Honors Advanced Algebra DATE: 9/26

Polynomial Functions

Target 2B. Graph, transform and identify the key features of a polynomial function and make connections between algebraic and graphical forms

Basic Shapes of the Graph of Polynomial Functions

Standard Form of a Polynomial Function arranges the terms by degree in descending numerical order:

$$P(x) = \boxed{a_n} x^{n} + a_{n-1}x^{n-1} + \cdots + a_1x^1 + \underbrace{a_0}_{\text{constant}}$$

$\boxed{a_n}$: Leading Coefficient

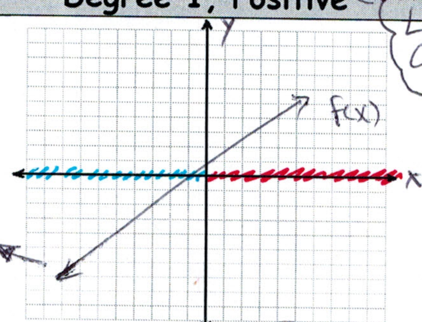
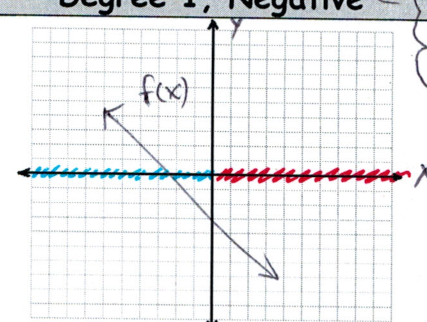
n : degree

a_0 : constant term

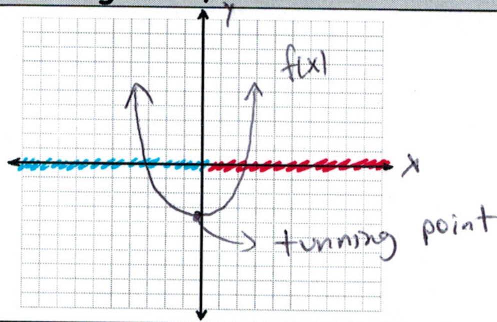
Ex: $f(x) = \boxed{3}x^{6} + x^5 - 3x^4 + 10x^3 - x \underbrace{+1}$

Name	Form	Degree
Constant	$f(x) = a$	0
Linear	$f(x) = ax + b$ $(a \neq 0)$	1
Quadratic	$f(x) = ax^2 + bx + c$ $(a \neq 0)$	2
Cubic	$f(x) = ax^3 + bx^2 + cx + d$ $(a \neq 0)$	3
Quartic	$f(x) = ax^4 + bx^3 + cx^2 + dx + e$ $(a \neq 0)$	4

- Degree is determined by term with largest exponent.
- Leading coefficient is the number infront of term with largest degree.

Degree 1, Positive ← refers to Leading Coefficient	Degree 1, Negative ← refers to Leading Coefficient
end behavior 	
End Behavior: $\lim_{x \to -\infty} f(x) = -\infty$ → Think: "where is the graph going? Up or down?" $\lim_{x \to \infty} f(x) = \infty$	End Behavior: $\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = -\infty$
Maximum number of turning points: 0	Maximum number of turning points: 0
Maximum number of real zeros: 1	Maximum number of real zeros: 1

Degree 2, Positive



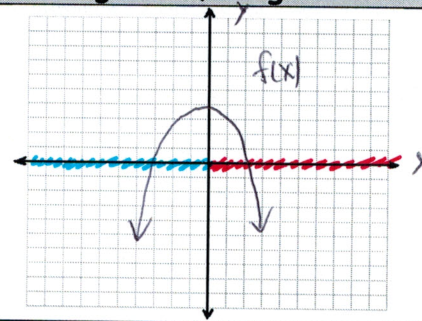
End Behavior: $\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = \infty$

Maximum number of turning points: 1

Maximum number of real zeros: 2

Degree 2, Negative



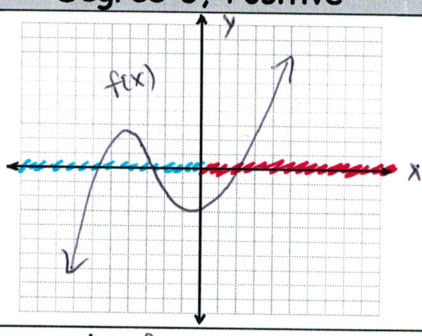
End Behavior: $\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = -\infty$

Maximum number of turning points: 1

Maximum number of real zeros: 2

Degree 3, Positive



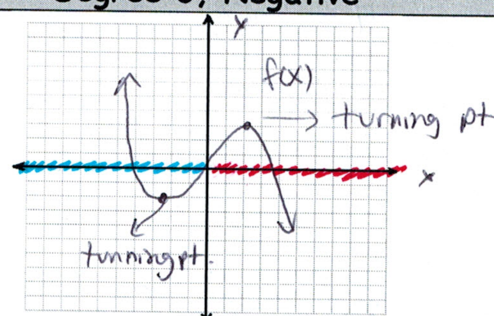
End Behavior: $\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = \infty$

Maximum number of turning points: 2

Maximum number of real zeros: 3

Degree 3, Negative



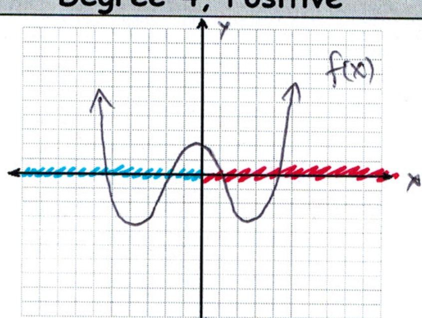
End Behavior: $\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = -\infty$

