begin{array}{c} x-5=0 \\ \textcircled{x=5} \end{array}$$

*undefined
(not included)*

Domain: $(-\infty, 5) \cup (5, \infty)$

$\leftarrow \underset{5}{\times} \rightarrow$

• Vertical Asymptote:

V.A. @ $x = 5$

• No holes

b) $g(x) = \frac{3x-5}{3x^2-11x+10}$

$$\begin{aligned} &= \frac{3x-5}{(3x-5)(x-2)} \\ &= \frac{1}{x-2} \end{aligned}$$

• Domain: $(-\infty, \frac{5}{3}) \cup (\frac{5}{3}, 2) \cup (2, \infty)$

• V.A. @ $x = 2$

• Hole @ $x = \frac{5}{3}$

$$\begin{array}{c} 3x^2 - 11x + 10 \\ \textcircled{3} \quad \textcircled{-11} \quad \textcircled{10} \\ (x-5)(x-2) \\ (3x-5)(x-2) \end{array}$$

$$\begin{array}{c} 3x-5=0 \text{ or } x-2=0 \\ \textcircled{+5} \quad \textcircled{+5} \\ 3x=5 \\ \textcircled{x}=\frac{5}{3} \\ (x=\frac{5}{3}) \end{array}$$

$\leftarrow \underset{\frac{5}{3}}{\times} \rightarrow$

Vertical Asymptotes

Recall:

Vertical asymptotes occur when the denominator is equal to zero
(the function must be in simplest form)

Horizontal Asymptotes occur when end behavior approaches a #, c. H.A. is @ $y = c$.



NOTATION: $\lim_{x \rightarrow \infty} f(x) = c$ or $\lim_{x \rightarrow -\infty} f(x) = c$



Using your graphing calculator, find the vertical and horizontal asymptotes.

a) $f(x) = \frac{3x-5}{x-4}$

b) $g(x) = \frac{3x^2-5}{x^2-4}$

• V.A. @ $x = 4$

• H.A. @ $y = 3$

$\lim_{x \rightarrow \infty} f(x) = 3$ or

$\lim_{x \rightarrow \infty} f(x) = 3$

• V.A. @ $x = \pm 2$

• H.A. @ $y = 3$

$\lim_{x \rightarrow -\infty} g(x) = 3$

$\lim_{x \rightarrow \infty} g(x) = 3$

Using your graphing calculator, find the horizontal asymptotes (if any).

a) $f(x) = \frac{3x^2 - 5x + 1}{x^2 - 4}$

H.A. @ $y = 3$

b) $g(x) = \frac{3x - 5}{x^2 - 4}$

H.A. @ $y = 0$

c) $h(x) = \frac{3x^2 - 5x + 1}{x - 4}$

No H.A.

Can you find a pattern? If yes, then find the horizontal asymptotes (if any) without using your graphing calculator.

a) $f(x) = \frac{2x^3 + x^2 - 5x + 1}{x^3 - 4}$

H.A. @ $y = 2$

b) $g(x) = \frac{2x - 5}{x^3 - 4}$

H.A. @ $y = 0$

c) $h(x) = \frac{2x^3 + x^2 - 5x + 1}{x - 4}$

No H.A.

Conclusion about Horizontal Asymptotes:

- ① If the degree of the numerator is equal to the degree of the denominator, then the H.A. is $y = \frac{a}{b}$, where a is the leading coefficient of the term with largest exponent and b is the leading coefficient of the term with largest exponent.
- ② If the degree of the numerator is $<$ the degree of the denominator, then the H.A. is @ $y = 0$.
- ③ If the degree of the numerator is $>$ the degree of the denominator, then we have NO H.A.