DATE: _____

2.8 Solving Equations in One Variable (Target 2F)

Review of Prior Concepts

Find the least common denominator (LCD) in the following equations

a) $\frac{1}{3x} + \frac{5}{9x^2} = \frac{2}{27}$

$\frac{1}{3x} + \frac{5}{3\cdot 3x} = \frac{2}{3\cdot 3\cdot 3}$

$3\cdot 3\cdot 3\cdot x$

$\boxed{\text{LCD: } 27x}$

b) $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$

$2 - \frac{3}{x+4} = \frac{12}{x(x+4)}$

$x(x+4)$

$\boxed{\text{LCD: } x(x+4)}$

c) $\frac{3x}{x+2} + \frac{2}{x-1} = \frac{5}{x^2+x-2}$

$\frac{3x}{x+2} + \frac{2}{x-1} = \frac{5}{(x+2)(x-1)}$

$(x+2)(x-1)$

$\boxed{\text{LCD: } (x+2)(x-1)}$

Solving Rational Equations

① Multiply by the Least Common Denominator, LCD. (to eliminate the fractions/rational expressions)

② Solve for x.

③ Check for extraneous solutions.

Example

$x + \frac{3}{x} = 4$

LCD: x

$x(x) + \frac{3}{x}(x) = 4(x)$

$x^2 + 3 = 4x$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\boxed{x = 3 \quad x = 1}$

check: $3 + \frac{3}{3} = 4$ $\quad$ $1 + \frac{3}{1} = 4$

$3 + 1 = 4$ $\quad$ $1 + 3 = 4$

$4 = 4$ ✓ $\quad$ $4 = 4$ ✓

both check... so both $x=1$, $x=3$ work

Examples:

a) $\frac{3}{x+1} + \frac{2}{x} = 2$ $\quad$ LCD: $x(x+1)$

$x(x+1)\left(\frac{3}{x+1}\right) + \frac{2}{x}(x)(x+1) = 2(x)(x+1)$

$x\cancel{(x+1)}\left(\frac{3}{\cancel{x+1}}\right) + \frac{2}{\cancel{x}}(\cancel{x})(x+1) = 2(x)(x+1)$

$3x + 2x + 2 = 2(x^2 + x)$

$5x + 2 = 2x^2 + 2x$

$0 = 2x^2 - 3x + 2$

$0 = (2x+1)(x-2)$

$\boxed{x = -\frac{1}{2},\ x = 2}$

check: $\frac{3}{-\frac{1}{2}+1} + \frac{2}{-\frac{1}{2}} = 2$ $\quad$ $\frac{3}{2+1} + \frac{2}{2} = 2$

$\frac{3}{\frac{1}{2}} + 2\cdot\frac{-2}{1}$ $\quad$ $\frac{3}{3} + 1$

$3\cdot 2 - 4$ $\quad$ $1 + 1$

$6 - 4$ ✓ $\quad$ 2 ✓

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b) $2 - \frac{3}{x+4} = \frac{12}{x^2+4x}$

LCD: $x(x+4)$

$x(x+4)\cdot 2 - \frac{3}{x+4}x(x+4) = \frac{12}{x(x+4)}\cdot x(x+4)$

$x(x+4)\cdot 2 - \frac{3}{\cancel{x+4}}\cdot x\cancel{(x+4)} = \frac{12}{\cancel{x(x+4)}}\cancel{\cdot x(x+4)}$

$(x^2+4x)2 - 3x = 12$

$2x^2 + 8x - 3x = 12$

$2x^2 + 5x = 12$

$2x^2 + 5x - 12 = 0$

$(2x-3)(x+4) = 0$

$\boxed{x = 3/2}$, $x = -4$ extraneous

Check: $2 - \frac{3}{3/2+4} = \frac{12}{(3/2)^2 + 4(3/2)}$

$\frac{16}{11} = \frac{16}{11}$ ✓

$2 - \frac{3}{-4+4} = \frac{12}{(-4)^2 + 4(-4)}$

$2 - \frac{3}{0} = \frac{12}{0}$

undefined

does n't check

Solve the equation algebraically. Support your answer graphically.

