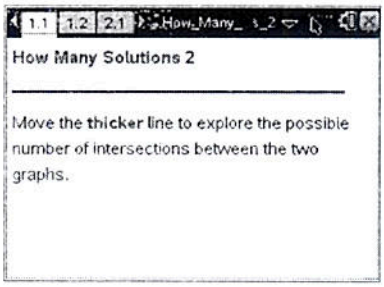
How Many Solutions 2

Student Activity

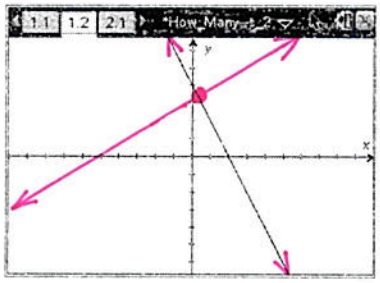
Open the TI-Nspire document *How_Many_Solutions_2.tns*.

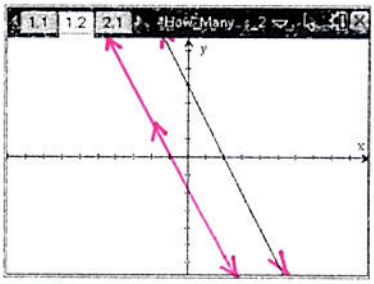


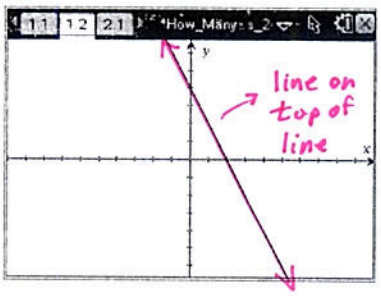
Systems of equations can be solved by the graphing method. This activity lets you manipulate one of two graphs to find the number of possible intersections of a system. When a system of linear equations is represented by two lines, you can see how the number of solutions to that system corresponds to the relationship between the lines.

Move to page 1.2.

1. Move the thicker line to show all possible numbers of solutions to the system of equations. Sketch each possibility below.

A. 

B. 

C. 

|| lines ⟺ slopes equal

2. Given two lines, at how many points can they intersect?

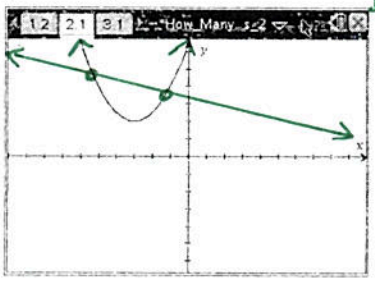
A. One intersection point
∴ One solution

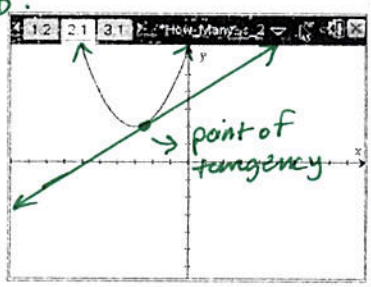
B. No intersection
∴ No solution

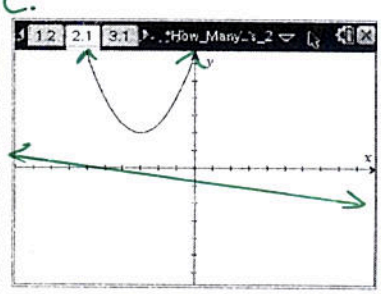
C. Infinitely many intersection points
∴ Inf. many solutions

Move to page 2.1.

3. Move the line to show all possible numbers of solutions to the system of equations. Sketch each possibility below.

A. 

B. 

C. 

4. Given a parabola and a line, at how many points can they intersect?

A. Two intersection points
∴ Two solutions

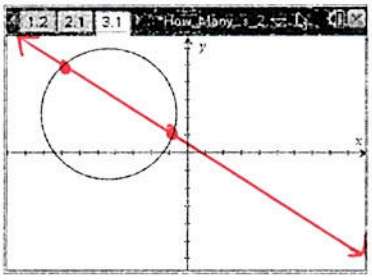
B. One intersection point
∴ One solution
(Line is tangent to parabola @ pt. of tangency)

C. No intersection
∴ No solution

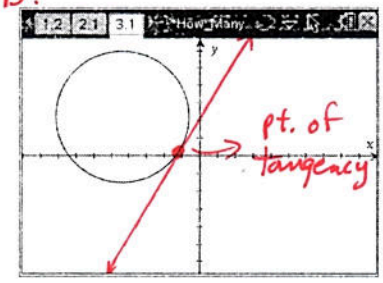
Move to page 3.1.

5. Move the line to show all possible numbers of solutions to the system of equations. Sketch each possibility below.

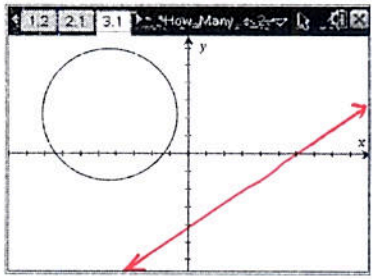
A.



B.



C.



