

Operations on Functions

Target 2C. Add, subtract, and multiply polynomials and explain why solutions are equivalent

A polynomial equation used to represent a function is called a polynomial function.

Examples:

$$\begin{aligned}f(x) &= 4x^2 - 5x + 2 \\p(x) &= 2x^3 + 4x^2 - 5x + 7 \\g(x) &= 3x - 4\end{aligned}$$

Evaluating PolynomialsFind each value if $f(x) = 4x - 7$ and $g(x) = 2x^2 - 3x + 1$ 1. $f(-3)$ "f of -3"

$$f(-3) = 4(-3) - 7 = -12 - 7 = \boxed{-19}$$

"Replace every x in
the function with value
of expression"

2. $g(2a)$ "g of 2a"

$$\begin{aligned}g(\quad) &= 2(\underline{\underline{2a}})^2 - 3(\underline{\underline{2a}}) + 1 = 2 \cdot 2^2 a^2 - 6a + 1 \\&= 2 \cdot 4a^2 - 6a + 1 \\&= \boxed{8a^2 - 6a + 1}\end{aligned}$$

3. $f(g(1))$ "f of g of 1"

Find $g(1)$ first! $g(1) = 2(1)^2 - 3(1) + 1$
 $= 2 - 3 + 1$
 $= 0$
 $\therefore g(1) = 0$

Last find $f(0)$.
 $f(0) = 4(0) - 7$
 $= 0 - 7$
 $= \boxed{-7}$

4. $g(f(1))$ "g of f of 1"

Find $f(1)$ first. $f(1) = 4(1) - 7$
 $= 4 - 7$
 $= -3$

Last find $g(-3)$.
 $g(-3) = 2(-3)^2 - 3(-3) + 1$
 $= 2 \cdot 9 + 9 + 1$
 $= 18 + 9 + 1$
 $= \boxed{28}$

5. $f(4b^2) + g(b)$ "f of $4b^2$ plus g of b"

$$\begin{aligned}f(4b^2) + g(b) &= 4(4b^2) - 7 + 2(b)^2 - 3(b) + 1 \\&= \underline{\underline{16b^2}} - \underline{\underline{7}} + \underline{\underline{2b^2}} - \underline{\underline{3b}} + \underline{\underline{1}} \\&= \boxed{18b^2 - 3b - 6}\end{aligned}$$

Operations with Functions

Operation	Definition	Examples if $f(x) = x + 2$, $g(x) = 3x$
Sum	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = f(x) + g(x)$ $= x + 2 + 3x$ $= 4x + 2 \quad \checkmark$
Difference	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = f(x) - g(x)$ $= x + 2 - 3x$ $= -2x + 2 \quad \checkmark$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$	$(f \cdot g)(x) = f(x) \cdot g(x)$ $= (x + 2)3x$ $= 3x^2 + 6x \quad \checkmark$

Given $h(x) = x^2 - 3x + 1$ and $k(x) = 4x + 5$, find each function.

6. $(h + k)(x)$ "h plus k of x"

$$\begin{aligned} &= h(x) + k(x) \\ &= x^2 - 3x + 1 + 4x + 5 \\ &= [x^2 + x + 6] \end{aligned}$$

7. $(k - h)(x)$ "k minus h of x"

$$\begin{aligned} &= k(x) - h(x) \\ &= 4x + 5 - (x^2 - 3x + 1) \quad \text{Distribute } “-“ \\ &= 4x + 5 - x^2 + 3x - 1 \\ &= [-x^2 + 7x + 4] \end{aligned}$$

8. $(h \cdot k)(x)$ "h times k of x"

$$\begin{aligned} &= h(x) \cdot k(x) \\ &= [4x^3 - 7x^2 - 11x + 5] \end{aligned}$$

$$\begin{array}{c|ccc} & x^2 & -3x & +1 \\ \hline 4x & | 4x^3 & -12x^2 & 4x \\ +5 & | 5x^2 & -15x & 5 \\ \hline & \text{Like} & \text{Like} & \end{array}$$

Given $f(x) = 3x^2 + 7x$ and $g(x) = 2x^2 - x - 1$, find each function.

9. $(g + f)(x)$

$$\begin{aligned} &= g(x) + f(x) \\ &= 2x^2 - x - 1 + 3x^2 + 7x \\ &= [5x^2 + 6x - 1] \end{aligned}$$

10. $(g - f)(x)$

$$\begin{aligned} &= g(x) - f(x) \quad \text{Distribute } “-“ \\ &= 2x^2 - x - 1 - (3x^2 + 7x) \\ &= 2x^2 - x - 1 - 3x^2 - 7x \\ &= [-x^2 - 8x - 1] \end{aligned}$$

11. $(f \cdot g)(x)$

$$f(x) \cdot g(x)$$

$$= [6x^4 + 11x^3 - 10x^2 - 7x] / 3x^2$$

$$\begin{array}{c|ccc} & 2x^2 & -x & -1 \\ \hline 7x & | 14x^3 & -7x^2 & -7x \\ & | 6x^4 & -3x^3 & -3x^2 \\ \hline & \text{Like} & \text{Like} & \end{array}$$

