DATE: 10/10

Target 2E: Remainder Theorem

Review of Prior Concepts

Write the factored form of the polynomial function and solve: $f(x) = x^3 - 2x^2 + 2x$

Set $f(x)$ equal to zero:

Factor out GCF

$0 = x^3 - 2x^2 + 2x$

$0 = x(x^2 - 2x + 2)$ $\quad a=1,\ b=-2,\ c=2$

$0 = x$ or $0 = \underbrace{x^2 - 2x + 2}$

Can't factor ☹

∴ so use quadratic formula ☺

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = \frac{2}{2} \pm \frac{2}{2}i = 1 \pm i$$

∴ zeros:
$x = 0$
$x = 1+i$
$x = 1-i$

Perform long division to find the remainder of: $1272 \div 7$

$$\frac{1272}{7} \Rightarrow$$

$$\begin{array}{r} 181 \\ 7 \overline{)1272} \\ -7 \\ \hline 57 \\ -56 \\ \hline 12 \\ -7 \\ \hline 5 \end{array}$$

181 → Quotient
7 → divisor
1272 → Divident
5 → Remainder

$$\therefore \frac{1272}{7} = 181 + \frac{5}{7}$$

Divide $f(x) = \underbrace{x^3 + 4x^2 + 7x - 9}_{\text{original}}$ by $d(x) = x + 3$

<u>Long Division</u>

$$\begin{array}{r} x^2 + x + 4 \\ x+3 \overline{) x^3 + 4x^2 + 7x - 9} \\ -(x^3 + 3x^2) \\ \hline x^2 + 7x \\ -(x^2 + 3x) \\ \hline 4x - 9 \\ -(4x + 12) \\ \hline -21 \end{array}$$

Remainder

- $x^2(x+3) = x^2 + 3x$
- $x(x+3) = x^2 + 3x$
- $4(x+3) = 4x + 12$

$$\therefore \frac{f(x)}{d(x)} = x^2 + x + 4 + \frac{-21}{x+3}$$

<u>Synthetic Division</u>

- Set divisor equal to zero & solve for *x*. $x + 3 = 0$, $x = -3$
- Write coefficients of dividend
  $\boxed{\text{(use 0 for any missing term)}}$ Very important: see example 1 on next page

$$\begin{array}{r|rrrr} -3 & 1 & 4 & 7 & -9 \\ & \downarrow & -3 & -3 & -12 \\ \hline & 1 & 1 & 4 & -21 \end{array}$$

multiply / multiply / multiply

$1x^2 \quad +x \quad +4$ — Quotient

-21 → remainder

1. Bring down Leading coefficient
2. multiply by $x = k$ (In our case k is -3)
3. Let the sign do the work for you (Add)
4. Repeat ② + ③

First number is the coefficient of the x^2 term: when dividing a polynomial by a linear factor $(x-k)$, the resulting polynomial will be one degree less than original.

$\rightarrow$ x^2 term is missing, so fill in with 0.

Example: Divide $f(x) = x^3 + 3x - 4$ by $g(x) = x - 2$ using synthetic division. Write the resulting polynomial.

Think: $x^3 + 0x^2 + 3x - 4$

$x - 2 = 0$
$x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 3 & -4 \\ & \downarrow & 2 & 4 & 14 \\ \hline & 1 & 2 & 7 & 10 \end{array}$$

10 → Remainder

$x^2 + 2x + 7$

$$\therefore \frac{f(x)}{g(x)} = x^2 + 2x + 7 + \frac{10}{x-2}$$

Example: Is $(x-1)$ a linear factor of the polynomial $2x^3 + 7x^2 - 5x - 4$? If so, find the remaining factors.

To answer question, use synthetic division.

$x - 1 = 0$
$x = 1$

$$\begin{array}{r|rrrr} 1 & 2 & 7 & -5 & -4 \\ & \downarrow & 2 & 9 & 4 \\ \hline & 2 & 9 & 4 & 0 \end{array}$$

$\boxed{2x^2 + 9x + 4}$

If the remainder is 0, then $(x-k)$ is a factor

So in our case, $(x-1)$ IS a factor!

For remaining factors, factor $\boxed{2x^2 + 9x + 4}$

multiply

$2 \cdot 4 = 8$
$1 \cdot 8$
$1 + 8 = 9$

$$\left(x + \frac{1}{2}\right)\left(x + \frac{8}{2}\right)$$

$$= \underbrace{(2x+1)(x+4)}_{\text{Remaining two factors}}$$

☺

Example: If you multiply $(x+4)$ by a quadratic polynomial, then $x^3 - 2x^2 - 40x - 64$ is the result. Determine the unknown quadratic polynomial.

Again, synthetic division:

$x + 4 = 0$
$x = -4$

$$\begin{array}{r|rrrr} -4 & 1 & -2 & -40 & -64 \\ & \downarrow & -4 & 24 & 64 \\ \hline & 1 & -6 & -16 & 0 \end{array}$$

$x^2 - 6x - 16$

The unknown quadratic is:

$x^2 - 6x - 16$ ✓

→ missing $0x$

Example (Challenge): Find two factors of $f(x) = 3x^4 - 2x^3 - 9x^2 + 4$ using the graphing calculator and find the other factors using synthetic division.

Use any of the zeros, either $\boxed{x = -1}$ or $\boxed{x = 2}$, and perform synthetic division (twice)!

$$\begin{array}{r|rrrrr} -1 & 3 & -2 & -9 & 0 & 4 \\ & \downarrow & -3 & 5 & 4 & -4 \\ \hline & 3 & -5 & -4 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 3 & -5 & -4 & 4 \\ & \downarrow & 6 & 2 & -4 \\ \hline & 3 & 1 & -2 & 0 \end{array}$$

multiply

$3x^2 + x - 2$

-6
$3 \cdot -2$

$$\left(x + \frac{3}{3}\right)\left(x - \frac{2}{3}\right)$$

$$= \underbrace{(x+1)(3x-2)}_{\text{other factors}}$$