6. Given a circle and a line, at how many points can they intersect?

A. Two intersection points ∴ Two solutions

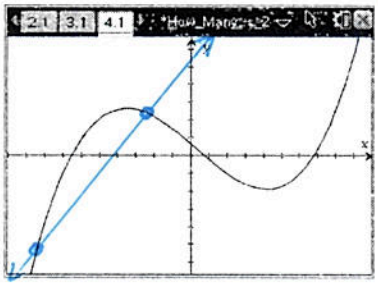
B. One intersection pt. ∴ One solution (Line is tangent to ⊙)

C. No intersection ∴ No solution

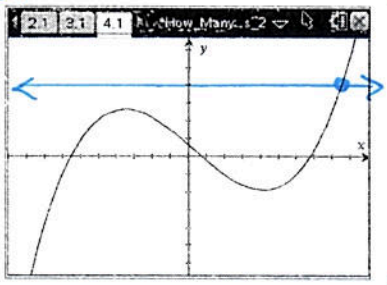
Move to page 4.1.

7. Move the line to show all possible numbers of solutions to the system of equations. Sketch each possibility below.

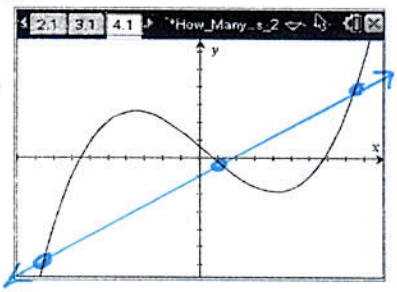
A.



B.



C.



8. Given a cubic and a line, at how many points can they intersect?

A. Two intersection pts ∴ Two solutions

B. One intersection pt. ∴ One solution

C. Three intersection pts. ∴ Three solutions

9. A real root exists at the point where a graph intersects the *x*-axis.

   a. On page 1.2, rotate the thicker line until it appears to be horizontal. Consider this line to be a moveable *x*-axis. Translate the line up and down to determine the possible numbers of real roots. How many possible real roots exist? Exactly 1 real root

   b. Repeat this process for page 2.1. How many possible real roots exist?
   There are 2, 1, or 0 real roots

   c. Repeat this process for page 3.1. How many possible real roots exist?
   There are 2, 1, or 0 real roots

   d. Repeat this process for page 4.1. How many possible real roots exist?
   There are 3, 2, or 1 real roots

10. How do the answers in question 9 relate to the answers in questions 1–8?

The answers are the same! This means that the possible # of real roots of a relation is the same as solving for the possible # of solutions of the system of equations when one of the equations is a horizontal line, i.e., $y = 0$.

11. The vertical line test is used to determine whether a relation is a function. If a vertical line passes through more than one point on the graph of a relation, the relation is not a function.

a. On page 1.2, rotate the thicker line until it appears to be vertical, and then translate it from left to right across the screen. Is the graph a function? Explain.

Yes, passes the vertical line test; i.e., * never intersects the other line @ more than one point.

b. Repeat this process for page 2.1. Is the graph a function? Explain.

Yes, passes the vertical line test; i.e., * never intersects the parabola @ more than one point.

c. Repeat this process for page 3.1. Is the graph a function? Explain.

No, fails the vertical line test; i.e., * the vertical line intersects the ⊙ @ more than one points

d. Repeat this process for page 4.1. Is the graph a function? Explain.

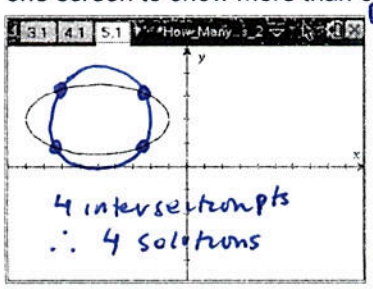
Yes, passes the vertical line test; i.e., * never intersects the cubic function @ more than one point.

* the vertical line

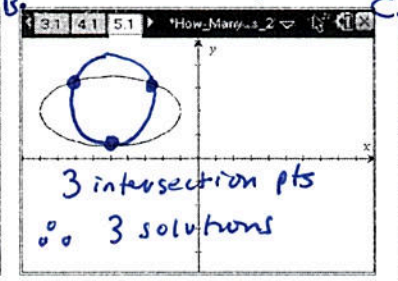
Move to page 5.1.

12. Drag the point to move the circle to show all possible numbers of solutions to the system of equations. Sketch each possibility below. (It is possible to change the diameter of the circle.) Use one screen to show more than one answer if necessary.

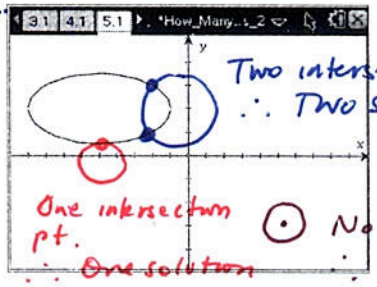
A.



B.



C.



13. Given the following systems, determine all possible numbers of solutions:

a. the graphs of a circle and a parabola

4, 3, 2, 1, or 0 solutions

b. the graphs of two circles

∞ many, 2, 1, or 0 solutions (If one ⊙ or both is an ellipse, then we have 4, 3, 2, 1 or 0 solutions)

* ∞ many

c. the graphs of two parabolas where one parabola is sideways and the other opens up

4, 3, 2, 1, or 0