DATE: _____

2.4 Polynomial Functions (Target 2B)

Review of Prior Concepts

Find the degree and leading coefficient of: $f(x) = 5x^2 - 4x^3 + 2 - 7x$.

degree: 3
leading coefficient: −4 (l.c.)
↑ highest exponent

End Behavior of polynomials:

→ What happens to the graph as $x \to -\infty$ and $x \to \infty$

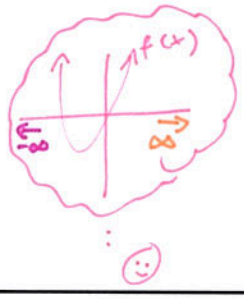
Notation	Meaning of the Notation
$\lim_{x \to -\infty} f(x)$	as x goes to $-\infty$, f(x) approaches some y-value
$\lim_{x \to \infty} f(x)$	as x goes to ∞, f(x) approaches some y-value

Using a graphing calculator, describe the end behavior of the function.

1. $f(x) = x^2 + 3x - 1$

$\lim_{x \to -\infty} f(x) = \infty$

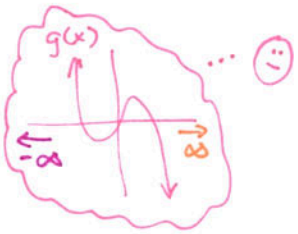
$\lim_{x \to \infty} f(x) = \infty$



2. $g(x) = -x^3 + 2x$

$\lim_{x \to -\infty} f(x) = \infty$

$\lim_{x \to \infty} f(x) = -\infty$

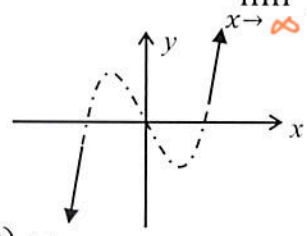
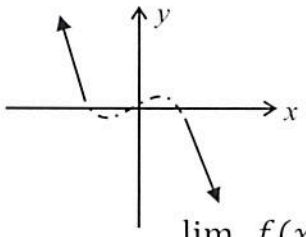
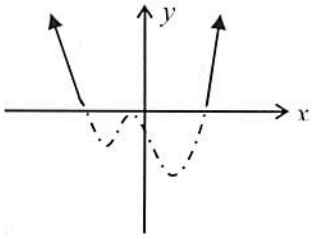
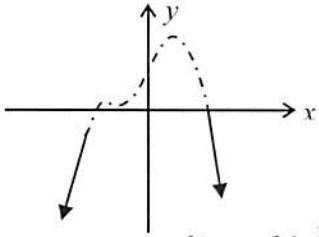


Leading Term Test for Polynomial End Behavior (p.197)

For any polynomial function $f(x) = a_n x^n + \cdots + a_1 x + a_0$, the $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$ are determined by the degree n of the polynomial and its leading coefficient a_n.

highest exponent → degree

→ coefficients paired w/ highest exponent

	leading coefficient positive $a_n > 0$	leading coefficient negative $a_n < 0$
degree → n is odd	$\lim_{x \to \infty} f(x) = \infty$  $\lim_{x \to -\infty} f(x) = -\infty$	$\lim_{x \to -\infty} f(x) = \infty$  $\lim_{x \to \infty} f(x) = -\infty$
degree → n is even	$\lim_{x \to -\infty} f(x) = \infty$ $\lim_{x \to \infty} f(x) = \infty$ 	 $\lim_{x \to -\infty} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = -\infty$

Conclusions about Leading Term Test

equal

$\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x)$

1. When n (degree) is even, the end behaviors are the same ☺
2. When n is odd, the end behaviors are the opposite
3. Whenever the leading coefficient is positive, $\lim_{x\to\infty} f(x) =$ ∞
   - In other words, the graph ends by approaching the positive direction
4. Whenever the leading coefficient is negative, $\lim_{x\to\infty} f(x) =$ $-\infty$
   - In other words, the graph ends by approaching the negative direction

Examples

Describe the end behavior of each function WITHOUT using a graphing calculator

1. $f(x) = x^4 - 2x$

L.C. positive ↗ degree even ↖

$\lim_{x\to-\infty} f(x) = \infty$

$\lim_{x\to\infty} f(x) = \infty$

2. $g(x) = -4x^5$

L.C. negative ↗ degree odd ↖

$\lim_{x\to-\infty} g(x) = \infty$

$\lim_{x\to\infty} g(x) = -\infty$

3. $h(x) = 7 - 3x^6$

L.C. negative ↗ degree even ↖

$\lim_{x\to-\infty} h(x) = -\infty$

$\lim_{x\to\infty} h(x) = -\infty$

4. $k(x) = -\frac{1}{2}x^2 + 5x^7$

L.C. positive ↘ degree odd ↖

$\lim_{x\to-\infty} k(x) = -\infty$

$\lim_{x\to\infty} k(x) = \infty$