Name: Key Period:______

Honors Advanced Algebra

Study Guide Key Concept 2—Polynomials

Use the given functions to perform the operations.

$h(x) = 3x - 4$ $\quad g(x) = -2x^2 + 7x - 8$ $\quad f(x) = 8 - 3i$ $\quad b(x) = 12 - i$ $\quad d(x) = -3 + 4i$

1) Find $(h-g)(x)$

$(h-g)(x) = h(x) - g(x)$

Distribute "–" sign

$= 3x - 4 - (-2x^2 + 7x - 8)$

$= 3x - 4 + 2x^2 - 7x + 8$

$= 2x^2 - 4x + 4$

So, $\boxed{(h-g)(x) = 2x^2 - 4x + 4}$

2) Find $(h \cdot g)(x)$

$\boxed{(h\cdot g)(x) = h(x)\cdot g(x) = -6x^3 + 29x^2 - 52x + 32}$

	$-2x^2$	$+7x$	-8
$3x$	$-6x^3$	$+21x$	$-24x$
-4	$8x^2$	$-28x$	32

3) Find $(f + 2b - d)(x) = f(x) + 2\cdot b(x) - d(x)$

$= (8 - 3i) + 2(12 - i) - (-3 + 4i)$

$= 8 - 3i + 24 - 2i + 3 - 4i$

$= \boxed{35 - 9i}$

Write the following in standard form.

4) $(3-8i)^2 =$

$= (3 - 8i)(3 - 8i)$

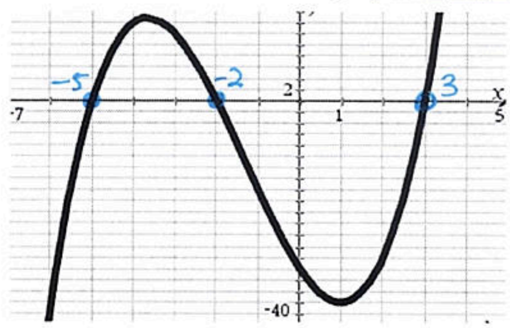
$= \boxed{-55 - 48i}$

	3	$-8i$
3	9	$-24i$
$-8i$	$-24i$	$64i^2$ $64(-1) = -64$

Recall: $i^2 = -1$

Determine the factors of the graphed polynomial

5)



Zeros:

$x = -5$

$x = -2$

$x = 3$

Factors: $(x+5)$, $(x+2)$, $(x-3)$

Create a polynomial of least degree in factored form using the information given below.

6) $x = 8 \quad x = -6 \quad x = -13$

Factors: $(x-8)$, $(x+6)$, $(x+13)$

$\boxed{f(x) = (x-8)(x+6)(x+13)}$ Factored form

7) $x = -2 \quad x = -9i$, $x = 9i$ → complex conjugate

Factors: $(x+2)$, $(x+9i)$, $(x-9i)$

$\boxed{f(x) = (x+2)(x+9i)(x-9i)}$ Factored form

Solve using algebra AND check your work using a graphing calculator.

8) $x^3 - 81x = 0$ Factor GCF x

$x(x^2 - 81) = 0$

$\boxed{x = 0}$ or $x^2 - 81 = 0$

$(x+9)(x-9) = 0$

$x + 9 = 0$ or $x - 9 = 0$

$\boxed{x = -9}$ or $\boxed{x = 9}$

CHECK USING NSPIRE!

9) $2x^3 + 20x^2 = 48x$

$-48x \quad -48x$

$2x^3 + 20x^2 - 48x = 0$ Factor GCF $2x$

$2x(x^2 + 10x - 24) = 0$

$\frac{2x}{2} = \frac{0}{2}$ or $x^2 + 10x - 24 = 0$

$\boxed{x = 0}$ $\quad (x - 2)(x + 12) = 0$

$x - 2 = 0$ or $x + 12 = 0$

$\boxed{x = 2}$ or $\boxed{x = -12}$

-24 ^ $-1\cdot 24$ $-2 \cdot 12$ ✓

10) You know that $2x^3 - 17x^2 + 19x + 14$ has a factor of $(x-2)$. What are the other two factors?

Synthetic Division! $\quad x - 2 = 0$, $x = 2$

$$\begin{array}{r|rrrr} 2 & 2 & -17 & 19 & 14 \\ & & 4 & -26 & -14 \\ \hline & 2 & -13 & -7 & 0 \end{array}$$

$2x^2 - 13x - 7$

Lizzy method

$2\cdot(-7) = -14$ ^ $-14 \cdot 1$ ✓

$(x - \frac{14}{2})(x + \frac{1}{2})$

OTHER TWO FACTORS $\boxed{(x-7)(2x+1)}$

11) Multiplying $(x+4)$ by what quadratic expression gives us $x^3 - 2x^2 - 15x + 36$?

$x + 4 = 0$, $x = -4$

Synthetic Division!

