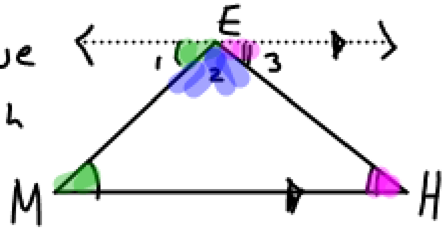
7.1. Honors Geometry

DATE: 1/30

Target 6A. Use and apply interior and exterior angle theorems

Interior Angle Theorem for a Δ: The sum of measures of three angles of a Δ is 180°.

Proof: According to the parallel line postulate, we can construct a line parallel to $\overline{MH}$ through point E. Since ∠1, ∠2, ∠3 make up a st. ∠,



∠1+∠2+∠3 = 180. But ∥ lines ⟹ AI ∠s ≅.

Given: △MEH

So ∠1 ≅ ∠M and ∠3 ≅ ∠H. By substitution,

Prove: ∠M + ∠E + ∠H = 180

∠M + ∠2 + ∠3 = 180. Hence, ∠M + ∠E + ∠H = 180. ✓

↳ ∠2 is the same as ∠E

Exterior Angle Theorem for a Δ: The measure of an exterior angle of a Δ is equal to the sum of the measures of the remote interior angles.

Proof:

Exterior Angle Theorem for a Δ: The measure of an exterior angle of a Δ is equal to the sum of the measures of the remote interior angles.

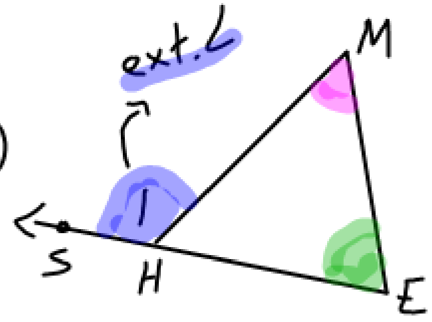
Proof: $\angle 1 + \angle MHE = 180°$ (def. supp. $\angle$s.)

$\angle M + \angle E + \angle MHE = 180°$ (Sum of $\angle$s in $\Delta = 180°$)

So $\angle 1 + \cancel{\angle MHE} = \angle m + \angle E + \cancel{\angle MHE}$ (subst)

$-\cancel{\angle MHE}$ $\quad -\cancel{\angle MHE}$

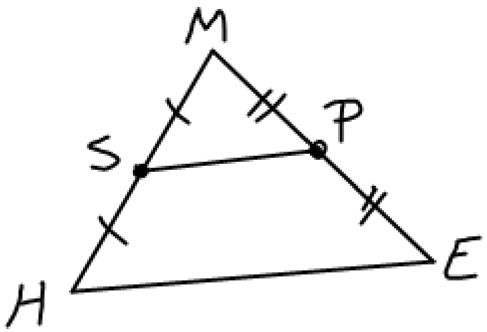
$\therefore \angle 1 = \angle M + \angle E$ ✓



Given: ΔMEH with $\overline{HE}$ extented to form $\angle 1$

Prove: $\angle 1 = \angle m + \angle E$

Midline Theorem for a Δ: A segment joining the midpoints of two sides of a Δ is parallel to the third side, and its length is one-half the length of the third side. (Proof on page 296)



Given: ΔMHE

S midpt of $\overline{HM}$

P midpt of $\overline{ME}$

Prove: $\overline{SP} \parallel \overline{HE}$ and

$\frac{1}{2}\overline{HE} = SP$ (or $2\overline{SP} = \overline{HE}$)

7.1 Interior Angle Formulas [Compatibility Mode] - Microsoft Word

Examples

1) The measures of three angles of a triangle are in the ratio of 3:4:5. Find the measure of the largest angle.

$$3x + 4x + 5x = 180$$

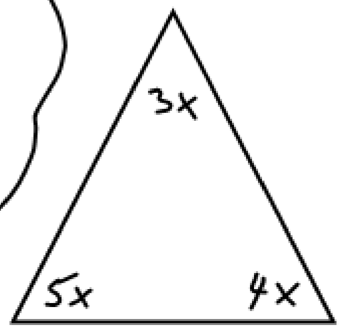
$$\frac{12x}{12} = \frac{180}{12}$$

$$x = 15$$

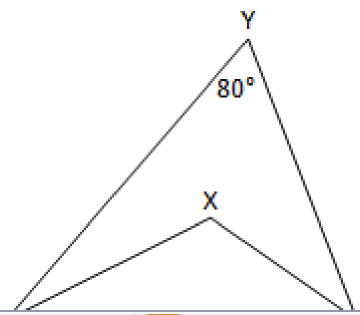
Largest $\angle$ =

$$5(15)$$

$$= 75^\circ$$



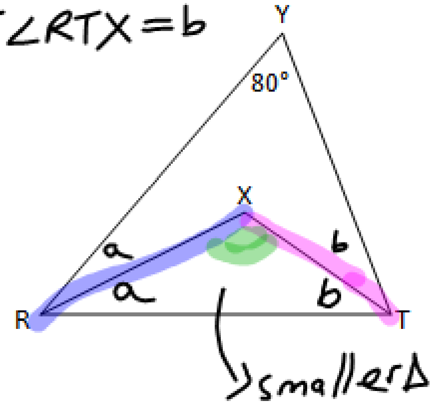
2) If one of the angles of a triangle is 80°, find the measure of the angle formed by the bisectors of the other two angles.



2) If one of the angles of a triangle is 80°, find the measure of the angle formed by the bisectors of the other two angles.

$\overrightarrow{RX}, \overrightarrow{TX}$ bisectors $\Rightarrow \angle YRX \cong \angle TRX = a$

$\angle YTX \cong \angle RTX = b$



$80 + a + a + b + b = 180$

$80 + 2a + 2b = 180$

$-80 \qquad\qquad -80$

$\dfrac{2a}{2} + \dfrac{2b}{2} = \dfrac{100}{2}$

$\boxed{a+b} = 50$

$\boxed{a+b} + \angle X = 180$

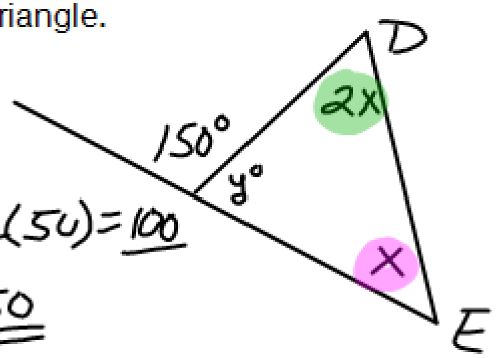
$50 + \angle X = 180$

$-50 \qquad -50$

$\angle X = 130$ ✓

3) The exterior angle of ΔDEF is 150°. If ∠D and ∠E are the remote interior angles and the measure of ∠D is twice that of ∠E, find the measure of each angle of the triangle.

By exterior ∠ thm:



$2x + x = 150$

$3x = 150$

$x = 50$

$\angle D = 2x = 2(50) = \underline{100}$

$\angle E = x = \underline{\underline{50}}$

$\angle y° = 180 - 150 = \underline{\underline{30}}$