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3.4. Advanced Algebra
Linear Programming

DATE: 11/15

Target 3A. Translate a verbal model into an algebraic model
Target 3D. Graph a system of inequalities to determine the feasible region and maximize or minimize the objective function
Target 3E. Problem solve using Linear Programming

Linear Programming: the process of finding the maximum or minimum values of a function for a region defined by inequalities

Constraints: conditions given to variables, often expressed as linear inequalities

Feasible Region: the intersection of the graphs in a system of constraints

Bounded: a region is bounded when the graph of a system of constraints is a polygonal region

Unbounded: a system of inequalities that forms a region that is open

Vertices: the maximum or minimum value that a linear function has for the points in a feasible region

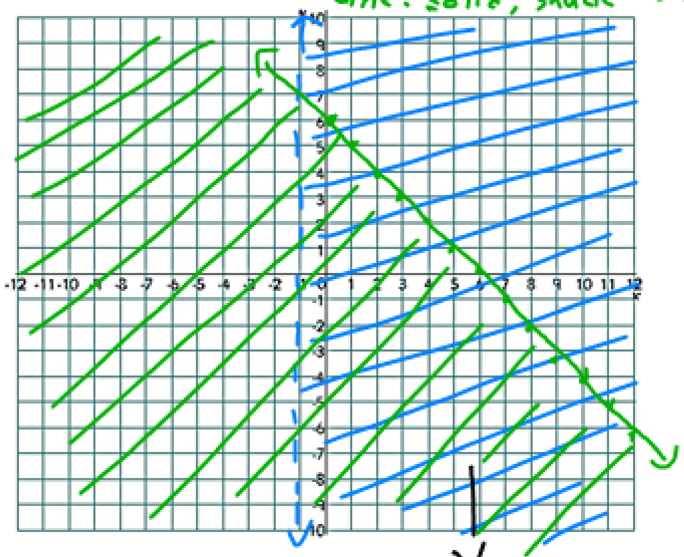
Examples:
Is each system of inequalities bounded or unbounded? Where are the vertices?

Examples:

Is each system of inequalities bounded or unbounded? Where are the vertices?

1. $x \geq -1$ — vertical line / dashed, shade: above (right)

$y \leq -x + 6$ — Slope: $\frac{-1\downarrow}{1\rightarrow}$ y-int: 6, Line: solid, shade: below



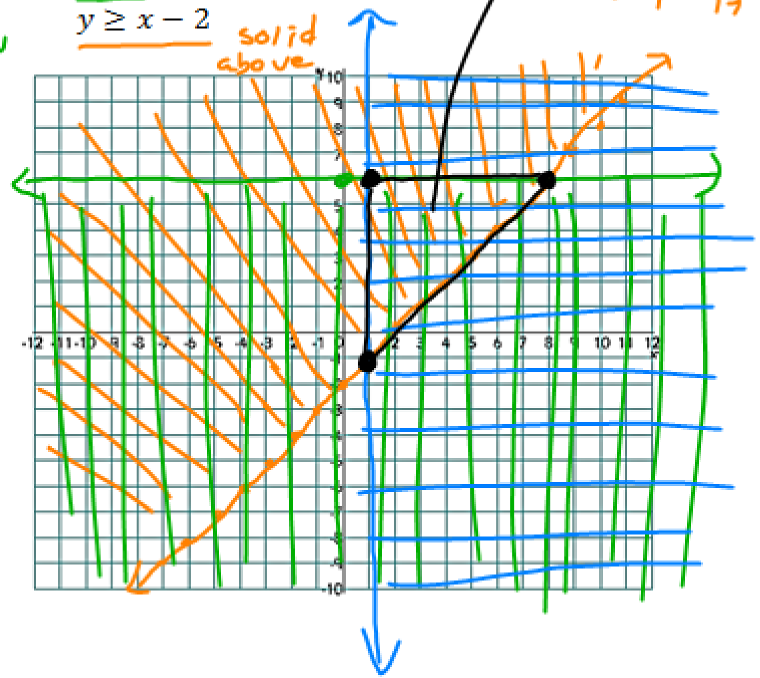
unbounded solution

2. $x \geq 1$ — vertical / solid / above

$y \leq 6$ — horiz / solid / below

$y \geq x - 2$ — solid, above; slope: $\frac{1\uparrow}{1\rightarrow}$ y-int: -2

bounded



Vertices: (1,-1); (8,6), (1,6)

3. Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 3x + y$ for this region.

