Rational Expressions

Target 3A. Perform operations with rational expressions to demonstrate the analogy with integers.

Rational Expression is a ratio of two polynomial expressions.

Example: $\dfrac{x+8}{x+13}$

Just like with numbers we can't divide by zero, so we have to avoid "plugging in" values to our rational expressions for which the denominator will be zero. We say that the expression is *undefined* at those values.

Undefined Values

Find the values at which the following expressions are undefined.

(handwritten note in cloud) Set denominator equal to zero and solve if needed ... so just look at denominator

1. $\dfrac{5}{x}$

$\boxed{X=0}$

So the value of 0 makes the expression $\frac{5}{x}$ undefined

2. $\dfrac{6x^2-7x+2}{x-4}$

$x-4=0$

$\boxed{x=4}$

So 4 makes the expression $\frac{6x^2-7x+2}{x-4}$ undefined

3. $\dfrac{3x}{x^2-8x+12}$ $\overset{12}{\underset{-6\cdot-2}{\wedge}}$

factor

$x^2-8x+12=0$

$(x-6)(x-2)=0$

$x-6=0$ or $x-2=0$

$\boxed{x=6}$ or $\boxed{x=2}$

4. $\dfrac{x^2+x-12}{x^2+7x+12}$

$x^2+7x+12=0$

$(x+3)(x+4)=0$

$\boxed{X=-3}$ or $\boxed{X=-4}$

Notice for #3 and #4 we have two undefined values

Simplifying Rational Expressions

To simplify rational expressions, we factor the numerator and denominator, then check if there are common factors. If the numerator and denominator have a common factor, those factors can be divided out. However, not all expressions will simplify.

Simplify.

1. $\dfrac{21a^2}{7a^3} = \dfrac{3\cdot 7 \cdot a \cdot a}{7\cdot a \cdot a \cdot a}$ divide out

$= \boxed{\dfrac{3}{a}}$

You try it ☺

2. $\dfrac{4m^3}{28m^4}$

Ans: $\dfrac{1}{7m}$

3. $\dfrac{2x-3}{6x-9} = \dfrac{2x-3}{2\cdot 3 \cdot x - 3\cdot 3}$

$= \dfrac{2x-3}{3(2x-3)}$ → divide out

$= \boxed{\dfrac{1}{3}}$

4. $\dfrac{3x^2-9x}{x-3}$

Ans: $3x$

5. $\frac{2x^2-8x-10}{(x-5)(x^2-1)}$

②$x \cdot x$ -②$2 \cdot 2 \cdot x$ -②$5$

$= \frac{②(x^2-4x-5)}{(x-5)(x^2-1)}$ factor

Think 🙂
x^2-1
↓
$x^2 + 0x - 1$

$\begin{array}{cc} -5 & -1 \\ \wedge & \wedge \\ -5\cdot1 & -1\cdot1 \end{array}$

$= \frac{2(x-5)(x+1)}{(x-5)(x+1)(x-1)}$ → Divide out

$= \boxed{\dfrac{2}{x-1}}$

6. $\frac{c^2-6c+8}{c^2+c-6}$

Ans: $\underline{\dfrac{c-4}{c+3}}$

7. $\frac{n^2-1}{3n-3}$ factor

$= \frac{(n+1)(n-1)}{3(n-1)}$ → divide out

$= \boxed{\dfrac{n+1}{3}}$

8. $\frac{m+4}{m^2+2m-8}$

Ans: $\underline{\dfrac{1}{m-2}}$

Recognizing Opposite Factors

Sometimes you must factor -1 from numerator and/or denominator. When should you do this?
Here are a few examples.

9. $\frac{4-x^2}{7x-14} = \frac{-1(-4+x^2)}{⑦(x-2)}$

⑦x -⑦$2$

$= \frac{-1(x^2-4)}{7(x-2)}$ factor $= \frac{-1(x-2)(x+2)}{7(x-2)}$ → Divide out

$= \boxed{\dfrac{-(x+2)}{7}}$

10. $\frac{y^2-16}{4-y}$

Ans: $-y-4$ or $-(y+4)$

11. $\frac{2x-5}{5-2x} = \frac{2x-5}{-1(-5+2x)}$

$= \frac{1(2x-5)}{-1(2x-5)}$ → divide out

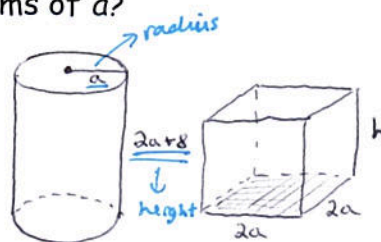$= \frac{1}{-1} = \boxed{-1}$

12. $\frac{3-3z}{2z^2-2}$

Ans: $\dfrac{-3}{2(z+1)}$

Application

You are choosing between two wastebaskets: One is cylindrical with radius a and height 2a + 8
and the other is a rectangular prism with dimensions 2a by 2a by h. They both have the same
volume. What is the height, h, of the rectangular wastebasket in terms of a?

$\text{Volume}_{cylinder} = \pi r^2 h$

$\text{Volume}_{prism} = B \cdot h$, where B = area of base

radius

STEP 1: Find volume of cyliner

$V = \pi r^2 h$

$= \pi (a)^2 (2a+8)$

$= \boxed{\pi a^2 (2a+8)}$

STEP 2: Find height in terms of a.

$B = 2a \cdot 2a = \boxed{4a^2}$ (area of rectangular base)

$V = Bh \Rightarrow \frac{V}{B} = h$

So $h = \frac{V}{B} = \frac{\pi a^2(2a+8)}{4a^2} = \frac{\pi \cdot a \cdot a(a+4)}{2 \cdot 2 \cdot a \cdot a}$

Final ans: $\boxed{= \pi(a+4)}$ → height in terms