P.6 Complex Numbers

Target 2B: Find Real and Complex Zeroes of Polynomials by Synthetic and Long Division

Review of Prior Concepts

Without a graphing calculator, find the zeroes of $f(x) = x^3 - x^2 - x - 2$. Identify if the zeroes are rational or irrational.

$p = \{\pm 1, \pm 2\}$

$q = \{\pm 1\}$

p/q = possible rational zeroes $= \{\pm 1, \pm 2\}$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -1 & -2 \\ & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & -3 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -1 & -2 \\ & & 2 & 2 & 2 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

∴ x = 2 is a zero

$x^2 + x + 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{-1 \pm \sqrt{-3}}{2}$

$= \dfrac{-1 \pm i\sqrt{3}}{2}$ ← irrational

recall: $i = \sqrt{-1}$

Zeroes: $x = 2$ rational

$x = \dfrac{-1 \pm i\sqrt{3}}{2}$ irrational (and imaginary)

More Practice

Rational Zeroes Theorem

http://www.sparknotes.com/math/algebra2/polynomials/section4.rhtml

http://www.math-prof.com/Alg2/Alg2_Ch_16.asp

https://www.youtube.com/watch?v=7mNBBBspqUc

SAT Connection

Passport to Advanced Math

4. Create an equivalent form of an algebraic expression

Example: For $i = \sqrt{-1}$, what is the sum $(7 + 3i) + (-8 + 9i)$?

A) $-1 + 12i$

B) $-1 - 6i$

C) $15 + 12i$

D) $15 - 6i$

$7 + -8 + 3i + 9i$

$-1 + 12i$

Solution

Standard Form of Complex Numbers

Fill in the key ideas from the definition p. 49

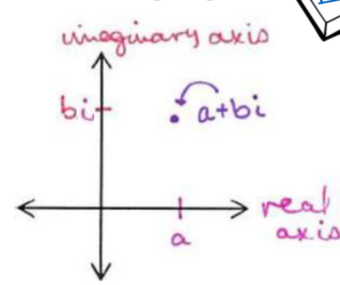
$a + bi$, where a and b are real #s

and $i = \sqrt{-1}$

a: real part

bi: imaginary part

Label the complex plane p. 503-504



https://www.mathsisfun.com/numbers/complex-numbers.html

p. 49-50

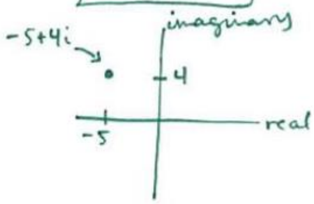
Examples:
Simplify and graph the solution.

1) $(2+i)-(7-3i)$ distribute "−"

$2+i-7+3i$

$2-7+i+3i$

$\boxed{-5+4i}$



2) $(4+i)(3-2i)$ distribute

$12-8i+3i-2i^2$

$12-5i-2i^2$

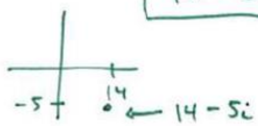
$12-5i-2(-1)$

$12-5i+2$

$12+2-5i$

$\boxed{14-5i}$

$i=\sqrt{-1}$
$i^2=-1$



3) $(2+i)^3$

$(2+i)(2+i)(2+i)$ distribute

$(4+2i+2i+i^2)(2+i)$

$(4+4i+-1)(2+i)$ simplify

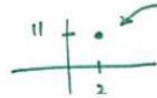
$(3+4i)(2+i)$ distribute

$6+3i+8i+4i^2$

$6+11i+4(-1)$

$6+11i-4$

$\boxed{2+11i}$



Complex Conjugates and Division

p. 51

Vocabulary Term	In my own words…	Examples
Complex Conjugate	answers vary by student * product of complex # and its conjugate is a real # * real part sign stays same, imaginary part sign changes	$a+bi$ conjugate → $a-bi$ $3+2i$ conjugate → $3-2i$ $-4+i$ conjugate → $-4-i$ $7-5i$ conjugate → $7+5i$

Examples:
Write the complex number in standard form.

