1.6. Advanced Algebra

DATE: 9/30

Solving Absolute Value Inequalities

Target 2A. Utilize a graph to illustrate the solution set of an absolute value inequality.

Absolute Value Inequalities

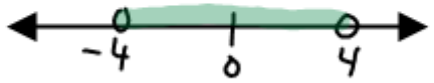
For all real numbers a and b, $b > 0$, the following statements are true:

1. If $|a| \leq b$, then $-b \leq a \leq b$ "AND"
2. If $|a| \geq b$, then $a \geq b$ "OR" $a \leq -b$

Also true for: $\leq$, $\geq$

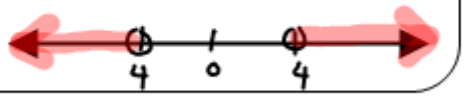
Examples based on the above statements:

1. If $|a| \leq 4$, then $-4 \leq a \leq 4$ "AND"



The distance from zero is less than 4.

2. If $|a| \geq 4$, then $a \geq 4$ "or" $a \leq -4$



The distance from zero is greater than 4.

Recall:

The absolute value of a # is the distance from zero on the # line.

Solve a Multi-Step Absolute Value Inequality

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Solve a Multi-Step Absolute Value Inequality

1. Solve $|2x - 1| < 5$ Graph the solution on a number line. Then give the solution set in both set-builder and interval notation.

Like #1 above. So $|2x-1|<5 \Rightarrow -5 < 2x-1 < 5$

Solve $-5 < 2x-1 < 5$

$+1 \quad +1 \quad +1$

$-4 < 2x < 6$

$\frac{-4}{2} < \frac{2x}{2} < \frac{6}{2}$

$-2 < x < 3$

Set builder: $\{x \mid -2 < x < 3\}$

Graph:

Interval: $(-2, 3)$

2. Solve $|3x - 12| \geq 6$. Graph the solution on a number line. Then give the solution set in both set-builder and interval notation.

Like #2 above. So $|3x-12| \geq 6 \Rightarrow 3x-12 \geq 6$ "OR" $3x-12 \leq -6$. Now Solve.

$3x-12 \geq 6$

$+12 \quad +12$

$\frac{3x}{3} \geq \frac{18}{3}$

$x \geq 6$

OR

$3x-12 \leq -6$

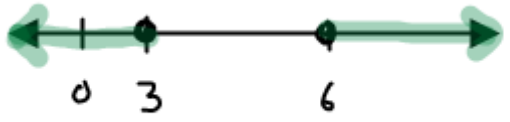
$+12 \quad +12$

$\frac{3x}{3} \leq \frac{6}{3}$

$x \leq 3$

Set builder: $\{x \mid x \geq 6 \text{ or } x \leq 3\}$

Graph:



Interval: $(-\infty, 3] \cup [6, +\infty)$

3. Solve $|6r - 3| < 21$. Graph the solution on a number line. Then give the solution set in both set-builder and interval notation.

$|6r-3| < 21 \Rightarrow -21 < 6r-3 < 21$

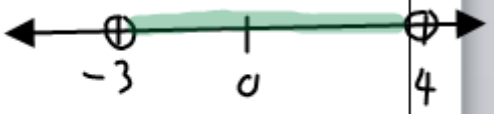
$+3 \quad +3 \quad +3$

$-18 < 6r < 24$

$\frac{-18}{6} < \frac{6r}{6} < \frac{24}{6}$

$-3 < r < 4$

Graph:



Interval notation: $(-3, 4)$

Set-builder: $\{r \mid -3 < r < 4\}$

4. Solve $|b - 4| > 6$. Graph the solution on a number line. Then give the solution set in both set-builder and interval notation.

$|b-4| > 6 \Rightarrow b - 4 > 6$ OR $b - 4 < -6$

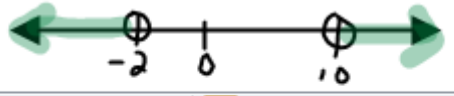
$+4 \quad +4 \qquad +4 \quad +4$

$b > 10$ OR $b < -2$

Interval: $(-\infty, -2) \cup (10, +\infty)$

Set-builder: $\{b \mid b > 10 \text{ OR } b < -2\}$

Graph:



5. Solve $|7x| + 4 < 0$. Graph the solution on a number line. Then give the solution set in both set-builder and interval notation.

$|7x| + 4 < 0$

$-4 \quad -4$

$|7x| < -4 \Rightarrow$ Looks like an "AND". But the distance on the # line from zero cannot be -4. So NO SOLUTION!