Pre-Calculus Semester 2 Final Exam Review Name: _____

Key Concept 5: Trigonometric Functions

Non-calculator

1) Express 250° in radians.

2) Express $\frac{7\pi}{3}$ in degrees.

3) What are the coordinates of the point on the terminal side of $\frac{5\pi}{6}$?

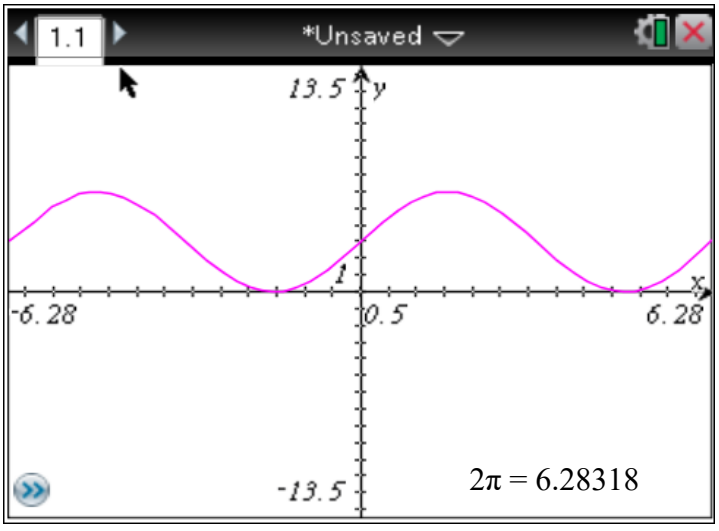
4) Evaluate the trigonometric expression using its period as an aid: $\cos 5\pi$

5) Given $\cot\theta = \frac{5}{12}$ and $\cos\theta > 0$, find $\sin\theta$.

6) Find the amplitude and period of $y = 3.25\cos 3x$.

7) Evaluate: $\cos\left(\arcsin\left(\frac{1}{2}\right)\right)$

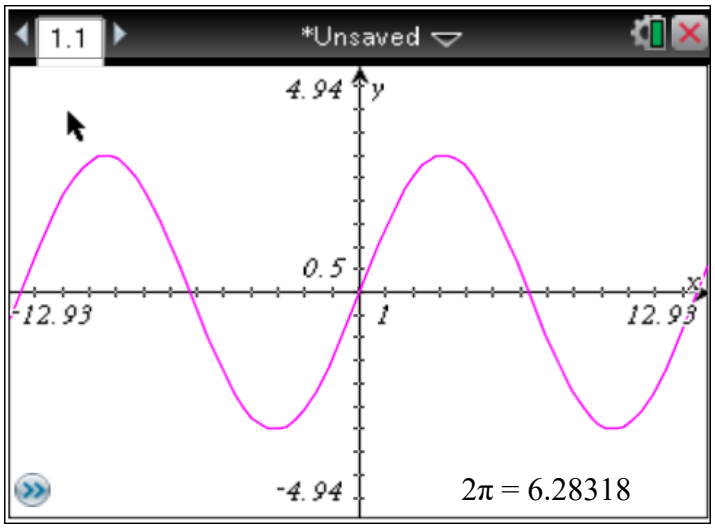
8) If $f(x) = d + a\cos(bx - c)$, find $a, b, c,$ and d such that the function matches the graph below.



9) Find the amplitude and period of the function graphed below.

Amplitude: _____

Period: _____



10) Write an equation of the function using the graph above.

11) Determine the quadrant in which the following angle lands: $\frac{13\pi}{3}$

12) Express 445° in radians.

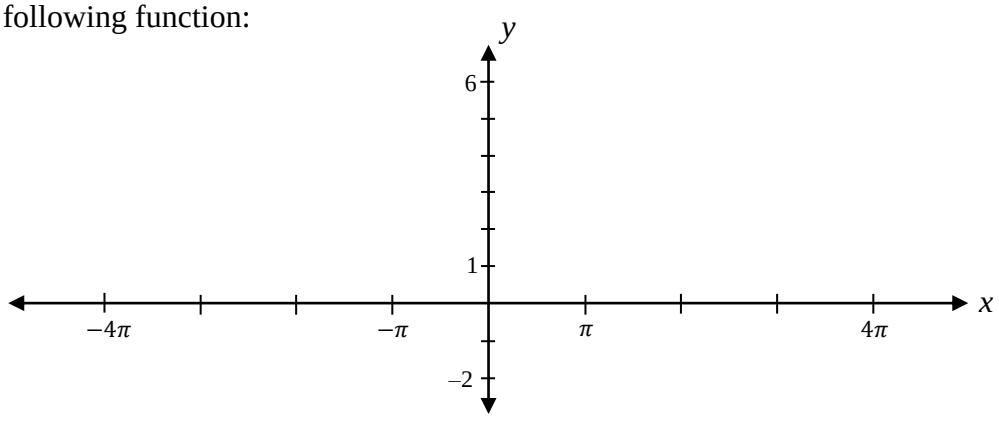
13) Find the point (a, b) on the unit circle that corresponds to the real number $t = \frac{5\pi}{3}$. Then, find $\sin t$, $\cos t$, and $\tan t$.

