Honors Advanced Algebra DATE:________________

6-2 & 6.3 Radical Expressions

Target 4A. Use properties of integer exponents and apply them to rational exponents

Product Property of Radicals
For any real numbers a and b and any integer $n > 1$, 1. if n is even and a and b are both nonnegative, the $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ 2. if n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Simplify each expression.

1. $\sqrt[2]{16p^8q^7}$

$= \sqrt[2]{2^4} \cdot \sqrt[2]{p^8} \cdot \sqrt[2]{q^7}$

$= 2^2 \cdot p^4 \cdot \sqrt[2]{q^6} \cdot \sqrt[2]{q}$

$= 4p^4|q^3|\sqrt{q}$

2. $\sqrt[2]{75a^6b^{13}}$ $= \sqrt[2]{75} \cdot \sqrt[2]{a^6} \cdot \sqrt[2]{b^{13}}$

$= \sqrt[2]{5^2 \cdot 3} \cdot \sqrt[2]{a^6} \cdot \sqrt[2]{b^{12}} \cdot \sqrt[2]{b}$

$= \sqrt[2]{5^2} \cdot \sqrt[2]{3} \cdot \sqrt[2]{a^6} \cdot \sqrt[2]{b^{12}} \cdot \sqrt[2]{b}$

$= 5|a^3|b^6\sqrt[2]{3} \cdot \sqrt[2]{b} = 5|a^3|b^6\sqrt{3b}$

Quotient Property of Radicals
For any real numbers a and $b \neq 0$, and any integer $n > 1$, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

Simplify each expression and rationalize the denominator.

↳ 1st ↳ 2nd

" No radicals in denominator"
So we need to get rid of radicals in denominator. ☺

3. $\sqrt[2]{\frac{x^4}{y^5}} = \frac{\sqrt[2]{x^4}}{\sqrt[2]{y^5}} = \frac{x^2}{\sqrt[2]{y^4} \cdot \sqrt[2]{y}}$

$= \frac{x^2}{y^2\sqrt[2]{y}} \cdot \frac{\sqrt[2]{y}}{\sqrt[2]{y}}$

$= \frac{x^2\sqrt[2]{y}}{y^2\sqrt[2]{y^2}} = \frac{x^2\sqrt[2]{y}}{y^2|y|} = \frac{x^2\sqrt{y}}{y^3}$

4. $\sqrt[5]{\frac{5}{4a}} = \frac{\sqrt[5]{5}}{\sqrt[5]{4a}}$

$= \frac{\sqrt[5]{5}}{\sqrt[5]{2^2 a}} \cdot \frac{\sqrt[5]{2^3 \cdot a^4}}{\sqrt[5]{2^3 \cdot a^4}} = \frac{\sqrt[5]{5 \cdot 2^3 \cdot a^4}}{\sqrt[5]{2^5 \cdot a^5}} = \frac{\sqrt[5]{40a^4}}{2a}$

You can use the Product and Quotient Properties to multiply and divide some radicals, respectively.

Multiply Radicals

5. $6\sqrt[3]{9n^2} \cdot 3\sqrt[3]{24n}$

$= 6 \cdot \sqrt[3]{3^2 \cdot n^2} \cdot 3\sqrt[3]{2^3 \cdot 3 \cdot n}$

$= 18\sqrt[3]{3^3 \cdot 2^3 \cdot n^3}$

$= 18 \cdot 3 \cdot 2 \cdot n$

$= 108n$

6. $5\sqrt[3]{100a^2} \cdot \sqrt[3]{10a}$

$= 5 \cdot \sqrt[3]{10^2 \cdot a^2} \cdot \sqrt[3]{10 \cdot a}$

$= 5\sqrt[3]{10^3 \cdot a^3}$

$= 5 \cdot 10 \cdot a$

$= 50a$

You cannot add radicals in the same manner in which you multiply them. You can only add/subtract radicals if they are "like radical expressions". This means that both the indices and the radicands are alike.

