

PreCal Review for 1st Semester Final

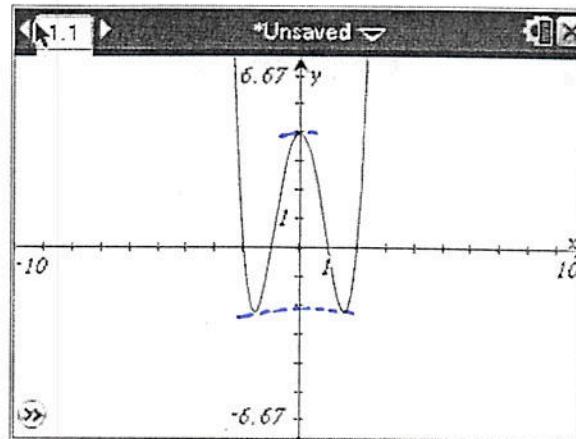
Name: _____

Non-Calculator

- 1) List all relative max or min.

relative max: 4

relative min: -2



- 2) Find the Domain & Range of
- $f(x) = \sqrt{x-1} + 3$
- .

$$D: [1, \infty)$$

$$R: [3, \infty)$$

- 3) Find
- $(f+g)(x)$
- if
- $f(x) = x^3 - 3x + 5$
- &
- $g(x) = x^2 - 5x - 6$

$$\begin{aligned}(f+g)(x) &= x^3 - 3x + 5 + x^2 - 5x - 6 \\ &= x^3 + x^2 - 8x - 1\end{aligned}$$

- 4) Find
- $(f \cdot g)(x)$
- if
- $f(x) = (x+3)^2$
- &
- $g(x) = x-3$
- .

$$\begin{aligned}(f \cdot g)(x) &= (x+3)^2(x-3) \\ &= (x^2+6x+9)(x-3) \\ &= x^3+3x^2-9x-27\end{aligned}$$

- 5) Find
- $(f \circ g)(x)$
- if
- $f(x) = x^2 - 7$
- &
- $g(x) = \sqrt{x+3}$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = (\sqrt{x+3})^2 - 7 \\ &= x+3-7 = x-4\end{aligned}$$

- 6) Find the inverse of
- $h(x) = (x-3)^2 + 9$
- .

$$\begin{aligned}x &= (y-3)^2 + 9 \quad \rightarrow \quad \sqrt{x-9} = \sqrt{(y-3)^2} \\ x-9 &= (y-3)^2 \quad \rightarrow \quad \pm \sqrt{x-9} = y-3 \quad \rightarrow \quad y = \pm \sqrt{x-9} + 3 \\ h^{-1}(x) &= 3 \pm \sqrt{x-9}\end{aligned}$$

- 7) Graph 3 functions that are NOT continuous over Real Numbers.

Sample answers:
Greatest int function

$y = \frac{1}{x}$

$y = \ln x$

- 8) Graph 3 functions that are increasing on the interval
- $(-\infty, 0)$
- .

Sample answers:
 $y = x^3$

$y = x$

$y = -x^2$

- 9) Graph 3 functions that are bounded below.

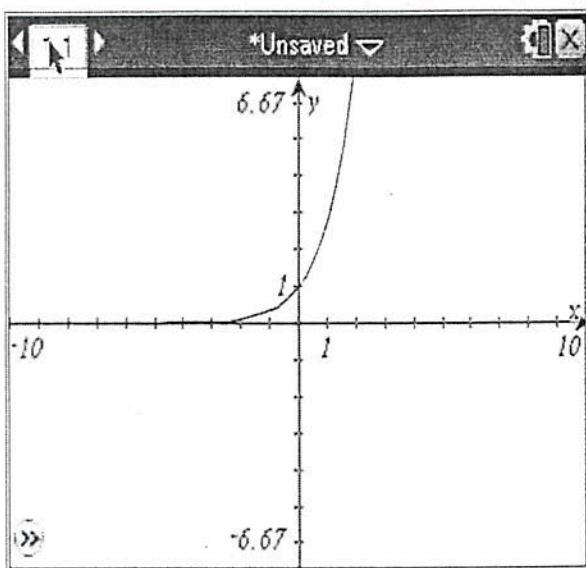
sample answers :

- 10) Find the function that produces this graph.

