Honors Advanced Algebra DATE:________________

6-1 Roots of Real Numbers

Target 4A. Use properties of integer exponents and apply them to rational exponents

Finding the square root of a number and squaring a number are inverse operations.

Definition of Square Root
For any real numbers a and b, if $a^2 = b$, then a is a square root of b. "a is sqrt of b" $a = \sqrt{b}$
Example: $5^2 = 25 \Rightarrow \sqrt{25} = 5$

Since finding the square root of a number and squaring a number are inverse operations, it makes sense that the inverse of raising a number to the nth power is finding the nth root of a number.

Powers	Factors	Roots
$a^3 = 125$	$125 = 5 \cdot 5 \cdot 5$	5 is a cube root of 125
$a^4 = 81$	$81 = 3 \cdot 3 \cdot 3 \cdot 3$	3 is a fourth root of 81
$a^5 = 32$	$32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2 is a fifth root of 32

Definition of nth Root
For any real numbers a and b, and any positive integer n, if $a^n = b$, then $a = \sqrt[n]{b}$. (Note: If n is odd, there is one real nth root of b. If n is even and b is positive, there are two real nth roots of b. The positive root is called the principal root. The negative root is its opposite.)
Example: $2^3 = 8 \Rightarrow \sqrt[3]{8} = 2$ "one real nth root of 8" ; $2^4 = 16 \Rightarrow \sqrt[4]{16} = 2$, $(-2)^4 = 16 \Rightarrow \sqrt[4]{16} = -2$ "two real nth roots of 16" ; $\sqrt[n]{b}$: n index, radical sign, b radicand

Property of nth Roots of nth Powers
For any real number a, $$\sqrt[n]{a^n} = \begin{cases} a \text{ if } n \text{ is odd} \\ \lvert a \rvert \text{ if } n \text{ is even} \end{cases}$$ Example 1: $\sqrt[3]{(-4)^3} = -4$ ✓ Example 2: $\sqrt[4]{(-4)^4} = \lvert -4 \rvert = 4$ ✓

Find Roots

Simplify. Assume all variables are positive. Since we are making this assumptions, we don't worry

1. $\sqrt{25x^4}$

$\sqrt[2]{5^2 x^4}$

$5x^2$

2. $-\sqrt[2]{(y^2+2)^8}$

$-(y^2+2)^4$

3. $\sqrt[5]{32x^{15}y^{20}}$

$= \sqrt[5]{2^5 x^{15} y^{20}}$

$= 2x^3y^4$

4. $\sqrt{-9}$ $\longrightarrow$ Not a real #

$= 3i$

5. $\pm\sqrt{16x^6}$

$= \pm\sqrt[2]{2^4 x^6}$

$= \pm\ 2^2 x^3$

$= \pm 4x^3$

6. $\sqrt[4]{81(a+1)^{12}}$

$= \sqrt[4]{3^4(a+1)^{12}}$

$= 3(a+1)^3$

7. $\sqrt[3]{-27c^9d^{12}}$

$= \sqrt[3]{(-3)^9c^9d^{12}}$

$= (-3)^3c^3d^4$

$= -27c^3d^4$

8. $\sqrt[2]{x^2+8x+16}$

$= \sqrt[2]{(x+4)(x+4)}$

$= \sqrt[2]{(x+4)^2}$

$= (x+4)$

Think $1^2 = 1\cdot1 = 1$

$1^3 = 1\cdot1\cdot1 = 1$

9. $\sqrt[3]{(2y)^6}$

$= (2y)^2$ $\quad\quad$ 2^2y^2

$= (2y)(2y)$ $\quad$ or $\quad$ $4y^2$

$= 4y^2$

10. $\sqrt[3]{\frac{1}{125}}$

$= \sqrt[3]{\frac{1^3}{5^3}}$

$= \frac{1}{5}$

Under Menu and Algebra:

factor(448)

11. $\sqrt[3]{56x^5y}$

$= \sqrt[3]{2^3\cdot 7\cdot x^3x^2\cdot y}$

$= 2x\sqrt[3]{7x^2y}$

56

$2\cdot 28$

$2\cdot 14$

$2\cdot 7$

$2^3\cdot 7$

12. $\sqrt[6]{448x^7y^7}$

$= \sqrt[6]{2^6\cdot 7\cdot x^6x\cdot y^6y}$

$= 2xy\sqrt[6]{7xy}$

Simplify. Use absolute value signs when necessary. (Note that in these examples we are NOT assuming the variables are positive)

1) $3\sqrt[2]{16hj^2k^4}$

$= 3\cdot\sqrt[2]{2^4 h j^2 k^4}$

$= 3\cdot 2^2|j|k^2\sqrt{h}$

$= 12|j|k^2\sqrt{h}$

2) $5\sqrt[3]{108x^6y^7z^3}$

$= 5\cdot\sqrt[3]{3^3 2^2\cdot x^6\cdot y^6\cdot y\cdot z^3}$

$= 5\cdot 3\cdot x^2\cdot y^2\cdot z\sqrt[3]{4y}$

$= 15x^2y^2z\sqrt[3]{4y}$

3) $-2\sqrt[2]{36x^4y^2z^3}$

$= -2\sqrt[2]{6^2 x^4 y^2 z^2\cdot z}$

$= -2\cdot 6\cdot x^2|y||z|\sqrt{z}$

$= -12x^2|y||z|\sqrt{z}$

4) $3\sqrt[3]{216m^8pq^3}$

$= 3\cdot\sqrt[3]{2^3 3^3 m^6 m^2 p q^3}$

$= 3\cdot 2\cdot 3\cdot m^2 q\sqrt[3]{m^2p}$

$= 18m^2q\sqrt[3]{m^2p}$