$\hookrightarrow 3\sqrt{7} + 5\sqrt{7} - 1\sqrt{7} = 7\sqrt{7}$

"Not like radicals"

$3\sqrt{5} \quad - 2\sqrt{3}$

Add and Subtract Radicals

7. $2\sqrt{12} - 3\sqrt{27} + 2\sqrt{48}$

$= 2\cdot\sqrt[2]{2^2\cdot 3} - 3\sqrt[2]{3^2\cdot 3} + 2\sqrt[2]{2^4\cdot 3}$

$= 2\cdot 2\sqrt{3} - 3\cdot 3\cdot\sqrt{3} + 2\cdot 2^2\cdot\sqrt{3}$

$= 4\sqrt{3} - 9\sqrt{3} + 8\sqrt{3}$ (Add/subtract coefficients)

$= 3\sqrt{3}$

8. $3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$

$3\sqrt[2]{3^2\cdot 5} - 5\sqrt[2]{2^4\cdot 5} + 4\cdot\sqrt[2]{2^2\cdot 5}$

$= 3\cdot 3\sqrt{5} - 5\cdot 2^2\sqrt{5} + 4\cdot 2\sqrt{5}$

$= 9\sqrt{5} - 20\sqrt{5} + 8\sqrt{5}$ (Add/subtract coefficients)

$= -3\sqrt{5}$

Multiply Radicals Using the Distributive Property

9. $(3\sqrt{5} - 2\sqrt{3})(2 + \sqrt{3})$

$= 3\sqrt{5}\cdot 2 + 3\sqrt{5}\cdot\sqrt{3} - 2\sqrt{3}\cdot 2 - 2\sqrt{3}\cdot\sqrt{3}$

$= 6\sqrt{5} + 3\sqrt{15} - 4\sqrt{3} - 2\sqrt{9}$

$= 6\sqrt{5} + 3\sqrt{15} - 4\sqrt{3} - 2\cdot 3$

$= \boxed{6\sqrt{5} + 3\sqrt{15} - 4\sqrt{3} - 6}$

No like terms ☹

10. $(5\sqrt{3} - 6)(5\sqrt{3} + 6)$

	$5\sqrt{3}$	$+6$
$5\sqrt{3}$	$25\sqrt{9}$ $25\cdot 3$ $=75$	$30\sqrt{3}$
-6	$-30\sqrt{3}$	-36

$75 + \cancel{30\sqrt{3}} - \cancel{30\sqrt{3}} - 36$

$= 75 - 36$

$= \boxed{39}$

Use a Conjugate to Rationalize a Denominator

11. $\frac{1-\sqrt{3}}{5+\sqrt{3}}$

$= \frac{(1-\sqrt{3})}{(5+\sqrt{3})}\cdot\frac{(5-\sqrt{3})}{(5-\sqrt{3})}$

$\hookrightarrow$ conjugate

	5	$-\sqrt{3}$
5	25	$-5\sqrt{3}$
$+\sqrt{3}$	$\cancel{5\sqrt{3}}$	$-\sqrt{9}$ -3

$= \frac{5+3-5\sqrt{3}-\sqrt{3}}{25-3}$

	5	$-\sqrt{3}$
1	5	$-\sqrt{3}$
$-\sqrt{3}$	$-5\sqrt{3}$	$+\sqrt{9}$ $+3$

$= \frac{8-6\sqrt{3}}{22}$

$= \frac{8}{22} - \frac{6\sqrt{3}}{22}$

$= \boxed{\frac{4}{11} - \frac{3\sqrt{3}}{11}}$

12. $\frac{2+\sqrt{3}}{4-\sqrt{3}}\cdot\frac{(4+\sqrt{3})}{(4+\sqrt{3})}$

$\hookrightarrow$ conjugate

	4	$+\sqrt{3}$
2	8	$2\sqrt{3}$
$+\sqrt{3}$	$4\sqrt{3}$	$\sqrt{9}$ $=3$

$= \frac{8+3+4\sqrt{3}+2\sqrt{3}}{16-3}$

	4	$+\sqrt{3}$
4	16	$\cancel{4\sqrt{3}}$
$-\sqrt{3}$	$\cancel{-4\sqrt{3}}$	$-\sqrt{9}$ -3

$= \frac{11+6\sqrt{3}}{13}$

$= \frac{11}{13} + \frac{6\sqrt{3}}{13}$