$$y = x^2$$

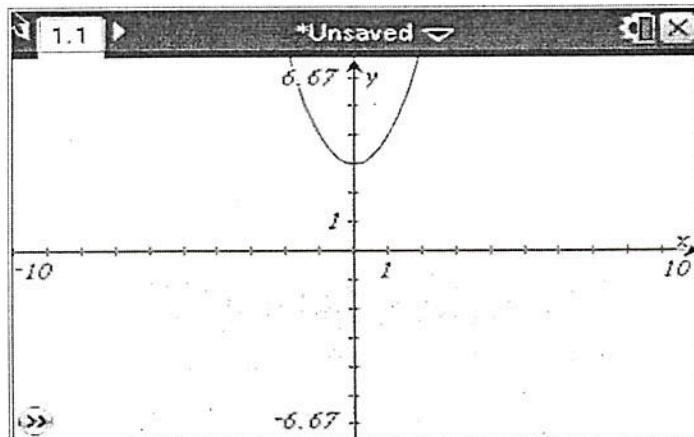
$$y = \sin x$$

$$y = |x|$$



$$y = e^x$$

$$y = x^2 + 3$$



- 11) Find the function that produces this graph.

Calculator

- 12) Find the zeroes of $f(x) = x^2 - 5x + 3$

$$0.697 \text{ and } 4.303$$

- 13) Determine the function that has zeroes @ $\frac{2}{3}, 3, 5$.

$$f(x) = (3x-2)(x-3)(x-5)$$

$$f(x) = 3x^3 - 26x^2 + 61x - 30$$

- 14) What is the end behavior of #13?

$$\lim_{x \rightarrow \infty} f(x) = \infty \quad \lim_{x \rightarrow -\infty} f(x) = -\infty$$

15) Determine the approximate intervals on which #13 is decreasing.

on $(1.637, 4.141)$

16) Reflect across the x-axis: $g(x) = (x - 3)^2 - 5$

\leftarrow ~~smallest negative function~~ $g(x) = -[(x - 3)^2 - 5] = -[(x - 3)^2 - 5]$
or $-[(x - 3)^2 + 5]$

17) What type of function does this describe?

quadratic

18) What type and what is the equation of the best fit regression curve?

# of Minutes	3	4	5	6	8
# of cars	8	15	24	35	63

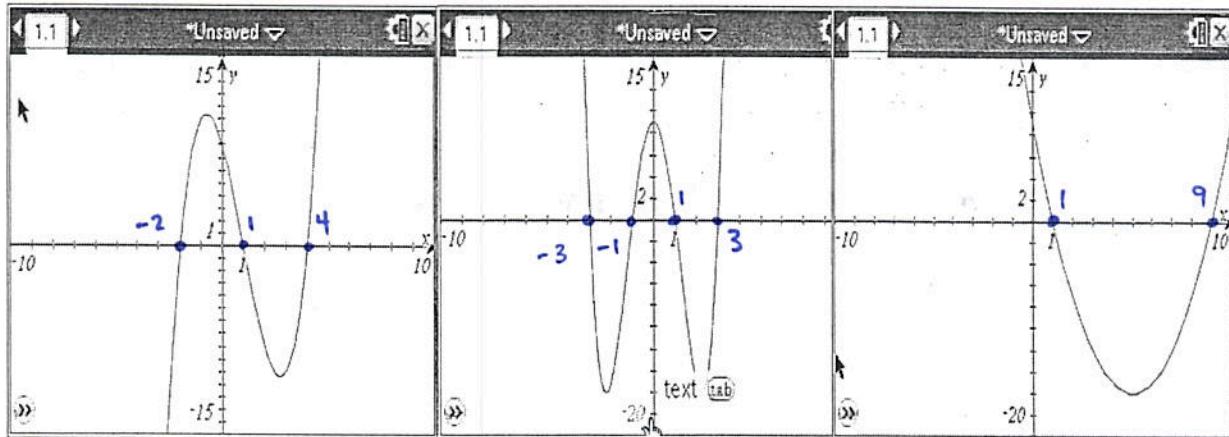
exponential or quadratic
 $y = 2.801(1.498)^x$ $y = x^2 - 1$

19) Graphite Inc. makes tennis racquets. If each racquet costs \$53 to make with fixed overhead costs of \$567,000, what is best fit regression curve?