1. $\frac{4x}{x+4} + \frac{3}{x-1} = \frac{15}{x^2+3x-4}$ LCD: $(x+4)(x-1)$

$(x+4)(x-1)\ \frac{4x}{x+4} + \frac{3}{x-1}(x+4)(x-1) = \frac{15}{(x+4)(x-1)}(x+4)(x-1)$

$\cancel{(x+4)}(x-1)\ \frac{4x}{\cancel{x+4}} + \frac{3}{\cancel{x-1}}(x+4)\cancel{(x-1)} = \frac{15}{\cancel{(x+4)(x-1)}}\cancel{(x+4)(x-1)}$

$(x-1)(4x) + 3(x+4) = 15$

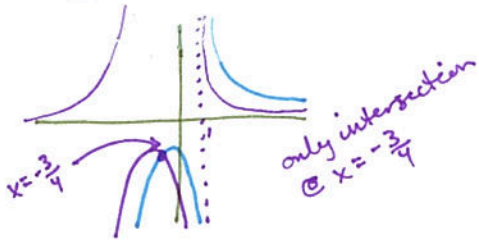
$4x^2 - 4x + 3x + 12 = 15$

$4x^2 - x + 12 = 15$

$4x^2 - x - 3 = 0$

$(4x+3)(x-1) = 0$

$\boxed{x = -3/4}$, $x = 1$ extraneous



2. $\frac{3}{x+2} + \frac{6}{x^2+2x} = \frac{3-x}{x}$ LCD: $x(x+2)$

$x(x+2)\cdot\frac{3}{x+2} + \frac{6}{x(x+2)}\cdot x(x+2) = \frac{3-x}{x}\cdot x(x+2)$

$x\cancel{(x+2)}\cdot\frac{3}{\cancel{x+2}} + \frac{6}{\cancel{x(x+2)}}\cdot\cancel{x(x+2)} = \frac{3-x}{\cancel{x}}\cdot\cancel{x}(x+2)$

$3x + 6 = (3-x)(x+2)$

$3x + 6 = 3x + 6 - x^2 - 2x$

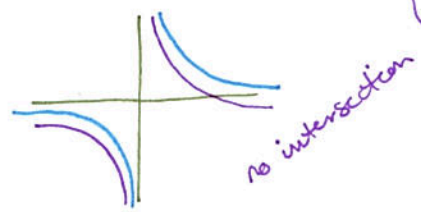
$3x + 6 = -x^2 + x + 6$

$x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0$, $x = -2$ extraneous

$\boxed{\therefore \text{ no solution}}$



Application of Rational Functions

Example: Consider all rectangles with an area of 182 square feet. Let x be the length of one side of such a rectangle.

a) Express the perimeter P as a function of x.

$x \to$ length

$w \to$ width

$A = lw$

$182 = x \cdot w$

$\frac{182}{x} = w$

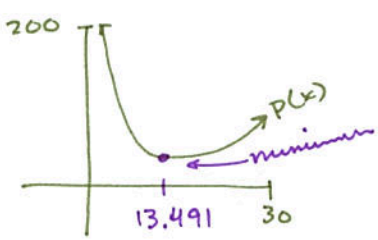
$P = 2l + 2w$

$P = 2x + 2w$

$P = 2x + 2\left(\frac{182}{x}\right)$

$P(x) = 2x + \frac{364}{x}$

b) Find the dimensions of the rectangle that has the least perimeter. What is the least perimeter?



$x = 13.491$

$w = \frac{182}{13.491} = 13.491$

$\boxed{\text{rectangle with least perimeter : } 13.491 \text{ ft} \times 13.491 \text{ ft}}$

least Perimeter : $2(13.491) + 2(13.491)$

$= \boxed{53.963 \text{ ft}}$