

Target 8A: Experimental & Theoretical Probability & Probability Distributions and Frequency Tables

Problem: Sophia has three tickets to a concert. Yolanda, Michael, Kevin, and Marissa have all stated they would like to go to the concert with Sophia. To be fair, Sophia decides to randomly select the two people who can go to the concert with her.

- (a) Determine the sample space of the experiment. In other words, list all possible simple random samples of size $n = 2$.
- (b) Compute the probability of the event "Michael and Kevin attend the concert."
- (c) Compute the probability of the event "Marissa attends the concert."
- (d) Interpret the probability in part (c).

b) $P(\text{Mi and Kevin attend show}) = \frac{1}{6} \checkmark$

c) $P(\text{Marissa attends show}) = \frac{3}{6} = \frac{1}{2}$

One way of thinking about sample space

a) Sample Space

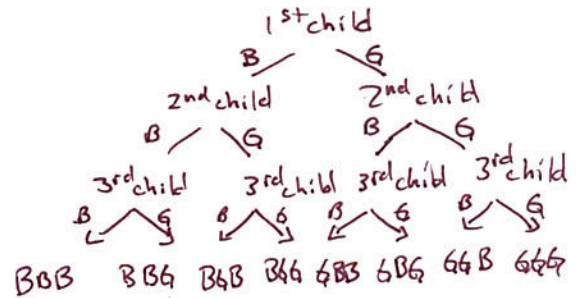
	Y	Mi	K	Ma
Y	YY	YMi	YK	YMa
Mi	MiY	MiMi	MiK	MiMa
K	KY	KMi	KK	KMa
Ma	MaY	MaMi	MaK	MaMa

$S = \{YMi, YK, YMa, MiK, MiMa, KMa\}$

"\ - repeats! since YMi is same as MiY.

d) Conducting this experiment many times would result in Marissa attending show about 50% of time.

Sample Space



Problem: Suppose that a survey is conducted in which 500 families with three children are asked to disclose the gender of their children. Based on the results, it was found that 180 of the families had two boys and one girl.

- (a) Estimate the probability of having two boys and one girl in a three-child family using the empirical method.
- (b) Compute and interpret the probability of having two boys and one girl in a three-child family using the classical method, assuming boys and girls are equally likely.

a) Empirical Probability = Experimental Prob.

Let $E = \text{"two boys and one girl"}$

$P(E) \approx \text{relative frequency of } E = \frac{180}{500} = 0.36 = 36\%$

b) $S = \{BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG\}$

Problem:

Classical Prob = Theoretical Prob

The table below shows the relative frequencies of the ages of the students at a high school.

Age (in years)	Relative frequency
13	0.15
14	0.31
15	0.21
16	0.19
17	0.14
Total	1

a A student is randomly selected from this school. Find the probability that

- i the student is 15 years old, 0.21
- ii the student is 16 years of age or older. $0.19 + 0.14 = 0.33$

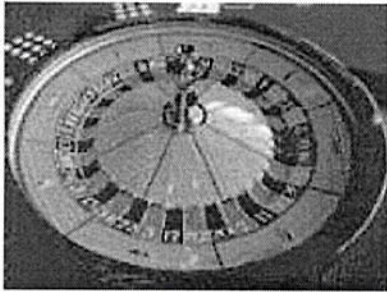
There are 1200 students at this school.

b Calculate the number of 15-year-old students. $= 1200 \cdot 0.21 = 252$

$\therefore P(E) = \frac{3}{8} = 37.5\%$
 \therefore we have 37% prob. that a family of three children will have "two boys and one girl." If we repeat exp. 1000 times (and outcomes are equally likely), we would expect about 375 of the trials to result in "2 boy & 1 girl."

More Practice:

Roulette In the game of roulette, a wheel consists of 38 slots numbered 0, 00, 1, 2, ..., 36. (See the photo.) To play the game, a metal ball is spun around the wheel and is allowed to fall into one of the numbered slots.



- Determine the sample space.
- Determine the probability that the metal ball falls into the slot marked 8. Interpret this probability.
- Determine the probability that the metal ball lands in an odd slot. Interpret this probability.

$$a) S = \underbrace{\{00, 0, 1, 2, 3, 4, \dots, 35, 36\}}_{38 \text{ slots}}$$

$$b) P(\#8 \text{ occurring}) = \frac{1}{38} \quad ; \quad \text{A ball has prob. } 2.6\% \text{ to land in slot \#8.}$$

$$c) \text{ Odd \#s} = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35\}$$

$$\therefore 18 \text{ odd \#s} \quad \text{or} \quad \frac{36}{2} = 18$$

$$\therefore P(\text{odd slot}) = \frac{18}{38} = \frac{9}{19}$$

The sides of a six-sided spinner are numbered from 1 to 6. The table shows the results for 100 spins.

Number on spinner	1	2	3	4	5	6
Frequency	27	18	17	15	16	7

Total 100

- What is the relative frequency of getting a 1?
- Do you think the spinner is fair? Give a reason for your answer.
- The spinner is spun 3000 times. Estimate the number of times the result will be a 4.

$$a) \text{ relative freq.} = \frac{\text{frequency of event}}{\text{\# of trials of experiment}} = \frac{27}{100} = 0.27$$

b) No, frequencies are very different

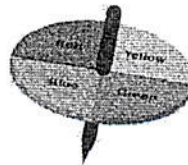
c) A 4 happens $\frac{15}{100} = 0.15 = 15\%$ of time
 So $0.15 \cdot 3000 = 450$ is the number that gives a good indicator of a 4 happening if the spinner is spun 3000 times.

Each letter of the word CONSECUTIVE is written on a separate card. The 11 cards are placed face downwards. A card is drawn at random.

- What is the probability of picking a card with
- the letter C two "Cs" $\frac{2}{11}$
 - the letter P No Ps $\frac{0}{11} = 0$
 - a vowel? 2 Es, 1 I, 1 O, 1 U $\frac{5}{11}$

The spinner shown is biased. The probabilities of getting red and getting blue are shown in the table. The probability of getting green is twice that of getting yellow.

Color	red	yellow	blue	green
Frequency	0.4	X	0.3	2X



Prob of getting green

Find the probability of getting green.

Let x be yellow. Since Frequency must add to 1, we have

$$0.4 + x + 0.3 + 2x = 1 \Rightarrow 3x + 0.7 = 1 \Rightarrow 3x = 0.3 \Rightarrow x = 0.1$$

$\therefore \text{Green} = 2x = 2(0.1) = 0.2$

A bag contains 40 discs numbered 1 to 40. A disc is selected at random. Find the probability that the number on the disc

- is an even number,
- has the digit 1 in it.

1-40 has 20 even #s, so $\frac{20}{40} = \frac{1}{2}$

So the #1 has a digit 1. Then we have 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 31, all have a digit of 1 in it.

There are 13 #s in 1-40 that have a digit 1.
 $\therefore \text{Prob} = \frac{13}{40}$