Linear $y = 53x + 567000$

Non-Calculator

20) Name a possible function that fits each graph.



$f(x) = x^3 - 3x^2 - 6x + 8$

$f(x) = x^4 - 10x^2 + 9$

$f(x) = x^2 - 10x + 9$

21) Given $x = 4$ is a root, find the rest of the zeroes for: $f(x) = x^3 + x^2 - 16x - 16$

$$\begin{array}{r} \boxed{4} & 1 & 1 & -16 & -16 \\ & 4 & 20 & 16 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$

$$\begin{aligned} x^2 + 5x + 4 &= 0 \\ (x+4)(x+1) &= 0 \\ x = -1, x = -4 & \end{aligned}$$

zeroes: $4, -1, -4$

22) Determine the polynomial of least degree given the roots: $3+i, -2$

$x = 3+i, x = 3-i, x = -2$

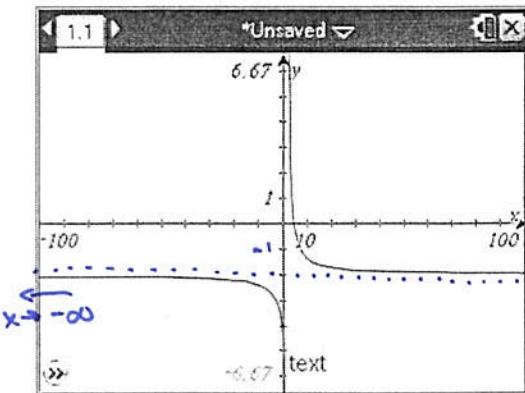
$$P(x) = (x-3-i)(x-3+i)(x+2) \rightarrow f(x) = (x^2 - 6x + 10)(x+2)$$

$$f(x) = x^3 - 4x^2 - 2x + 20$$

23) Determine how many complex zeroes there are for $g(x) = 3x^4 - 6x^2 + 5x - 11$

4 complex zeroes b/c degree of polynomial is 4

24) Determine the limit based upon the graph. $\lim_{x \rightarrow -\infty} g(x)$



$$\lim_{x \rightarrow -\infty} g(x) = -2$$

Calculator

25) Solve for q : $2q^3 - 10q = 5$ $q = -1.924, q = -0.530, q = 2.453$

26) Solve for c : $c^2 + 3 = c$ $c = \frac{1 \pm i\sqrt{11}}{2}$
Quadratic formula

27) Find the vertical and horizontal asymptotes for $h(x) = \frac{x-5}{x+3}$ V.A. @ $x = -3$
H.A. @ $y = 1$

28) Determine all complex zeroes for: $w(x) = x^4 - 8x^2 - 9$ $x = -3, 3, -i, i$

29) How many real zeroes there are for: $b(x) = 2x^3 + 3x^2 + 3x + 9$ one b/c there is one x-intercept.

30) Describe the End Behavior of $m(x) = -2x^3 - x + 1$

$$\lim_{x \rightarrow -\infty} m(x) = \infty \quad \lim_{x \rightarrow \infty} m(x) = -\infty$$

Non Calculator

31) Evaluate: $-7 \log 10^3 - 3 = -7(3)^{-3} = -24$

32) Write $\frac{9}{8} = a^{-2}$ in logarithmic form. $\log_a(\frac{9}{8}) = -2$

33) Solve for m : $\log_{\frac{1}{5}} \sqrt[3]{25}^5 = m$ $(\frac{1}{5})^m = (\sqrt[3]{25})^5$ $5^{-m} = (5^2)^{\frac{5}{3}}$
 $5^{-m} = 25^{\frac{5}{3}}$ $5^{-m} = 5^{\frac{10}{3}}$