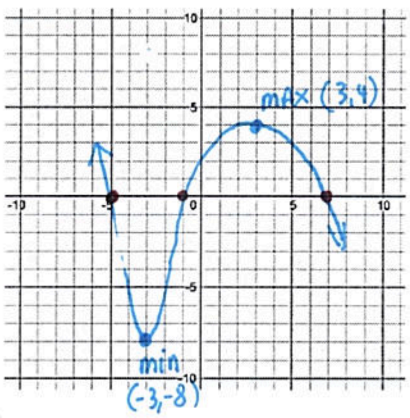
$$\begin{array}{r|rrrr} -4 & 1 & -2 & -15 & 36 \\ & & -4 & 24 & -36 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$\boxed{x^2 - 6x + 9}$ → Quadratic expression

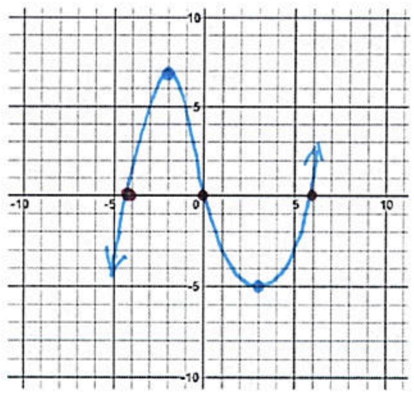
Sketch a polynomial with the following features.

Zeros: $x=-4$, $x=0$, $x=6$

12) Zeros: { -5, -1, 7 }
Min of y = -8 at (-3,-8)
Max of y = 4 at (3,4)



13) Factors: (x+4), x, (x-6)
Min of y = -5 at (3,-5)
Max of y = 7 at (-2,7)

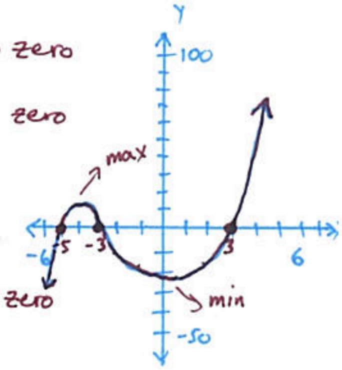


State the intervals that contain the relative min and relative max.

14) → x-intervals

x	f(x)
-6	-27
-5	0
-4	7
-3	0
-2	-15
-1	-32
0	-45
1	-48
2	-35
3	0
4	63
5	160

zero (-5), zero (-3), zero (3); max: -5 to -3; min: 0 to 2



x-interval containing rel. max: [-5,-3]

x-interval containing rel. min: [0,2]

Describe the end behavior in limit notation. Also state the domain and range.

15) $f(x)=x^3-3x+2$

$\lim_{x\to-\infty} f(x) = -\infty$

$\lim_{x\to\infty} f(x) = \infty$

Domain: $(-\infty,\infty)$

Range: $(-\infty,\infty)$

all odd deg. have range $(-\infty,\infty)$

16) $g(x)=-x^3+x^2+5x+1$

$\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = -\infty$

Domain: $(-\infty,\infty)$

Range: $(-\infty,\infty)$

it's odd deg, so range same as 15)

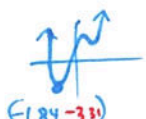
17) $d(x)=x^4+x^3-4x^2+5$

$\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = \infty$

Domain: $(-\infty,\infty)$

Range: $[-3.31,\infty)$



absolute minimum y-value

18) $h(x)=-x^4+4x^2+3x-4$

$\lim_{x\to-\infty} f(x) = -\infty$

$\lim_{x\to\infty} f(x) = -\infty$

Domain: $(-\infty,\infty)$

Range: $(-\infty, 4.49]$

absolute maximum y-value

19). The height of a box is 3 cm less than the width. The length is 2 cm less than the width. The volume is 50 cm cubed. What is the width of the box? Also determine the height and length.

Let x be the width of box.

So, width = x.

h = x − 3 (height of box 3 less than width)

l = x − 2 (length of box 2 less than width)

$V(x) = l\cdot w\cdot h = (x-2)\cdot x\cdot(x-3)$

$50 = x(x-2)(x-3)$
$-50 \quad -50$

$0 = x(x-2)(x-3) - 50$ Graph w/calculator

→ Only zero is $x \approx 5.54$

So width 5.54 cm
$h = 5.54-3 = 2.54$ cm
$l = 5.54-2 = 3.54$ cm

20) A box has the dimensions of x, 10-2x, and 12-2x. Find the maximum value of the box and the value of x that generates that volume.

$V(x) = x(10-2x)(12-2x)$

Graph function!

height x, volume
(1.81, 96.78)

Volume is: 96.78 units^3

height that generates volume is: 1.81 units

Extra Practice: Factor (one of these is NOT factorable)

A) $14x^2-7x$
$= \boxed{7x(2x-1)}$

B). x^2-36
-36: $-6\cdot6$
$= \boxed{(x-6)(x+6)}$

C) x^2+16
Not factorable over real #s

D) $x^2-5x-36$
$-9+4=-5$, -36: $-9\cdot4$
$= \boxed{(x-9)(x+4)}$

E). $2x^2+3x-20$
$2\cdot-20 = -40$: $8\cdot-5$, $8-5=3$
$= (x+\frac{8}{2})(x-\frac{5}{2})$
$= \boxed{(x+4)(2x-5)}$