14) Find $\cot x$ if $\cos x = \frac{7}{9}$ and $\sin x < 0$.

15) Find an algebraic expression that is equivalent to the following expression: $\cot\left(\arcsin\left(\frac{5x}{7}\right)\right)$

16) Sketch TWO full periods of the following function:

$f(x) = -2\sin(x) + 3$



17) Given the equation $y = 2\sin(3x - \pi) - 5$, find the following:

Amplitude: _____ Period: _____

Horizontal Shift: _____ Vertical Shift: _____

Domain: _____ Range: _____

Calculator

18) Evaluate the trigonometric expression: $\csc 17.2°$

19) Find the amplitude and period of $f(x) = 2\sin\frac{\pi x}{3}$.

Amplitude: _____

Period: _____

20) Using the equation below, approximate TWO values of θ such that $0° \leq \theta < 360°$. Round to three decimal places.

$\cot\theta = -0.5$

21) Evaluate the trigonometric expression with a calculator: $\sin\left(\frac{2\pi}{3}\right)$

Key Concept 6: Analytic Trigonometry

Non-calculator

Simplify each trigonometric expression:

1) $\sin\alpha\tan\alpha\sec\alpha\csc\alpha$

2) $\frac{(\cot\theta)^2}{1-(\sin\theta)^2}$

3) $\frac{\sin 2\beta}{\cos 2\beta - \cos^2\beta}$

4) $\frac{2}{1-\csc\gamma} - \frac{2}{1+\csc\gamma}$

5) $1 - 4\sin^2\theta\cos^2\theta$

6) $2\sin\alpha\cos^3\alpha + 2\sin^3\alpha\cos\alpha$

For problems 7−10, prove the identity.

7) $\tan^2 x - \sin^2 x = \sin^2 x\tan^2 x$

8) $\frac{\cos\sigma}{1-\tan\sigma} + \frac{\sin\sigma}{1-\cot\sigma} = \cos\sigma + \sin\sigma$

9) $\sec x - \sin x \tan x = \cos x$

10) $\cos^2 2\theta - \cos^2\theta = \sin^2\theta - \sin^2 2\theta$

Solve each equation on the interval $[0, 2\pi)$:

11) $\cos 2x = \cos x$

12) $\sqrt{2}\sec x \sin x = \sec x$

13) $3\tan^2\theta = 1$

Find the exact value of x:

14) $\sin\frac{5\pi}{12} = x$

15) $\cos\frac{11\pi}{12} = x$

Calculator

Solve each equation on the interval $[0, 2\pi)$. Round to the nearest thousandth (three decimal places).

16) $\sin^2 x + 0.5 = 3\cos x$

17) $x^2 = 10 - \sin^4 x$

Prove each identity algebraically and graphically (if possible).

18) $\sin 4\theta = 2\sin 2\theta \cos 2\theta$

19) $\csc x + \cot x = \frac{\sin x}{1+\cos x}$

For problems 20−23, in ΔABC, round to the nearest thousandth.

20) Find $b, c,$ and $m\angle C$ given $\angle A = 79°$, $\angle B = 33°$, and $a = 7$.

21) Find c, $m\angle A$, and $m\angle C$ given $a = 5$, $b = 8$, and $\angle B = 30°$.

22) Find $m\angle A$ given $a = 5$, $b = 7$, and $c = 6$.

23) Solve ΔABC given $a = 6$, $b = 7$, and $\angle A = 30°$.

Key Concept 7: Discrete Mathematics

Non-Calculator

1) Find the sum of the coefficients of $(4x - 5y)^3$.

2) Find the sum of the first 328 even natural numbers.

3) Find the 10th term of the geometric sequence if $a_3 = \frac{1}{3}$ and $a_7 = 27$.