Maximum number of turning points: 3

Maximum number of real zeros: 4

Degree 4, Positive



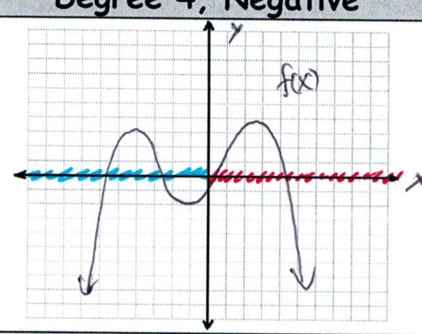
End Behavior: $\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = \infty$

Maximum number of turning points: 3

Maximum number of real zeros: 4

Degree 4, Negative

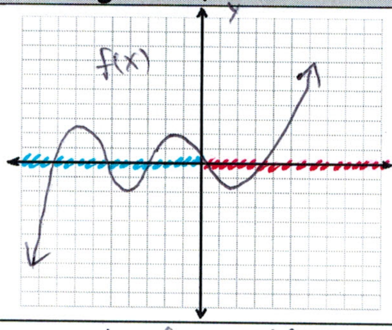
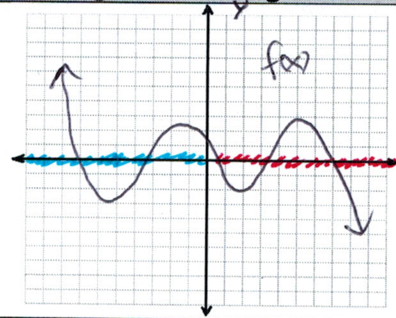


End Behavior: $\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to \infty} f(x) = -\infty$

Maximum number of turning points: 3

Maximum number of real zeros: 4

Degree 5, Positive	Degree 5, Negative
	
End Behavior: $\lim_{x\to-\infty} f(x) = -\infty$ $\lim_{x\to\infty} f(x) = \infty$	End Behavior: $\lim_{x\to-\infty} f(x) = \infty$ $\lim_{x\to\infty} f(x) = -\infty$
Maximum number of turning points: 4	Maximum number of turning points: 4
Maximum number of real zeros: 5	Maximum number of real zeros: 5

Answer the following questions based on the information above:

1) What do you notice when looking at end behavior of even or odd degree polynomials?

- Even degree polynomials have the same end behavior.
- Odd degree polynomials have the same end behavior.

2) What do you notice when looking at the degree of the polynomial and its maximum number of turning points?

Max # of turning points $= n-1$ (degree minus one)

3) What do you notice when looking at the degree of the polynomial and its maximum number of real zeros?

Max # of real zeros $= n$ (degree)

4) Can you predict the end behavior of a polynomial of degree 11 (positive)? How about the end behavior of a polynomial of degree 14 (negative)? What is the maximum number of turning points of each polynomial? How about the maximum number of real zeros?

(odd)
- Degree 11 (positive) has same behavior as all odd deg. poly. (positive)

(even)
- Degree 14 (negative) has same behavior as all even deg. poly. (negative)

•• Degree 11 has 10 max # of turning pts.
•• Degree 14 has 13 max # of turning pts.
* Degree 11 has 11 max # of real zeros.

P2-3

Do each of the following for every given function:

a. Sketch the general shape of each function.
b. Describe the end behavior of each function.
c. State the maximum number of turning points the graph can make.
d. State the maximum number of real zeros.
e. Then use the Nspire to check your work.

1. $f(x) = 2x + 7$

a. Graph:

b. End behavior:

c. Max. number of turning pts: ________

d. Max. real zeros: ________

2. $f(x) = 2x^2 + 7x - 1$

a. Graph:

b. End behavior:

c. Max. number of turning pts: ________

d. Max. real zeros: ________

3. $f(x) = 2x^3 + 7x^2 - x - 4$

a. Graph:

b. End behavior:

c. Max. number of turning pts: ________

d. Max. real zeros: ________

4. $f(x) = 2x^4 + 7x^3 - x^2 - 4x + 3$

a. Graph:

b. End behavior:

c. Max. number of turning pts: ________

d. Max. real zeros: ________

5. $f(x) = 2x^5 + 7x^4 - x^3 - 4x^2 + 3x + 6$

a. Graph:

b. End behavior:

c. Max. number of turning pts: ________

d. Max. real zeros: ________

6. $f(x) = -2x + 7$

a. Graph:

b. End behavior:

c. Max. number of turning pts: ________

d. Max. real zeros: ________

7. $f(x) = -2x^2 + 7x - 1$

a. Graph:

b. End behavior:

c. Max. number of turning pts: ________

d. Max. real zeros: ________

8. $f(x) = -2x^3 + 7x^2 - x - 4$

a. Graph:

b. End behavior:

c. Max. number of turning pts: ________

d. Max. real zeros: ________

9. $f(x) = -2x^4 + 7x^3 - x^2 - 4x + 3$

a. Graph:

b. End behavior:

c. Max. number of turning pts: ________

d. Max. real zeros: ________

10. $f(x) = -2x^5 + 7x^4 - x^3 - 4x^2 + 3x + 6$

a. Graph:

b. End behavior:

c. Max. number of turning pts: ________

d. Max. real zeros: ________

What determines the end behavior of a polynomial function? The leading coefficient, depending on whether it's positive or negative