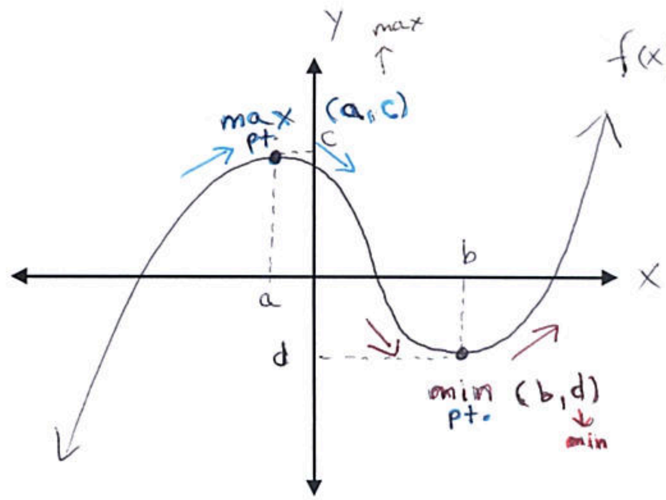
Honors Advanced Algebra **DATE:** 10 | 03

Creating Polynomial Functions

Target 2B. Graph, transform and identify the key features of a polynomial function and make connections between algebraic and graphical forms
Target 2D. Create polynomial functions given factors and zeros

Review of Prior Concepts

If the graph of a polynomial function has several turning points, the function can have a relative maximum and a relative minimum. A *relative maximum* is the value of the function at an __up – to – down__ turning point (the "max" y-value). A *relative minimum* is the value of the function at a __down – to – up__ turning point (the "min" y-value). Furthermore, when we describe weather a function is increasing or decreasing, we do this using __the interval notation__. This notation describes where a function is increasing or decreasing on the __x-interval(s)__.



Increasing : $(-\infty, a) \cup (b, \infty)$
Decreasing : (a, b)

Fundamental Connection

The following are equivalent statements about a real number b and a polynomial

$$f(x) = a_n x^n + \cdots + a_1 x + a_0.$$

- $x - b$ is a linear factor of the polynomial $f(x)$.
- b is a zero of the polynomial function $y = f(x)$.
- b is a root (or solution) of the polynomial equation $f(x) = 0$
- b is an x-intercept of the graph $y = f(x)$.

Writing Polynomials in Factored Form

Examples

$-15 = -5 \cdot 3$ ✓ $\quad -5+3=-2$ ✓

1) What is the factored form of $x^3 - 2x^2 - 15x$?

GCF (Greatest common factor)

$x^3 - 2x^2 - 15x = x \cdot x \cdot x - 2x \cdot x - 15x = x\underbrace{(x^2 - 2x - 15)}_{\text{Factor}} = x(x-5)(x+3)$

2) What is the factored form of $x^3 - x^2 - 12x$?

$x^3 - x^2 - 12x = x \cdot x \cdot x - x \cdot x - 12x = x\underbrace{(x^2 - x - 12)}_{\text{Factor}} = x(x-4)(x+3)$ GCF

$-12 = -4 \cdot 3 \quad -4+3=-1$ ✓

Write each polynomial in factored form. Check by multiplication.

3) $x^3 - 36x$ $= x\underbrace{(x^2 - 36)}_{\text{Factor}}$ GCF

$-36 = -6 \cdot 6 \quad -6+6$ ✓

$= x(x+6)(x-6)$

CHECK: $x(x+6)(x-6) = x(x^2 - 6x + 6x - 36)$
$= x(x^2 - 36) = x^3 - 36x$ ✓

4) $9x^3 + 6x^2 - 3x$ $= 3 \cdot 3x \cdot x \cdot x + 2 \cdot 3x \cdot x - 3x$
$= 3x\underbrace{(3x^2 + 2x - 1)}_{\text{factor}}$ GCF

$3 \cdot -1 = -3 = 3 \cdot -1$

$= 3x(x + \frac{3}{3})(x - \frac{1}{3})$

CHECK: You try it ☺ $= 3x(x+1)(3x-1)$

Finding Zeros of Polynomials

where graph crosses x-axis

Find the zeros of each function.

5) $f(x) = (x+2)(x-1)(x-3)$

set $f(x) = 0$

$0 = (x+2)(x-1)(x-3)$

$0 = x+2$ $0 = x-1$ $0 = x-3$

$-2 \quad -2$ $+1 \quad +1$ $+3 \quad +3$

$\boxed{-2 = x}$ $\boxed{1 = x}$ $\boxed{3 = x}$

Zeros

6) $f(x) = x(x+5)(x-2)(x-3)$

Set $f(x) = 0$

$0 = x(x+5)(x-2)(x-3)$

$0 = x$ $0 = x+5$ $0 = x-2$ $0 = x-3$

$-5 \quad -5$ $+2 \quad +2$ $+3 \quad +3$

$\boxed{0 = x}$ $\boxed{-5 = x}$ $\boxed{2 = x}$ $\boxed{3 = x}$

Zeros

Factor Theorem

The expression $x - b$ is a factor of a polynomial if and only if the value of b is a zero of the related polynomial function.

Creating a Polynomial Given Zeros

7) What is a cubic polynomial function in standard form with zeros -2, 2, and 3?

Zeros: $x = -2$, $x = 2$, $x = 3$

Factors: $\underbrace{(x+2), (x-2)}_{\text{multiply}}$, $(x-3)$

	x	-2
x	x^2	$-2x$
$+2$	$2x$	-4

like terms

$\Rightarrow (x^2 - 4)$ multiply

	x	-3
x^2	x^3	$-3x^2$
-4	$-4x$	12

∴ Cubic poly. function:

$\boxed{f(x) = x^3 - 3x^2 - 4x + 12}$

Write a polynomial function in standard form with the given zeros.

8) $x = 3, -3$ (*degree 2*)

Zeros: $x = 3$, $x = -3$

Factors: $\underbrace{(x-3), (x+3)}_{\text{multiply}}$

	x	-3
x	x^2	$-3x$
$+3$	$+3x$	-9

$\boxed{\therefore f(x) = x^2 - 9}$

9) $x = 1, -1, -2$ (*degree 3*)

Zeros: $x = 1$, $x = -1$, $x = -2$

factors: $\underbrace{(x-1), (x+1)}_{\text{multiply}}$, $(x+2)$

	x	$+1$
x	x^2	$-x$
-1	x	-1

$x^2 - 1$

multiply x^2

	x	$+2$
x^2	x^3	$2x^2$
-1	$-x$	-2

∴ $\boxed{g(x) = x^3 + 2x^2 - x - 2}$

can use any letter