# 10.1. Advanced Algebra

DATE: 11/26

## Solving Systems of Equations by Graphing

*Target 4A. Understand the relationships between exponential functions and their graphs*

**Exponential Functions:** a function in the form $y = ab^x$, where $a \neq 0, b > 0$, and $b \neq 1$

important to definition

**Asymptote:** a line that a graph approaches but never crosses.

### Graph an Exponential Function

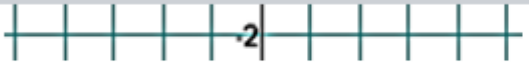
1. Sketch the graph of $y = 2^x$. Then state the function's domain and range.

| X | Y |
|---|---|
| -2 | 0.25 |
| -1 | 0.5 |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |

Domain: the set of all x-values.
Range: the set of y-values

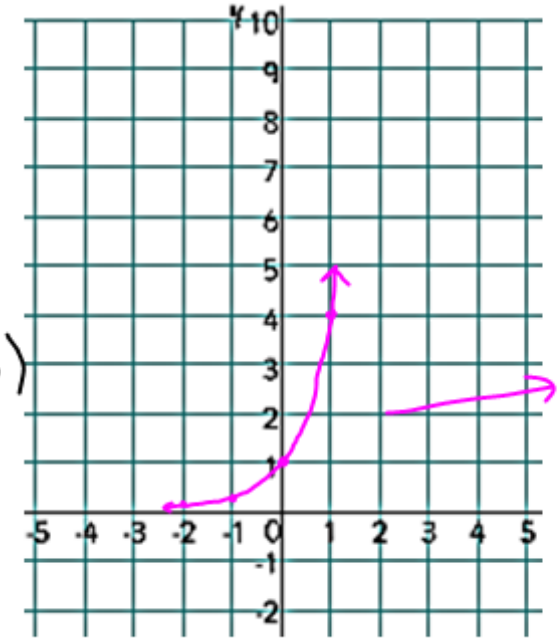

Domain: $\mathbb{R}$
All real #s

Range: $\{y \mid y > 0\}$
"All positive y-values"




2. Sketch the graph of $y = 4^x$. Then state the function's domain and range.

| x | y |
|---|---|
| -2 | 0.0625 ✓ |
| -1 | 0.25 ✓ |
| 0 | 1 ✓ |
| 1 | 4 ✓ |




Domain:
$\mathbb{R}$ (All reals)

Range:
$\{y \mid y > 0\}$
All pos. y #s

In general, an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$, is called an *exponential function* with base $b$.

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**Exponential Functions have the following characteristics:**

1. *The function is continuous and one-to-one*
2. *The domain is the set of all real numbers*
3. *The x-axis is an asymptote of the graph*
4. *The range is the set of all positive numbers if* $a > 0$ *and all negative numbers if* $a < 0$.
5. *The y-intercept is a.*
6. *The graphs of* $y = ab^x$ *and* $y = a\left(\frac{1}{b}\right)^x$ *are reflections across the y-axis.*

**Exponential Growth:** exponential growth occurs when a quantity increases exponentially over time.

- If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential growth.

**Exponential Decay:** exponential decay occurs when a quantity decreases exponentially over time.

- If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

***Determine whether each function represents growth or decay.***

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Left → Right

**Exponential Decay:** exponential decay occurs when a quantity decreases exponentially over time.

- If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

↳ A # less than 1

Left - Right

***Determine whether each function represents growth or decay.***

1. $y = \left(\frac{1}{5}\right)^x$ $\frac{1}{5} = 0.2$ Decay
2. $y = 3(4)^x$ Growth
3. $y = 7(1.2)^x$ Growth
4. $y = (0.7)^x$ Decay
5. $y = \frac{1}{2}(3)^x$ Growth
6. $y = 10\left(\frac{4}{3}\right)^x$ $\frac{4}{3} = 1.\overline{3}$ Growth

**Property of Equality for Exponential Functions:** If $b$ is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.

If b matches, then ready to equate exponents.

Example: If $2^x = 2^8$, then $x =$ 8

***Solve each exponential equation.***

1. $3^{2n+1} = 81$
2. $4^{2x} = 8^{x-1}$
3. $4^{9n-2} = 256$
4. $3^{5x} = 9^{2x-1}$

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***Solve each exponential equation.***

1. $3^{2n+1} = 81$

$3^{2n+1} = 3^4$

$2n + 1 = 4$

$-1 \quad -1$

$2n = 3$

$\frac{2n}{2} = \frac{3}{2}$

$n = 1.5$

2. $4^{2x} = 8^{x-1}$

$(2^2)^{2x} = (2^3)^{(x-1)}$

$2^{4x} = 2^{3(x-1)}$

$4x = 3(x-1)$

$4x = 3x - 3$

$-3x \quad -3x$

$x = -3$

3. $4^{9n-2} = 256$

$4^{9n-2} = 4^4$

$9n - 2 = 4$

$+2 \quad +2$

$9n = 6$

$\frac{9n}{9} = \frac{6}{9}$

$n = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$

$n = 2/3$

4. $3^{5x} = 9^{2x-1}$

$3^{5x} = (3^2)^{2x-1}$

$3^{5x} = 3^{2(2x-1)}$

$5x = 2(2x-1)$

$5x = 4x - 2$

$-4x \quad -4x$

$x = -2$