4) Find the sum of the infinite geometric series: $10 + 4 + \frac{8}{5} + \frac{16}{25} + \cdots$

5) Find the n^{th} term of the geometric sequence if $a_4 = 1$ and $a_8 = 81$.

6) Find the summation:
$$\sum_{n=1}^{6} -3\left(\frac{1}{2}\right)^{n-1}$$

7) Find a_n for the arithmetic sequence with $a_2 = -5,\ d = 4,\ \&\ n = 47$.

8) Find the fifth term in the expansion of $(5 - x)^7$.

9) If $f(x) = \frac{(x+2)!}{(x)!}$, find $f(4)$ by two different methods.

10) Find the summation:

$$\sum_{n=1}^{9,999} \log \frac{n}{n+1}$$

Calculator

11) Find the partial sum:

$$\sum_{x=1}^{79} \log_{\pi} x$$

12) Find the 12^{th} term in the expansion of $(1.5x - 2.1y)^{14}$.

13) Given $a_4 = -23$ and $a_8 = 95$, find the arithmetic sequence formula for a_n. Then, find a_1.

14) Find the summation by two different methods:

$$\sum_{x=24}^{95} 1.6\left(\frac{2}{3}\right)^x$$

15) Given $a_3 = \frac{25}{7}$ and $a_7 = \frac{15,625}{16,807}$, find the geometric sequence formula for a_n. Then, find a_1.

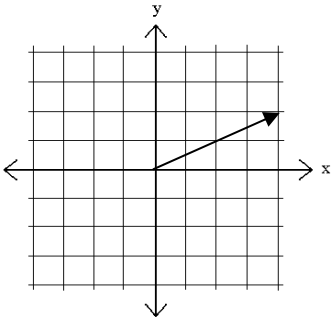
Key Concept 8: Matrix Algebra and Vectors

Non-Calculator

1) Let $\mathbf{a} = \langle -4, \frac{1}{2} \rangle$ and $\mathbf{b} = \langle \frac{2}{3}, -1 \rangle$. Find $4\mathbf{a} - 3\mathbf{b}$. Write your answer in linear combination form.

2) Given $Q = (7, 2)$ and $P = (-1, -2)$, find the magnitude of vector $\overrightarrow{PQ}$.

3) Find the component form of the vector in the diagram on the right.



4) Find the unit vector in the direction of the following vector: $\mathbf{w} = \langle -15, 8 \rangle$

5) Find the unit vector in the direction of the following vector: $\mathbf{w} = \langle 9, 3 \rangle$

6) Give two orthogonal vectors, one in the second quadrant and the other in the third quadrant. Then, verify that these two vectors are in fact orthogonal.

7) Find the dot product of **u** and **v**, where $\mathbf{u} = \langle \frac{2}{3}, -4 \rangle$ and $\mathbf{v} = \langle -2, \frac{2}{5} \rangle$.

8) Find the dot product of **u** and **v**, where $\mathbf{u} = -5\langle 1,0 \rangle + 2\langle 0,1 \rangle$ and $\mathbf{v} = 7\langle 1,0 \rangle - 9\langle 0,1 \rangle$.

9) Find the work done by a crane lifting a 585 pound girder 72 feet.

Calculator

10) Given $O = (11, -12)$ and $P = (-5,4)$, find the component form of vector $\overrightarrow{PO}$.

11) Let $\mathbf{v} = \langle -1,1 \rangle$ and $\frac{1}{2}\mathbf{u} - 6\mathbf{v} = \langle 7, 4 \rangle$. Find **u**.

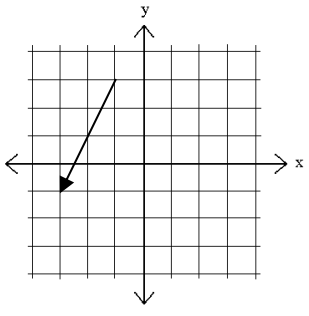
12) Find the direction angle of vector $\mathbf{u} = \langle 7, -2 \rangle$.

