Honors Advanced Algebra DATE: _____

QUADRATIC WORD PROBLEMS: PROJECTILE MOTION

$0 = -16t^2 + 29t + 6$

$a = -16$, $b = 29$, $c = 6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-29 \pm \sqrt{29^2 - 4(-16)(6)}}{2(-16)}$

$= \frac{-29 \pm \sqrt{1225}}{-32}$

$\therefore t \approx 0.1875$ ~~$\approx$~~ $t = 2$

disard "−" time

$\therefore$ Baseball will hit the ground after 2 seconds.

RESOURCES: Two websites which include examples and explanations (there are many)

1) http://www.purplemath.com/modules/quadprob.htm
2) http://www.algebralab.org/Word/Word.aspx?file=Algebra_MaxMinProjectiles.xml

PROJECTILE: an object thrown, shot, or dropped (usually straight up or down)

The problems below are classical applications of quadratic functions. The **FUNCTION**, $s(t)$, representing the object's height at any time t (in seconds) depends on the following:

1. The force of gravity *(In the formula below, the coefficient of the leading term, -16, is a constant based on the gravitational force of the earth and represents ½g = ½(32 ft/sec²) ≈ 16 ft/sec². The value is negative since gravity pulls downward. Also the value of g (≈32 ft/sec² or 9.8 m/sec² valid at sea level), the acceleration due to gravity, is being measured in ft/sec², we must also measure h(t), v_0, and h_0 in terms of feet and seconds.)*
2. The initial velocity, v_0, at which it was thrown/dropped *(coefficient of the middle term)*
3. The initial height, h_0, from which it was thrown/dropped *(constant term)*

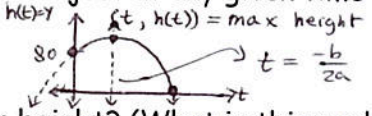
$$h(t) = -16t^2 + v_0 t + h_0$$

Note: Time is our variable of interest because it's usually the one we are trying to solve for. The formula disregards air resistance.

Examples

a) An object is launched directly upward at 64 feet per second (v_0) from a platform 80 feet high (h_0). Write the function for the height of this object at any given time t (in seconds).

$h(t) = -16t^2 + 64t + 80$



When will the object reach its maximum height? (What is this really asking?) time

At what time will the object reach maximum height?

Utilize $t = \frac{-b}{2a}$ (we derived this formula in class).

$a = -16$, $b = 64$, $c = 80$

$t = \frac{-64}{2(-16)} = \frac{-64}{-32} = 2$ $\therefore$ 2 seconds after launch, the object will reach its maximum height.

What will that maximum height be? (What is this really asking?)

This is asking to evaluate h(t) @ t=2.

Substitute 2 for every t in h(t).

$\therefore h(2) = -16(2)^2 + 64(2) + 80 = 144$ $\therefore$ 2 seconds after launch, the max height of the object is 144ft.

b) A baseball is thrown straight up in the air 6 feet off the ground (h_0) with an initial velocity of 29 feet per second (v_0). Write the function for the height of this object at any given time t in seconds.

$h(t) = -16t^2 + 29t + 6$

Again we can quickly analyze graph using Nspire to get our answer below.

When will this object return and hit the ground? (What is this really asking?) time

① $t = \frac{-b}{2a} = \frac{-29}{2(-16)} = \frac{-29}{-32} = \frac{29}{32} \approx 0.90625$ sec

$\therefore$ The object will come back down to earth after 0.90625 seconds.

Two questions here: ① when will object return? ② when does it hit the ground?

② Object hits ground @ h(t)=0. (see above)

$\therefore$ Solve $0 = -16t^2 + 29t + 6$ using appropriate

The ① asks to find time @ max height: $t = \frac{-b}{2a}$

P7-8

SHOW ALL WORK!

PROBLEMS: Use separate sheet of paper to work out the problems below.

1) Some fireworks are fired vertically into the air from the ground (h_0) at an initial velocity of 80 feet per second (v_0). Find the highest point reached by the firework – just as it explodes.

Function: $h(t) = -16t^2 + 80t + 0$ Follow example a) on front page

Ans: 100ft

2) A ball is thrown vertically upward with an initial velocity of 48 feet per second. If the ball started from a height of 8 feet off the ground, determine the time it will take for the ball to hit the ground. Follow example 2b on front page

Ans: ≈3.16 seconds

3) A pistol is accidentally discharged vertically upward from a height of 3 feet above the ground. If the bullet has an initial velocity of 200 feet per second, what maximum height will it reach before it starts to fall back down to the ground?

Follow example a) on front page

Ans: 628 ft

4) A tennis ball is propelled upward from the face of a racket at 40 feet per second. The racket face is 3 feet above ground when it makes contact with the ball. At what time will the ball be at its highest point? How high is that highest point?

Follow example a) on front page

Ans 1: ≈1.25 sec

Ans 2: 28ft

5) After the semester is over, you discover that the math department has changed textbooks (again) so the bookstore won't buy back your nearly-new book. You and your friend Herman decide to get creative. You go to the roof of a twelve-story building and look over the edge to the reflecting pool 160 feet below. You drop your book over the edge at the same instant that Herman chucks his book straight down at 48 feet per second. By how many seconds does his book beat yours into the water? You have to write TWO equations

Mine: $h(t) =$ _____

His: $h(t) =$ _____

Ans: ≈1.16 sec

6) The International Space Agency has finally landed a robotic explorer on an extra-solar planet. To demonstrate the crushing weight of gravity on this planet, the camera is aimed at a probe's ground-level ejection port which launches a baseball directly upwards at 147 feet per second, about the top speed of professional baseball pitcher. The force due to gravity on this planet is 98 ft/s^2. Assuming no winds and the probe can move out of the way in time, how long will it take the ball to smack back down to the surface of this planet?

Hint: $98 ft/s^2 \div 2 = 49$. So $h(t) = -49t^2 + 147t + 0$

Follow example b)

Ans: 3 seconds

What does t=0 mean in context?

Why don't we want t=0?