Honors Advanced Algebra **DATE:**________________

Rational Exponents

Target 4A. Use properties of integer exponents and apply them to rational exponents

Review of Properties of Powers	
Suppose a and b are real numbers and m and n are integers. Then the following properties hold:	
Example:	**Rule:**
1. Negative exponents: $2^{-3} = \frac{1}{2^3}$, $2x^{-5} = \frac{2}{x^5}$	$a^{-m} = \frac{1}{a^m}$
2. Product of Powers: $3^2 \cdot 3^5 = 3^{2+5} = 3^7$, $x^2 \cdot x^3 = x^{2+3} = x^5$	$a^m \cdot a^n = a^{m+n}$
3. Quotient of Powers: $\frac{x^{10}}{x^4} = x^{10-4} = x^6$, $\frac{5^3}{5^2} = 5^{3-2} = 5^1$	$\frac{a^m}{a^n} = a^{m-n}$
4. Power of a Power: $(a^3)^2 = a^{3\cdot2} = a^6$, $(8^4)^2 = 8^{4\cdot2} = 8^8$	$(a^m)^n = a^{m\cdot n}$
5. Power of a Product: $(x^2y^4)^3 = x^{2\cdot3}y^{4\cdot3} = x^6y^{12}$, $(2\cdot3)^5 = 2^5\cdot3^5$	$(a\cdot b)^m = a^m\cdot b^m$
6. Power of a Quotient: $\left(\frac{3^2}{4^5}\right)^6 = \frac{3^{2\cdot6}}{4^{5\cdot6}} = \frac{3^{12}}{4^{30}}$, $\left(\frac{x}{y^2}\right)^{-3} = \frac{x^{-3}}{y^{-6}} = \frac{y^6}{x^3}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $\left(\frac{a}{b}\right)^{-m} = \frac{a^{-m}}{b^{-m}} = \frac{b^m}{a^m}$

$b^{\frac{1}{n}}$
For any real number b and any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even. Index
Example: $\sqrt[3]{5^1} = 5^{\frac{1}{3}}$, $\sqrt[6]{5} = 5^{\frac{1}{6}}$

Write each expression in radical form.

1. $a^{\frac{1}{4}}$ $\sqrt[4]{a^1} = \sqrt[4]{a}$

2. $x^{\frac{2}{5}}$ $\sqrt[5]{x^2}$

Write each radical using rational exponents.

3. $\sqrt[3]{y}$ $y^{\frac{1}{3}}$

4. $\sqrt[8]{c^3}$ $c^{\frac{3}{8}}$

Evaluate Expressions with Rational Exponents

Evaluate each expression.

5. $16^{-\frac{1}{4}}$

$$(2^4)^{-\frac{1}{4}} = 2^{4\cdot -\frac{1}{4}} = 2^{\frac{4}{1}\cdot -\frac{1}{4}} = 2^{\frac{-4}{4}} = 2^{-1} = \frac{1}{2^1}$$

$$= \frac{1}{2}$$

6. $243^{\frac{3}{5}}$

$$(3^5)^{\frac{3}{5}} = 3^{\frac{5}{1}\cdot\frac{3}{5}} = 3^{\frac{15}{5}} = 3^3 = 27$$

Rational Exponents
For any nonzero real number b, and any integers m and n, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$, except when $b < 0$ and n is even **Example:** $5^{\frac{2}{3}} = \sqrt[3]{5^2} = \left(\sqrt[3]{5}\right)^2$

Simplify Expressions

All of the properties of powers we have previously learned still apply to rational exponents. When simplifying expressions containing rational exponents, leave the exponents in rational form rather than writing the expression as a radical. To simplify, you must write the expression with all positive exponents. Furthermore, any exponents in the denominator of a fraction must be positive integers. So it may be necessary to rationalize a denominator.

Simplify Expressions with Rational Exponents

Simplify each expression.

7. $x^{\frac{1}{5}} \cdot x^{\frac{7}{5}}$ $= x^{\frac{1}{5}+\frac{7}{5}} = x^{\frac{8}{5}}$

or $\sqrt[5]{x^8} = \sqrt[5]{x^3\cdot x^5} = x\sqrt[5]{x^3}$

8. $y^{-\frac{3}{4}}$ $= \frac{1}{y^{3/4}} \cdot \frac{y^{1/4}}{y^{1/4}} = \frac{y^{1/4}}{y^{3/4+1/4}}$

$= \frac{y^{1/4}}{y^{4/4}} = \frac{y^{1/4}}{y}$ or $\frac{\sqrt[4]{y}}{y}$

9. $y^{\frac{1}{7}} \cdot y^{\frac{4}{7}}$

$y^{\frac{1}{7}} \cdot y^{\frac{4}{7}} = y^{\frac{1}{7}+\frac{4}{7}} = y^{\frac{5}{7}}$

or $\sqrt[7]{y^5}$

10. $x^{-\frac{2}{3}}$

$= \frac{1}{x^{2/3}} \cdot \frac{x^{1/3}}{x^{1/3}} = \frac{x^{1/3}}{x^{3/3}} = \frac{x^{1/3}}{x}$

or $\frac{\sqrt[3]{x}}{x}$

11. $\frac{1}{a^{\frac{2}{5}}}$

$\frac{1}{a^{2/5}} \cdot \frac{a^{3/5}}{a^{3/5}} = \frac{a^{3/5}}{a^{2/5+3/5}} = \frac{a^{3/5}}{a^{5/5}} = \frac{a^{3/5}}{a}$

or $\frac{\sqrt[5]{a^3}}{a}$

12. $\frac{28}{7^{\frac{3}{4}}}$

$= 28 \cdot 7^{-3/4} = (2^2\cdot 7)\cdot 7^{-3/4}$

$= 2^2\cdot 7^1\cdot 7^{-3/4} = 4\cdot 7^{1+-3/4} = 4\cdot 7^{\frac{1}{4}}$

or $4\sqrt[4]{7}$

$1 + -\frac{3}{4}$

$\frac{4}{4} - \frac{3}{4}$

$\frac{4-3}{4} = \frac{1}{4}$

13. $\left(\frac{b^{-12}}{216}\right)^{-\frac{1}{3}}$

$= \frac{b^{-12\cdot -\frac{1}{3}}}{216^{-\frac{1}{3}}} = \frac{b^4}{(2^3\cdot 3^3)^{-\frac{1}{3}}} = \frac{b^4}{2^{3\cdot -\frac{1}{3}}\cdot 3^{3\cdot -\frac{1}{3}}}$

$= \frac{b^4}{2^{-1}\cdot 3^{-1}} = 2\cdot 3\cdot b^4 = 6b^4$

14. $a^{\frac{1}{4}} \cdot a^{\frac{3}{10}} \cdot a^{\frac{2}{5}}$ common denominator? LCD = 20

$a^{\frac{1}{4}+\frac{3}{10}+\frac{2}{5}} = a^{\frac{1}{4}\cdot\frac{5}{5}+\frac{3}{10}\cdot\frac{2}{2}+\frac{2}{5}\cdot\frac{4}{4}}$

$= a^{\frac{5}{20}+\frac{6}{20}+\frac{8}{20}}$

$= a^{\frac{5+6+8}{20}} = a^{\frac{19}{20}}$ or $\sqrt[20]{a^{19}}$