13) Find the unit vector in the direction of the following vector: $\mathbf{v} = \langle -9, -11 \rangle$

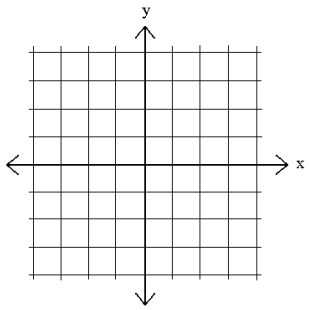
14) Find the angle between the following two vectors: $\mathbf{u} = \langle 6, -1 \rangle$ and $\mathbf{v} = \langle 2,12 \rangle$

15) Find the angle between the following two vectors: $\mathbf{u} = \langle 2,2 \rangle$ and $\mathbf{v} = \langle -1, -4 \rangle$

16) Find the component form of the vector in the diagram.



17) Given $\mathbf{u} = \langle 8.2, 3.7 \rangle$, draw $\mathbf{u}$ with magnitude and direction.



18) Find the work done by a force, F, of 72 lbs acting in the direction of $\langle 2, 1 \rangle$ in moving an object 5 feet along the *x*-axis starting at $(0, 0)$.

19) A car is parked on the side of a hill inclined at $7°$. The weight of the car is 2345 lbs. What force, F, is required to keep the car in place?

20) Find the inverse of A, if the inverse of A exists.

$$A = \begin{bmatrix} 4 & -2 \\ 1 & 5 \end{bmatrix}$$

20) Solve the system of equations.

$$x + z + w = 2$$
$$x + y + z = 3$$
$$3x + 2y + 3z + w = 8$$

21) Solve the system of equations.

$$x + 2y + z = -1$$
$$x - 3y + 2z = 1$$
$$2x - 3y + z = 5$$

22) Find the partial fraction decomposition: $\dfrac{-x+10}{x^2+x-12}$

23) Find the partial fraction decomposition: $\dfrac{x^2-2x+1}{(x-2)^3}$

25) Represent the problem using an augmented matrix and solve the problem.

A florist makes cut flower arrangements for Mother's Day, involving roses, carnations, and lilies. The florist prices the arrangement at \$50, where roses cost \$3.50, carnations cost \$1.50, and lilies cost \$2. If the arrangement can have 24 flowers and there needs to be twice as many carnations as roses, how many of each type of flower is needed to make the arrangement?

Key Concept 9: Limits

Calculator: 3−7, 12 & 14

1) Figure 5.2 shows the graph of $g(x)$. Find: $\lim_{x\to 3} g(x)$

2) Figure 5.2 shows the graph of $g(x)$. Find: $g(1)$

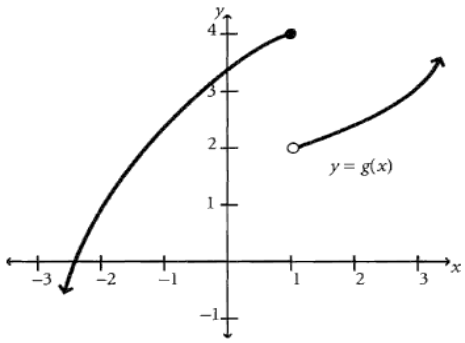


Figure 5.2

3) Graph $f(x) = \sin\frac{\pi}{x}$. Use the graph to help you find $\lim_{x\to 0} f(x)$.

4) Use the graph from #3 to help you find $\lim_{x\to\infty} \sin\frac{\pi}{x}$.

5) Find: $\lim_{x\to 0} \sin\frac{x}{x-1}$

6) Find: $\lim_{x\to\infty} \sin\frac{x}{x-1}$

7) Find: $\lim_{x\to 0} \sin(\sqrt{x}-2)$

8) Find: $\lim_{x\to 0} \frac{x^2+x-12}{x+4}$

9) Find: $\lim_{x\to -4} \frac{x^2+x-12}{x+4}$

10) Find $\lim_{x\to 5} f(x)$, where $f(x) = \begin{cases} x+2, x<5 \\ 2-2x, x \geq 5 \end{cases}$

11) If $\lim_{x\to 2} f(x) = -4$ and $\lim_{x\to 2} g(x) = 9$, find $\lim_{x\to 2} \frac{\sqrt{g(x)}}{(f(x))^2}$

12) Find: $\lim_{x\to -5^+} \frac{2|x+5|}{x+5}$

13) Find: $\lim_{x\to 0} \frac{\sin 3x}{x}$

14) Find: $\lim_{x\to 0} \frac{e^x - 3}{x}$

15) Find: $\lim_{x\to 4} \frac{\frac{1}{4} - \frac{1}{x}}{x-4}$

16) Find: $\lim_{x\to \infty} \frac{7x^2}{2x^2+7}$

17) Find: $\lim_{x\to \infty} \frac{x+3}{x^2-9}$