$x \geq 1$
$y \geq 0$
$2x + y \leq 6$

Step 1: Graph the inequalities (solve the last inequality for y). Then, find the vertices of the region.

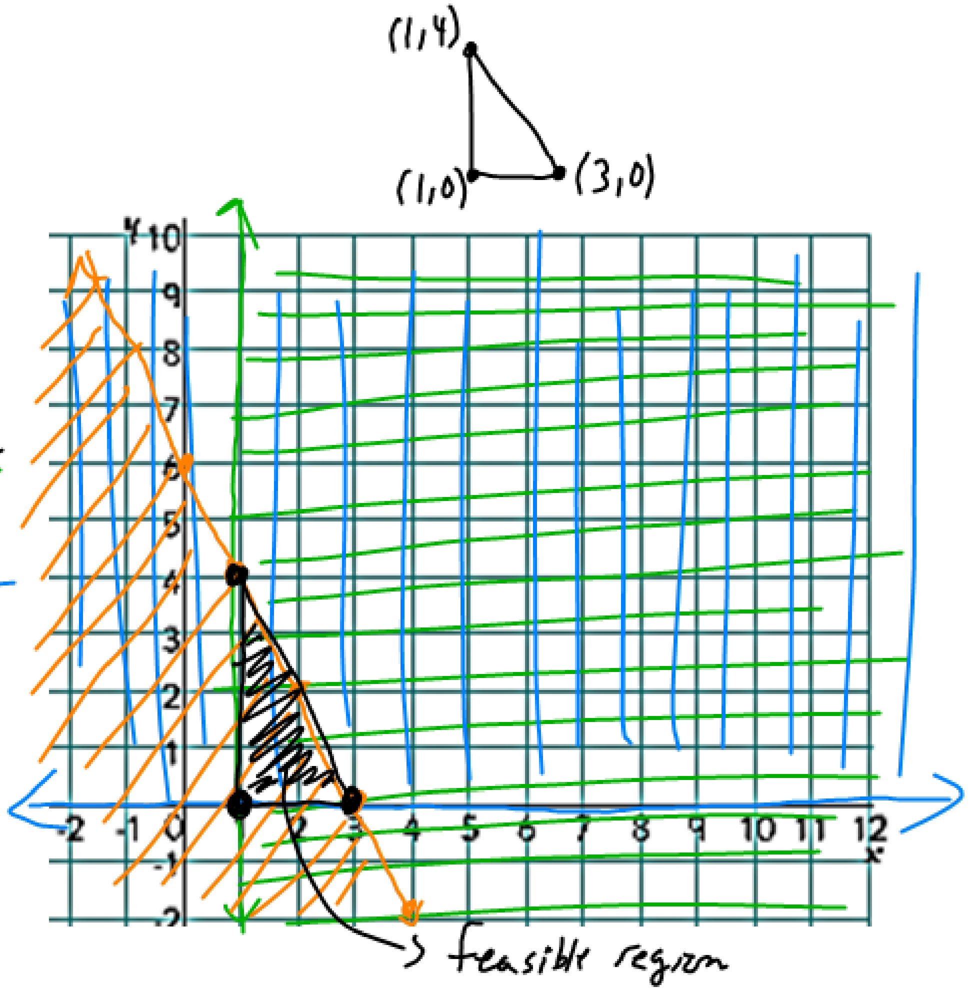
X ≥ 1 Vertical / solid / above

Y ≥ 0 Horiz / solid / above

$2x + y \leq 6$
$-2x \quad -2x$

$y \leq -2x + 6$

Slope: $\frac{-2}{1}$ y-int: 6 Solid Line
Shade below



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Step 2: Use a table to find the maximum and minimum values of $f(x,y)$. Substitute the coordinates of the vertices into the function.

(x,y)	$3x+y$	$f(x,y)$
(1,4)	3(1) + 4 = 3 + 4	7
(1,0)	3(1) + 0 = 3 + 0	3 min
(3,0)	3(3) + 0 = 9 + 0	9 MAX

The maximum is 9 at (3,0).

The minimum is 3 at (1,0).

4. Graph the following system of inequalities. Name the coordinates of the vertices of the

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4. Graph the following system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function $f(x, y) = 5x + 4y$ for this region.

