3.1 Quadratic Equations and Models

Quadratic Equations
Graphing Techniques
Completing the Square
The Vertex Formula
Quadratic Models

PEARSON
Education

Polynomial Function

A polynomial function of degree *n*, where *n* is a nonnegative integer, is a function defined by an expression of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where a_n, a_{n-1}, …, a_1, and a_0 are real numbers, with $a_n \neq 0$.

Polynomial Function

For the polynomial function defined by

$$f(x) = 2x^3 - \frac{1}{2}x + 5$$

Quadratic Function

A function f is a quadratic function if

$$f(x) = ax^2 + bx + c,$$

where a, b, and c are real numbers, with $a \neq 0$.

Simplest Quadratic

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

range
[0, ∞)

domain
(−∞, ∞)

$$f(x) = x^2$$

Simplest Quadratic

Parabolas are symmetric with respect to a line. The line of symmetry is called the **axis** of the parabola. The point where the axis intersects the parabola is the **vertex** of the parabola.

Opens up

Axis

Vertex

Vertex

Axis

Opens down

Example 1

GRAPHING QUADRATIC FUNCTIONS

Graph the function. Give the domain and range.

c. $F(x) = -\dfrac{1}{2}(x-4)^2 + 3$

Solution

Domain
$(-\infty, \infty)$

Range
$(-\infty, 3]$

$F(x) = -\dfrac{1}{2}(x-4)^2 + 3$

(4, 3)

$g(x) = -\dfrac{1}{2}x^2$

3

−2

−6

$x = 4$

Graph of a Quadratic Function

The quadratic function defined by $f(x) = ax^2 + bx + c$ can be written as

$$y = f(x) = a(x - h)^2 + k, \quad a \neq 0,$$

where

$$h = -\frac{b}{2a} \text{ and } k = f(h).$$

Graph of a Quadratic Function

The graph of f has the following characteristics.
1. It is a parabola with vertex (h, k) and the vertical line $x = h$ as axis.
2. It opens up if $a > 0$ and down is $a < 0$.
3. It is broader than the graph of $y = x^2$ if $|a| < 1$ and narrower if $|a| > 1$.
4. The y-intercept is $f(0) = c$.
5. If $b^2 - 4ac > 0$, the x-intercepts are $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

    If $b^2 - 4ac = 0$, the x-intercepts is $-\dfrac{b}{2a}$

    If $b^2 - 4ac < 0$, there are no x-intercepts.

Find the axis and vertex of the parabola having equation $f(x) = 2x^2 + 4x + 5$ using the vertex formula.

Solution Here $a = 2$, $b = 4$, and $c = 5$. The axis of the parabola is the vertical line

$$x = h = -\frac{b}{2a} = -\frac{4}{2(2)} = -1$$

The vertex is $(-1, f(-1))$.

Since $f(-1) = 2(-1)^2 + 4(-1) + 5 = 3$, the vertex is $(-1, 3)$.

Quadratic Models and Curve Fitting

Quadratic functions make good models for data sets where the data either increases, levels off, and then decreases, levels off, and then increases.

An application that models the path of the projectile is

$$s(t) = -16t^2 + v_0 t + s_0.$$

Example 5 **SOLVING A PROBLEM INVOLVING PROJECTILE MOTION**

A ball is thrown upward from an initial height of 100 ft with an initial velocity of 80 ft per sec.

a. Give the function that describes the height of the ball in terms of time t.

Solution Use the projectile height function with $v_0 = 80$ and $s_0 = 100$.

$$s(t) = -16t^2 + 80t + 100.$$

Example 5 **SOLVING A PROBLEM INVOLVING PROJECTILE MOTION**

A ball is thrown upward from an initial height of 100 ft with an initial velocity of 80 ft per sec.

b. Graph this function on a graphing calculator so that the *y*-intercept, the positive *x*-intercept, and the vertex are visible.

Solution One choice for a window is [-.3, 9.7] by [-60,300]

$$y = -16x^2 + 80x + 100$$

300

1

-.3   9.7

X=4.8   Y=115.36

-60

Example 5 **SOLVING A PROBLEM INVOLVING PROJECTILE MOTION**

A ball is thrown upward from an initial height of 100 ft with an initial velocity of 80 ft per sec.

c. The point (4.8, 115.36) lies on the graph of the function. What does this mean for this particular situation?

Solution When 4.8 seconds have elapsed, the projectile is at a height of 115.36 ft.

$$y = -16x^2 + 80x + 100$$

300

1

-.3

9.7

X=4.8   Y=115.36

−60

Example 5   **SOLVING A PROBLEM INVOLVING PROJECTILE MOTION**

A ball is thrown upward from an initial height of 100 ft with an initial velocity of 80 ft per sec.

d. After how many seconds does the projectile reach its maximum height? What is this maximum height?

Solution Find the coordinates of the vertex of the parabola. $a = -16$ and $b = 80$

$$x = -\frac{b}{2a} = -\frac{80}{2(-16)} = 2.5$$

Example 5 **SOLVING A PROBLEM INVOLVING PROJECTILE MOTION**

A ball is thrown upward from an initial height of 100 ft with an initial velocity of 80 ft per sec.

d. After how many seconds does the projectile reach its maximum height? What is this maximum height?

Solution Find the coordinates of the vertex of the parabola. $a = -16$ and $b = 80$

and $y = -16(2.5)^2 + 80(2.5) + 100$

$= 200$

After 2.5 sec the ball reaches its maximum height of 200 ft.

Example 5 SOLVING A PROBLEM INVOLVING PROJECTILE MOTION

A ball is thrown upward from an initial height of 100 ft with an initial velocity of 80 ft per sec.

f. After how many seconds will the ball hit the ground?

Solution The height is zero when the ball hits the ground. Find the positive solution…

$$-16x^2 + 80x + 100 = 0$$

$$x = \frac{-80 \pm \sqrt{80^2 - 4(-16)(100)}}{2(-16)}$$

Example 5 **SOLVING A PROBLEM INVOLVING PROJECTILE MOTION**

A ball is thrown upward from an initial height of 100 ft with an initial velocity of 80 ft per sec.

f. After how many seconds will the ball hit the ground?

Solution

$$x \approx \cancel{-1.04} \;\; \text{or} \;\; x \approx 6.04$$

The ball hits the ground after about 6.04 sec.