34) Solve for q : $\frac{1}{16} = 2^{q-3}$

$$\frac{1}{2^4} = 2^{q-3} \quad -4 = q-3 \\ 2^{-4} = 2^{q-3} \quad -1 = q$$

$$-m = \frac{10}{3} \\ m = -\frac{10}{3}$$

- 35) Condense the expression: $2[5\log(x+2) + \log x] - \log(x+4)$
- $$= 2(\log(x+2)^5 + \log x) - \log(x+4)$$
- $$= 2(\log x(x+2)^5) - \log(x+4)$$
- $$= \log x^2(x+2)^{10} - \log(x+4)$$
- $$= \log\left(\frac{x^2(x+2)^{10}}{x+4}\right)$$
- 36) Solve for w : $\log_5(2w-3) = 2 \rightarrow 5^2 = 2w-3$
- $$25 = 2w-3$$
- $$28 = 2w$$
- $$14 = w$$
- 37) Solve for a : $-4 = \log_a \frac{1}{16}$
- $$a^{-4} = \frac{1}{16} \rightarrow a^{-4} = -4 \rightarrow a = 2$$
- 38) Identify the domain, range, x&y intercepts, and any asymptotes for $y = 3^{x+2} - 1$
- D: $(-\infty, \infty)$ x-int: $0 = 3^{x+2} - 1$ y-int: $y = 3^{0+2} - 1$
R: $(-1, \infty)$ $1 = 3^{x+2}$ $y = 3^2 - 1$
Calculator $3^0 = 3^{x+2}$ $y = 9 - 1$
 $0 = x+2$ $y = 8$
 $-2 = x$
- asymptotes:
H.A. @ $y = -1$
no V.A.

39) Solve for x : $\ln(x+4) + \ln(x-3) = 2\ln 3 \quad x = 4.110$

40) Find the Domain & Range of: $f(x) = e^x + 7 \quad D: (-\infty, \infty)$
R: $(7, \infty)$

41) Find the Domain, Range, X&Y Intercepts, and Asymptotes of:

$$f(x) = -1 + \log_5(x+3)$$

D: $(-3, \infty)$ x-int: $x = -2$ asymptotes: $x = -3$ V.A.
R: $(-\infty, \infty)$ y-int: $y = -0.683$ no H.A.

42) The # of bacteria in a petri dish after "t" hours is $B = 100e^{kt}$ where $t = 0$ represents the time 12pm. At 6am the # of bacteria was 42.

a) Find "k"

b) Using "k", find the # of bacteria at 8pm.

c)

$$\begin{aligned} 42 &= 100e^{k(-6)} \\ \frac{42}{100} &= e^{-6k} \quad \ln\left(\frac{42}{100}\right) = -6k \\ .42 &= e^{-6k} \quad .145 = k \\ B &= 100e^{.145(8)} \quad \text{store the value of } k \\ B &= 317.932 \\ \text{so, } 318 \text{ bacteria} \end{aligned}$$

43) $pH = -\log[H^+]$

a) Find the pH if $[H^+] = 3.98 \times 10^{-9}$

b) Find $[H^+]$ if $pH = 2.0$

$$\begin{aligned} 2.0 &= -\log[H^+] \\ -2.0 &= \log[H^+] \\ 10^{-2.0} &= [H^+] \rightarrow .01 = [H^+] \end{aligned}$$

$$pH = -\log[3.98 \times 10^{-9}]$$

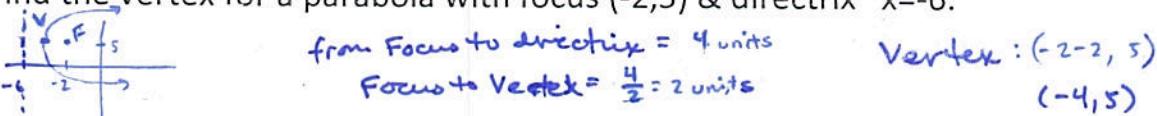
$$pH = 8.400$$

Non-Calculator

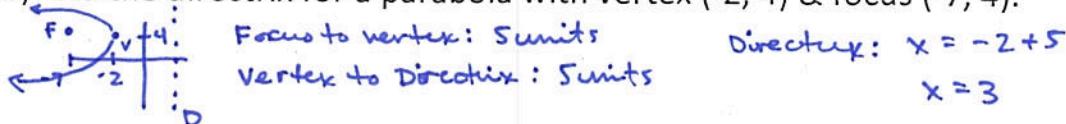
44) Find the focus for a parabola with vertex $(5, -2)$ & directrix $y = 3$.



45) Find the vertex for a parabola with focus $(-2, 5)$ & directrix $x = -6$.



46) Find the directrix for a parabola with vertex $(-2, 4)$ & focus $(-7, 4)$.