1) $\frac{3}{4-i}\cdot\frac{4+i}{4+i}$ multiply by 1, with conjugate

$=\frac{3(4+i)}{(4-i)(4+i)}$

$=\frac{12+3i}{16-i^2}$

$=\frac{12+3i}{16-(-1)}$

$=\frac{12+3i}{17} \rightarrow \boxed{\frac{12}{17}+\frac{3}{17}i}$

2) $\frac{2+5i}{3+i}\cdot\frac{3-i}{3-i}$ multiply num + denom. by conjugate of denom.

$=\frac{(2+5i)(3-i)}{(3+i)(3-i)}$

$=\frac{6-2i+15i-5i^2}{9-i^2}$

$=\frac{6+13i-5(-1)}{9-(-1)}$

$=\frac{6+13i+5}{10}$

$=\frac{11+13i}{10}$

$=\boxed{\frac{11}{10}+\frac{13}{10}i}$

Complex Solutions of Quadratic Equations

p. 51-52

When solving a quadratic equation, the discriminant (b^2-4ac) tells whether the solutions are real or imaginary.

Discriminant	Symbolically	# & type of solutions
Positive	$b^2-4ac>0$	2 real solutions
Zero	$b^2-4ac=0$	1 real solution (with multiplicity 2)
Negative	$b^2-4ac<0$	2 imaginary solutions

Examples:

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ … Quadratic Formula

Solve the quadratic equation.

1) $x^2+x+11=5x-8$

$-5x+8 \quad -5x+8$

$x^2-4x+19=0$

$x=\frac{4\pm\sqrt{(-4)^2-4(1)(19)}}{2(1)}$

$=\frac{4\pm\sqrt{-60}}{2}$

$=\frac{4\pm i\sqrt{60}}{2}$

$=\frac{4\pm i\sqrt{4\cdot 15}}{2}$

$=\frac{4\pm 2i\sqrt{15}}{2}$

$=\frac{4}{2}\pm\frac{2i\sqrt{15}}{2}$

$\boxed{x=2\pm i\sqrt{15}}$

$b^2-4ac<0$ 2 imag.

2) $3x^2+x+2=0$

$x=\frac{-1\pm\sqrt{1^2-4(3)(2)}}{2(3)}$

$=\frac{-1\pm\sqrt{-23}}{6}$

$=\frac{-1\pm i\sqrt{23}}{6}$

$\boxed{x=-\frac{1}{6}\pm i\frac{\sqrt{23}}{6}}$

$b^2-4ac<0$ 2 imag.

More Practice

Complex Numbers and Solutions

http://www.regentsprep.org/regents/math/algtrig/ate3/quadcomlesson.htm
http://www.coolmath.com/algebra/10-complex-numbers/03-quadratic-formula-01
https://www.mathsisfun.com/numbers/complex-numbers.html
http://www.virtualnerd.com/algebra-2/quadratics/formula-discriminant/quadratic-formula/complex-solutions-quadratic-formula-example
https://www.youtube.com/watch?v=kpywdu1afas
https://www.youtube.com/watch?v=SP-YJe7Vldo
https://www.khanacademy.org/math/algebra2/introduction-to-complex-numbers-algebra-2

Homework Assignment

p.52 #1,7,9,17,20,33,41,43, p.511#1

SAT Connection

Solution

Choice A is correct. To calculate $(7 + 3i) + (-8 + 9i)$, add the real parts of each complex number, $7 + (-8) = -1$, and then add the imaginary parts, $3i + 9i = 12i$. The result is $-1 + 12i$.

Choices B, C, and D are incorrect and likely result from common errors that arise when adding complex numbers. For example, choice B is the result of adding $3i$ and $-9i$, and choice C is the result of adding 7 and 8.