① $2x + y \geq 3$
② $3y - x \leq 9$
③ $2x + y \leq 10$

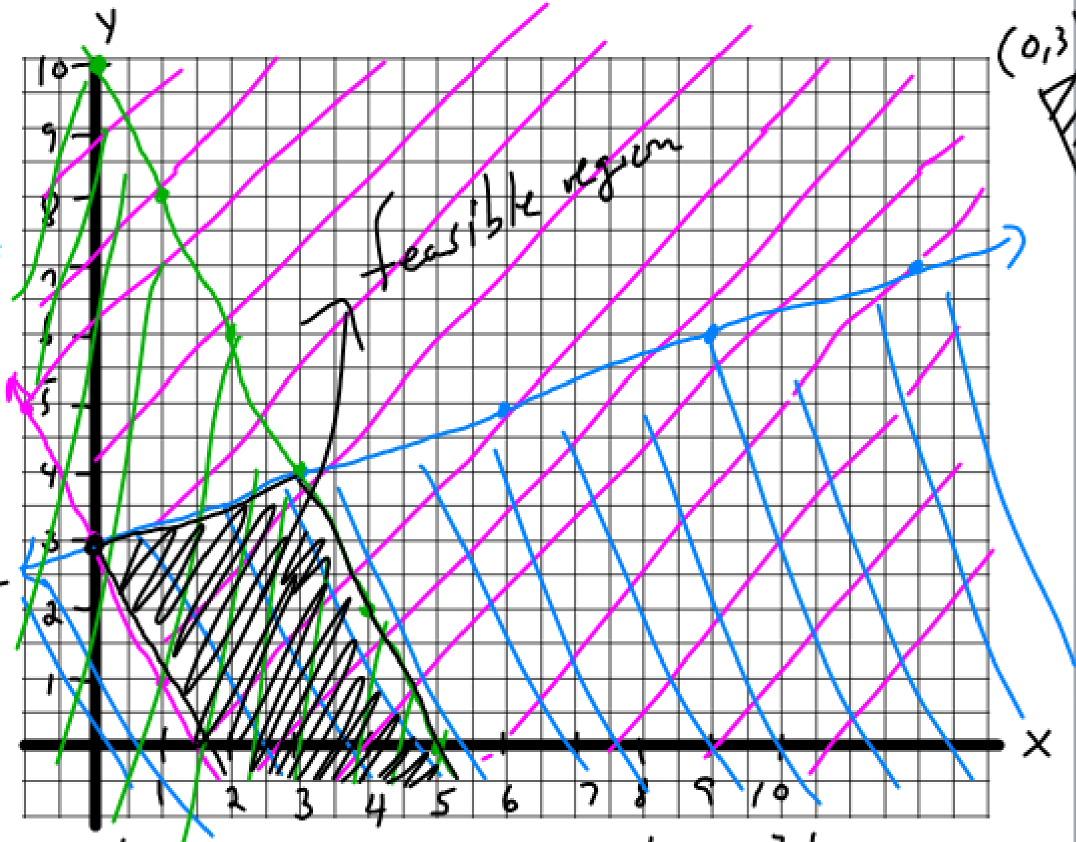
Step 1: Graph the inequalities (solve each inequality for y). Then, find the vertices of the region.

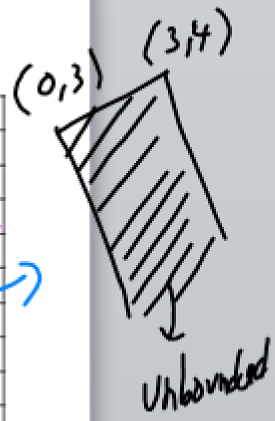
① $2x + y \geq 3$
$-2x \quad -2x$
$y \geq -2x + 3$

slope: $-\frac{2}{1}$ ↓ →
y-int: 3
solid / above

② $3y - x \leq 9$
$+x \quad +x$
$3y \leq x + 9$
$\frac{3y}{3} \leq \frac{x}{3} + \frac{9}{3}$
$y \leq \frac{1}{3}x + 3$

slope: $\frac{1}{3}$ ↑ →
y-int: 3
solid / below





③ $2x + y \leq 10$
$-2x \quad -2x$
$y \leq -2x + 10$

slope: $-\frac{2}{1}$ ↓ →
y-int: 10
solid / below

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Step 2: Use a table to find the maximum and minimum values of $f(x,y)$. Substitute the coordinates of the vertices into the function.

(x,y)	5x+4y	f(x,y)
(0,3)	5(0)+4(3) = 0+12	12
(3,4)	5(3) + 4(4) = 15 + 16	31 max

The maximum is 31 at (3,4).

Is there a minimum? Why? No min b/c if I take a pt. inside the feasible unbounded region, I will get a value lesser than 12: Ex (3,-1) → 5(3) + 4(-1) = 15 - 4 = 11

∴ No minimum exists.

11 < 12

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