47) Write the equation for a parabola with vertex (3, 2) & directrix $x=-2$.

$$\begin{array}{l} P=5 \\ \text{Diagram: A coordinate plane with vertex } V(3, 2) \text{ and focus } F(-2, 2). \end{array}$$

$$(y-k)^2 = 4p(x-h)$$

$$(y-2)^2 = 4(5)(x-3)$$

$$(y-2)^2 = 20(x-3)$$

48) Find the vertices of an ellipse with foci (1, 4), (1, -4) & major axis of 13.

$$\begin{array}{l} \text{center: } (1, 0) \\ \text{vertices: } (1, 6.5) \\ \quad (1, -6.5) \end{array}$$

$$\begin{array}{l} c = \text{distance from center to focus} \\ 2a = 13 \\ a = 6.5 \end{array}$$

$$\begin{array}{l} a^2 = b^2 + c^2 \\ (6.5)^2 = b^2 + 1^2 \\ b^2 = 42 \\ b = \sqrt{42} \end{array}$$

49) Write the equation for an ellipse with vertices (13, 3), (-13, 3) & foci (12, 3), (-12, 3).

$$\begin{array}{l} \text{center: } (0, 3) \\ a = 13 \\ a^2 = b^2 + c^2 \\ c = 12 \\ 13^2 = b^2 + 12^2 \\ 5^2 = b^2 \end{array}$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{x^2}{169} + \frac{y^2}{25} = 1$$

50) Draw the graph and write the equation of an ellipse with a major axis of 12, minor axis of 10 & center at (2, -1).

$$\begin{array}{l} 2b=10 \\ b=5 \\ 2a=12 \\ a=6 \end{array}$$

$$\frac{(x-2)^2}{36} + \frac{(y+1)^2}{25} = 1$$

$$\begin{array}{l} \text{center: } (2, -1) \\ \text{vertices: } (0, 2) (0, -2) \\ \text{foci: } (0, \sqrt{11}) (0, -\sqrt{11}) \end{array}$$

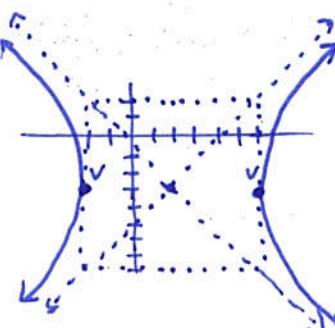
52) Find the eccentricity of #51.

$$e = \frac{c}{a} = \frac{\sqrt{2}}{2}$$

53) Draw & Label the graph of $\frac{25(x-2)^2}{400} - \frac{16(y+3)^2}{400} = 1$.

$$\frac{(x-2)^2}{16} - \frac{(y+3)^2}{25} = 1$$

$$\text{vertices: } (-2, -3) (6, -3)$$



54) Find the eccentricity of #53.

$$\begin{array}{l} c^2 = a^2 + b^2 \\ c^2 = 16 + 25 \\ c^2 = \sqrt{41} \end{array}$$

$$e = \frac{c}{a} = \frac{\sqrt{41}}{4}$$

55) Find the vertices & foci of $\frac{9y^2}{36} - \frac{6x^2}{36} = 1$.

$$\frac{y^2}{4} - \frac{x^2}{6} = 1$$

$$\begin{array}{l} a^2 = 4 \\ a = 2 \end{array}$$

$$\begin{array}{l} c^2 = 6 + 4 \\ c = \sqrt{10} \end{array}$$

$$\begin{array}{l} \text{vertices: } (0, 2) (0, -2) \\ \text{foci: } (0, \sqrt{10}) (0, -\sqrt{10}) \end{array}$$

56) Write the equation for a hyperbola with foci (10, 3), (-10, 3) & vertices (6, 3), (-6, 3).

$$\begin{array}{l} \text{center } (0, 3) \\ \frac{(x-0)^2}{6^2} - \frac{(y-3)^2}{8^2} = 1 \\ \frac{x^2}{36} - \frac{(y-3)^2}{64} = 1 \end{array}$$

$$\begin{array}{l} c = 10 \\ 10^2 = a^2 + b^2 \\ 100 = 36 + b^2 \\ 64 = b^2 \\ 8 = b \end{array}$$

57) Find the equation of the asymptotes of #56.

$$\text{asymptotes: } y - k = \pm \frac{b}{a}(x - h)$$

$$y - 3 = \pm \frac{8}{6}(x - 0)$$

$$y = \pm \frac{4}{3}x + 3$$